DESIGN OF MACHINE MEMBERS

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DESIGN OF MACHINE MEMBERS

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SECOND EDITION
SECOND IMPRESSION

McGRAW-HILL BOOK COMPANY, INC.
NEW YORK AND LONDON
1943

DESIGN OF MACHINE MEMBERS

PRINTED IN THE UNITED STATES OF AMERICA

THE MAPLE PRESS COMPANY, YORK, PA.

Preface to the Second Edition

In revising the text for the second edition, the authors have attempted to maintain most of the original text with such additions and revisions as are required by new developments in materials and design procedure. Additional information on nonferrous materials has been added because of their growing importance in present-day designs. Some information dealing with the recently developed nonmetallic materials such as plastics, synthetics, and so-called rubber substitutes has been included.

The order of the chapters has been changed to provide a better continuity of subject matter. Some chapters have been largely rewritten in an attempt to clarify some portions and to incorporate recent developments.

The authors are indebted to those manufacturing companies who have supplied material for this edition. Credit has been given throughout the text. The authors also wish to acknowledge the comments and helpful criticisms offered by those who used the former edition.

ALEX. VALLANCE,
VENTON LEVY DOUGHTIE.

HOUSTON, TEXAS,
AUSTIN, TEXAS,
August, 1943.

Preface to the First Edition

This text has been prepared for the use of students who have had some training in kinematics, mechanics, and factory processes. Using these subjects as a foundation, the author has attempted to explain the theory involved in the design of the elements of operating machines and to point out the variations from theory required by practical applications.

A number of selected derivations have been included to give the student practice in the analytical methods involved in design. Some design problems have been worked out in detail to illustrate practical application of theory to actual design. Numerous problems for class or home solution have been included at the end of the text. In many of these, some data have purposely been omitted, since the development of judgment in selecting materials and working stresses is more important than mere substitution of given data in a formula. Considerable space has been devoted to engineering materials, factors of safety, utilization factors, and the selection of design stresses, since good judgment in this phase of design is necessary to the designing engineer.

Although numerous tables have been included for illustrative purposes and as aids in the solution of problems, no attempt has been made to make this book a reference handbook for machine designers.

The student undertaking the study of machine design must realize that mathematical ability alone is not enough, and that this ability must be supported by experience, sound judgment, proper sense of proportions, and ability to analyze service requirements and to make logical compromises when necessary.

ALEX. VALLANCE.

HOUSTON, TEXAS,
April, 1938.

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DESIGN OF MACHINE MEMBERS

CHAPTER I

INTRODUCTION

The object of machine design is to create a machine or structure that will convert and control motions either with or without transformation of a supply of energy into a definite kind of work. The design may be that of a complete machine, an improvement of an existing machine, or an attachment to provide for special operation of an existing machine. To be of commercial value, not only must the design be capable of performing the required tasks, but it must be economical in material cost, manufacturing cost, and upkeep cost. The best design, therefore, for a given machine is one that will do the required work with the least total cost per unit, consistent with weight, size, portability, and similar considerations. As the young designer advances in his study of machine design, he will find that total cost is a very important factor and one that requires a great deal of practical experience in addition to a vast amount of technical information.

1. The Designer. As the designer develops his ideas into finished plans for a machine, he must bring into play an extensive knowledge of subjects that may be roughly classified as:

a. Technical factors
b. Experience factors
c. Human factors.

a. The technical information necessary to design a machine varies with the type and field of application; and no one designer can become expert in all types of design. However, an understanding of mechanisms, mechanics, structures, materials, mechanical processes, hydraulics, thermodynamics, and similar

technical subjects is essential to every type of design. Since the stresses and the deflections in all parts of a machine can not always be accurately determined, the designer is forced to rely upon experimental data. The ability to analyze and use experimental data is a very important characteristic of the designer.

b. Experience and a knowledge of existing designs are essential to a thorough understanding of machine design, but previous designs should not be considered in any way that will hinder the designer's creative ability. The young designer will find unlimited fields for original ideas. Experience includes not only technical experience, but also includes experience in the conduct of business, and a knowledge of commerce and economics.

c. The human element is receiving more and more attention in the design room. There has been considerable study leading to the simplification of the operating controls and to a reduction of the physical effort necessary to operate the machine. Safety devices are being built into the machines, thus reducing accidents to a minimum. Pleasing appearance and sense appeal are now being seriously considered as aids to the sales force. To develop his knowledge of the human element, the designer should develop the power of discussing problems with engineers, technicians, salesmen, operators, mechanics, and even unskilled labor, since the different viewpoints will always help in developing well-balanced designs.

2. Design Procedure. There are many ways of attacking the same problem, and no rigid rules can be laid down for the designer to follow. A logical procedure to be followed, in the design of a new machine, may be as follows:

a. Make a complete statement of the purpose for which the machine is to be designed.

b. Select the possible groups of mechanisms which will give the desired motion or group of motions.

c. Determine the energy transmitted by, and the forces acting on, each member of the selected mechanism.

d. Select the material best suited for each member.

e. Determine the size of each member, considering the forces acting, the permissible stress on the material, and the permissible deflection or deformation.

f. Modify the members to agree with previous experience and judgment, and to facilitate manufacture.

g. Make assembly and detail drawings of the machine and include complete specifications for the material and manufacturing methods.

Each of the preceding items is self-explanatory with the possible exception of *f*. In certain members, no exact analysis of stresses and deformations can be made, and it is then necessary for the designer to resort to his experience and that of others in order to obtain properly proportioned machine members. In many cases, empirical equations (based on experimental results) must be used. Some slight modification may result in an easier manufacturing method and, hence, result in lower production costs. It is also essential that the assembly of the various parts, the replacement of worn parts, the allowance for wear, provision for lubrication, and similar items be kept constantly in mind during the design of the machine.

In this text it is assumed that the student has a working knowledge of machine drawing, mechanics, mechanisms, machine shop, and foundry practice. Theories developed in these subjects will be used with little explanation, the main object of this work being to analyze intelligently the problems encountered in design and to apply the theories in a practical way.

3. Stress. When any solid body is subjected to external forces, there are set up within the body resisting forces called stresses. If the body is cut by an imaginary plane, each part of the body exerts forces on the adjoining part, these forces acting perpendicular and parallel to the plane. These forces per unit area are spoken of as unit stresses, and in this text the term stress is used to indicate unit stress in pounds per square inch. Unit stress is in general given by the equation

$$s = \frac{F}{A} \tag{1}$$

where s = unit stress, psi.
F = total force, lb.
A = area normal to the applied force, sq in.

The simple, or direct, stresses are tensile, where the forces tend to elongate or stretch the member; compressive, where the forces tend to shorten the member; and shear, where the forces tend to make adjacent planes in the member slide over each other. When the forces act so as to bend the member, some parts will be in

tension while other parts will be in compression. These stresses are generally referred to as bending or flexural stresses. When the forces act to twist the member, the stresses set up are shear but are usually referred to as torsional stresses.

4. Deformation. The forces acting on the body cause changes in the shape and size, which increase as the forces increase. These changes in shape or size are called deformations or strains, and if measured on a body of unit dimensions, are called unit strains. For most materials the deformation is proportional to the applied load until a certain unit stress is reached. The ratio of the unit stress to the unit strain within the proportional limit is called the modulus of elasticity, and is expressed by the equation

$$E = \frac{s}{\delta} = \frac{FL}{A\Delta} \tag{2}$$

where E = modulus of elasticity, psi.
F = total applied load, lb.
L = total length, in.
Δ = total deformation, in.
A = area normal to applied force, sq in.
δ = unit deformation, in. per in.
s = unit stress, psi.

The modulus of elasticity as defined applies to elongation and compression, *i.e.*, to tension and compression stresses. When the external forces cause adjacent planes of the body to have a sliding movement relative to each other, the deformations are shearing deformations, and the modulus of elasticity, or modulus of rigidity, is expressed by the equation

$$G = \frac{s_s}{b} = \frac{FL}{AB} \tag{3}$$

where G = modulus of rigidity or modulus of elasticity in shear, psi.
s_s = unit stress in shear, psi.
F = total force applied, lb.
L = total length, in.
A = area parallel to the shear forces, sq in.
B = lateral deformation in the length L, in.
b = unit lateral deformation, in. per in. length.

The relation between the modulus of elasticity in tension E and the modulus of rigidity G is expressed by the equation*

$$G = \frac{E}{2(1 + m)} \tag{4}$$

where m is Poisson's ratio (see Table 15, page 59).

5. Proportional and Elastic Limits. As the stress is increased beyond a certain value, different for each material, the deformation begins to increase faster than the applied stress. The stress at which this occurs is called the proportional limit, indicated by the point PL on the stress-strain curve in Fig. 1.

At a slightly higher stress, the curve for ductile materials becomes horizontal; *i.e.*, the deformation increases without an increase in stress, and in some cases the stress may even decrease.

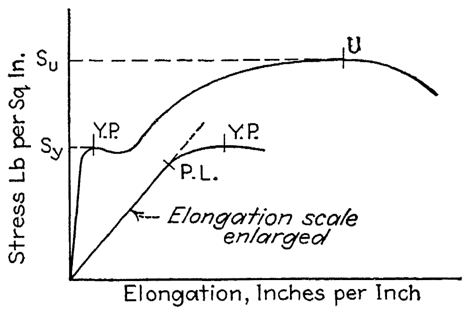


FIG. 1.—Typical stress-strain diagram for steel in tension.

The stress at which this occurs is called the yield stress or yield point and is indicated on the curve by the point YP. For materials having no well-defined yield point, the yield stress is taken as the stress that produces a permanent elongation of 0.2 per cent.

When the stress in an elastic body is removed, the body will return to its original shape and size. For most of the materials used in machine design, there is a stress beyond which a portion of the deformation will be retained when the stress is entirely removed. This limiting stress is called the elastic limit. The elastic limit for ductile materials is, in general, very nearly equal to the proportional limit; and, since the elastic limit is rather

* Proof of Eq. (4) may be found on pp. 9 and 10 of "Theory of Elasticity," by S. Timoshenko, McGraw-Hill Book Company, Inc., 1934.

difficult to determine accurately, whereas the proportional limit and the yield point are easily determined, we often find these used and called the elastic limit.

The ultimate strength s_u is the maximum unit stress reached before rupture occurs when a single application of load is applied without shock. This is indicated by the point U in Fig. 1.

6. Endurance Limit. The ultimate strength of the material is the stress that will cause rupture with one gradual application of the load. Experimental evidence proves that with repeated loading and unloading, failure will occur at stresses much below the ultimate, the number of repetitions of stress necessary to produce failure depending on the maximum stress imposed and on the range of stress variation. Experimental evidence also indicates that there is a limiting stress below which failure will not occur even when the stress is repeated an infinite number of times. This limiting stress is called the endurance limit of the material.*

* See Art. 74 for a more complete discussion of the endurance limit and fatigue failure.

CHAPTER II

ENGINEERING MATERIALS

7. General Discussion. After the general layout of the machine has been made, and the mechanisms to be used have been decided upon, it is necessary to select the proper material for each machine member. This involves the consideration of such factors as the engineering properties of the available materials, weight, size, shape of the member, material cost, fabricating cost, and any properties peculiar to the use to which the member is to be put. The engineering properties to be considered are: strength, stiffness, ductility, toughness, resilience, fatigue resistance, shock resistance, corrosion resistance, wear resistance, hardness, frictional qualities, machinability, casting properties, and forging properties.

The selection of the proper material has always been a difficult problem and is one that requires an increasing amount of experience as the variety of materials available is constantly extended. Pioneers of the machine-building industries were handicapped by the lack of ferrous and nonferrous materials suitable for their particular needs, but now the problem is not that of finding a material, but of selecting the most suitable one from the hundreds of materials available. Chemical analysis cannot be depended upon as the only criterion to be used in selecting and specifying materials. The method of manufacture, the amount of mechanical working, the heat-treatment, and the freedom from pipes, seams, and other defects must be considered. Initial cost should not be overemphasized, since this is generally a small percentage of the total cost of the finished part, and high-priced raw material may bring about large savings in machining and heat-treating costs. The proper material to use is one that meets the engineering requirements of the designers, the production and heat-treating requirements of the production department, and the cost requirements of the purchasing agent.

Various national organizations have established standard specifications of widely used materials, in an effort to assist

the designer in intelligently selecting materials. One of the most commonly used set of specifications is that of the S.A.E. In the S.A.E. specifications for steels, the materials are identified by an index number that indicates the approximate carbon content and the principal alloy content. The first figure indicates the class of steel as follows: carbon steels, 1; nickel steels, 2; chrome-nickel steels, 3; molybdenum steels, 4; chromium steels, 5; chrome-vanadium steels, 6; tungsten steels, 7; silico-manganese steels, 9. For example, S.A.E. 1020 is a plain carbon steel with 0.15 to 0.25 per cent carbon; S.A.E. 3120 is a chrome-nickel steel with 1.0 to 1.5 per cent nickel, and 0.15 to 0.25 per cent carbon; and S.A.E. 52100 is a chromium steel with 1.2 to 1.5 per cent chromium, and 0.95 to 1.10 per cent carbon. Other commonly used specifications are those of the American Society for Testing Materials (A.S.T.M.), the American Iron and Steel Institute (A.I.S.I.), and the American Standards Associations (A.S.A.). The Army, Navy, and Air Corps also have their own specifications. Certain National Emergency (N.E.) steels have been introduced recently as substitutes for the regular steels containing certain vital alloying materials.

Tables of the most important properties of materials, commonly used in machine design, are included for reference. The properties of materials vary with the mechanical and thermal treatment received during manufacture, and the tabulated values must be considered as average design values only. Higher values may be obtained by careful manufacture and heat-treatment, and the experienced designer must utilize his own experience with the material under consideration.

8. Fabrication of Materials. The designer must keep in mind the available fabrication methods and the effects of each on the properties of the finished member.

Castings are used for members of intricate shape that would be difficult to manufacture by other methods. Iron castings are generally made in sand or loam molds, but many nonferrous metals are cast in metal molds. When cast in metal molds under pressure, the products are called die castings and are more accurate in size and shape than are ordinary castings, and in many cases they require no machining. The expense of permanent molds prohibits their use except in quantity production. Centrifugal castings are made by rapidly revolving the molds

during the introduction and solidification of the molten casting material. In the case of most metals, the dirt and slag are lighter than the metal, and centrifugal action causes them to collect near the axis of rotation, where they may be later removed by machining.

Although castings are desirable for many parts of intricate shape, they present many problems of design. Shrinkage during the cooling period, combined with nonuniform cooling of irregular thick and thin sections, causes shrinkage or cooling stresses. These stresses may be severe, causing some castings to rupture before they can be put into service, or causing serious warping during subsequent machining operations. Important castings should always be annealed to relieve these stresses.

Castings with short-radius corners crystallize in such a way that cleavage planes are formed at these corners. The effect of these cleavage planes, and the highly localized stresses incident to sudden changes in section, may be greatly reduced by the use of generous fillets and well-rounded corners on all castings.

Large castings expose proportionately less cooling surface than similar small castings, and therefore cool more slowly and have a coarser grain structure. This results in less strength and ductility in the larger castings, unless they are properly heat-treated. Because of their finer grain structure, castings less than 1 in. in diameter may have 25 to 50 per cent more strength than castings several inches in diameter. The skin or outer surface of a casting is harder and stronger than the interior metal, this being especially true of cast iron, which contains considerable carbon.

Hot working of ductile materials by rolling, forging, and similar processes refines the grain and generally improves the properties. Rolling, pressing, and extruding processes work the metal throughout and produce nearly uniform structure in all parts of the material. In forging, the working of the material is more or less local, and the inner part is not affected unless the forging hammer is relatively heavy. After hot working, the material should be allowed to cool slowly and evenly to avoid cooling stresses and hardening. Brass, lead, and other soft materials are frequently extruded into intricate shapes that cannot be made by rolling or forging.

9. Cast Iron and Steel. Cast iron without the addition of alloying elements is weak in tension and shear, strong in com-

pression, and has low resistance to impact. The stress-strain diagram is curved all the way from zero to the rupture stress, there is no well-defined proportional limit, the ratio of stress to strain is not constant, and the modulus of elasticity decreases as the stress increases. Gray cast iron has the carbon present in the free or graphitic state and is soft, easily machined, and only moderately brittle. White cast iron, formed when the casting is rapidly cooled, has most of the carbon in the combined state and is therefore hard and brittle so that it cannot be machined with ordinary cutting tools, but requires grinding as a shaping process. This material is seldom used for complete castings, but is formed on wearing surfaces by the use of chill molds.

Malleable iron is made by heating white iron castings for a period of several days in airtight pots filled with an oxide of iron. The action of the heat and the iron oxide partially removes the carbon and reduces the remainder from the combined state to a globular form of free carbon, so that, after a slow cooling, a strong, soft, and somewhat ductile casting is obtained. Since the carbon change reaches only to a depth of about $\frac{3}{8}$ in., this process is not suitable for heavy castings. Malleable castings are used for cheap grades of tools, hardware, agricultural-machine parts, small pipe fittings, and as a substitute for light steel forgings.

Semisteel is a name given to metal made by melting 20 to 40 per cent of steel scrap with cast iron in the cupola. The product is not steel. It is a tough, close-grained cast iron.

Alloy cast irons have nickel, chromium, molybdenum, etc , added to produce castings of more uniform structure, improved hardness and wear resistance, and less chilling effect without increasing the grain structure. These alloy cast irons have better machining properties and wearing qualities than castings of ordinary composition and are being produced with tensile strength in excess of 60,000 psi.

Steel castings are more difficult to produce than iron castings and are more expensive, but are stronger and tougher. They are used for machine members of intricate shape that require high strength and impact resistance, such as locomotive frames, large internal-combustion-engine frames, and well-drilling tools. Alloy-steel castings have been developed to meet the demands of industry for greater strength and reliability of cast machine

TABLE 1.—PROPERTIES OF TYPICAL CAST IRONS

Material	Ultimate strength		Endurance limit in reversed bending s_{er}	Brinell hardness number	Modulus of elasticity		Elongation in 2 in., %	Remarks and suggested uses
	Tension s_t*	Compression s_c			Tension and compression E	Shear G		
Gray, ordinary	18,000	80,000	9,000	100–150	10–12,000,000	4,000,000	0–1	General industrial castings
Gray, good	24,000 16,000	100,000	12,000	100–150	12,000,000	4,800,000	0–1	Pump cylinders, etc.
Gray, high grade	30,000	120,000	15,000	100–150	14,000,000	5,600,000	0–1	Important castings
Malleable, S.A.E 32510	50,000	120,000	25,000	100–145	23,000,000	9,200,000	10	Substitute for unimportant forgings
Nickel alloys: **Ni-0.75, C-3.40, Si-1.75, Mn-0.55**	32,000 24,000	120,000	16,000	200 175	15,000,000	6,000,000	1–2	Light machine frames
Ni-2.00, C-3.00, Si-1.10, Mn-0.80	40,000 31,000	155,000	20,000	220 200	20,000,000	8,000,000	1–2	Heavy Diesel cylinders, pump, and valve bodies
Nickel-chromium alloys: **Ni-0.75, Cr-0 30, C-3 40, Si-1.90, Mn-0.65**	32,000	125,000	16,000	200	15,000,000	6,000,000	1–2	Light machine-tool tables, light engine cylinders
Ni-2.75, Cr-0 80, C-3 00, Si-1 25, Mn-0 60	45,000	160,000	22,000	300	20,000,000	8,000,000	1–2	Heavy forming dies

* Upper figures refer to arbitration test bars. Lower figures refer to the center of 4-in. round specimens.

Flexure: For cast irons in bending the modulus of rupture may be taken as 1.75 s_t (tension) for circular sections, 1.50 s_t for rectangular sections and 1.25 s_t for I and T sections.

Shear: The strength of cast iron in shear may be taken as 1 10 s_t (tension).

TABLE 2.—PROPERTIES OF TYPICAL CARBON STEELS

Material	Ultimate strength		Yield stress		Endurance limit in reversed bending	Brinell hardness number	Modulus of elasticity		Elongation in 2 in., %	Remarks and suggested uses
	Tension s_t	Shear s_s	Tension and compression s_y	Shear s_y	s_{er}		Tension and compression E	Shear G		
Wrought iron	48,000	50,000	27,000	30,000	25,000	100	28,000,000	11,200,000	30–40	
Cast steel, soft	60,000	42,000	27,000	16,000	26,000	110	30,000,000	12,000,000	22	General-purpose castings
medium	70,000	49,000	31,500	19,000	30,000	120	30,000,000	12,000,000	18	
hard	80,000	56,000	36,000	21,000	34,000	130	30,000,000	12,000,000	15	
S.A.E 1025, annealed	67,000	41,000	34,000	20,000	29,000	120	30,000,000	12,000,000	26	Machinery and general-purpose steel
water quenched	78,000 90,000	55,000 63,000	41,000 58,000	24,000 34,000	43,000 50,000	159 183	30,000,000	12,000,000	35 27	
S.A.E. 1045, annealed	85,000	60,000	45,000	26,000	42,000	140	30,000,000	12,000,000	20	Large forgings, axles, shafts
water quenched	95,000 120,000	67,000 84,000	60,000 90,000	35,000 52,000	53,000 67,000	197 248	30,000,000	12,000,000	28 15	
oil quenched	96,000 115,000	67,000 80,000	62,000 80,000	35,000 45,000	53,000 65,000	192 235	30,000,000	12,000,000	22 16	
S.A.E. 1095, annealed	110,000	75,000	55,000	33,000	52,000	200	30,000,000	12,000,000	20	Springs, cutting instruments
oil quenched	130,000 188,000	85,000 120,000	66,000 130,000	39,000 75,000	68,000 100,000	300 380	30,000,000	11,500,000	16 10	

Upper figures. steel quenched and drawn to 1300 F
Lower figures: steel quenched and drawn to 800 F.
Values for intermediate drawing temperatures may be approximated by direct interpolation.

members. The alloying elements usually used are nickel, chromium, manganese, molybdenum, vanadium, and combinations of these.

10. Wrought Steel. This is the most common material found in machine members, except cast iron. The mechanical working in the manufacturing process refines the structure and produces a more uniform steel having greater strength, greater toughness, and more durability than is obtained in castings. The properties of the steels vary greatly with the carbon content and with the form in which the carbon occurs, the beneficial effects of alloying materials depending to a large extent on their action on the carbon.

Low-carbon steels, containing up to 0.30 per cent of carbon. are soft, very ductile, easily machined, easily welded by any process, and, since the carbon content is low, unresponsive to heat-treatment. Medium-carbon steels, containing from 0.30 to 0.50 per cent carbon, are stronger and tougher than the low-carbon steels, machine well, and respond to heat-treatment. High-carbon steels, containing over 0.50 per cent carbon, respond readily to heat-treatment. In the heat-treated state, they may have very high strengths combined with hardness, but are not so ductile as the medium-carbon steels. In the higher carbon ranges, the extreme hardness is accompanied by excessive brittleness. The higher the carbon content, the more difficult it is to weld these steels.

11. Alloy Wrought Steel. The principal alloying elements used in steel are nickel, chromium, vanadium, molybdenum, manganese, and to a lesser extent, copper, tungsten, cobalt, beryllium boron, and silver. Alloys are used to effect increased strength increased elastic ratio, increased hardness without loss of ductility, more uniform structure, better machinability, and better resistance to fatigue and corrosion. The proper combination of these properties depends not only on the chemical composition, but largely on the heat-treatment, without which the alloy steels are not greatly superior to the plain carbon steels. The effects of heat-treatment on a typical alloy steel are shown in Fig. 2. With proper alloying and heat-treating, ultimate strengths of over 300,000 psi and yield stresses of over 250,000 psi. are obtainable. Many of the alloy steels are expensive, and their use is therefore limited.

TABLE 3.—PROPERTIES OF TYPICAL ALLOY STEELS

Material, alloy and S.A.E. No.	Ultimate strength		Yield stress		Endurance limit in reversed bending s_{er}	Brinell hardness number	Modulus of elasticity		Elongation in 2 in., %	Remarks and suggested uses
	Tension s_t	Shear s_s	Tension s_y	Shear s_y			Tension and compression E	Shear G		
Nickel: S.A.E. 2320, water quenched	77,000 140,000	55,000 98,000	50,000 110,000	30,000 65,000	49,000 68,000	143 277	30,000,000	12,000,000	30 18	Casehardening stock for heavy parts. Not desirable for thin sections
oil quenched	75,000 130,000	54,000 90,000	48,000 100,000	29,000 60,000	46,000 50,000	140 262	30,000,000	12,000,000	30 18	Gears
S.A.E. 2340, water quenched	95,000 175,000	60,000 110,000	65,000 150,000	38,000 87,000	53,000 75,000	183 340	30,000,000	12,000,000	30 16	Forgings, axles
oil quenched	93,000 165,000	59,000 105,000	62,000 148,000	36,000 85,000	55,000 75,000	183 330	30,000,000	12,000,000	17 14	Gears
Nickel-chromium. S.A.E. 3120, water quenched	86,000 140,000	60,000 98,000	57,000 115,000	34,000 68,000	55,000 58,000	174 269	30,000,000	12,000,000	34 15	Heavy sections requiring medium depth casehardening
oil quenched	77,000 120,000	55,000 82,000	48,000 96,000	29,000 57,000	46,000 48,000	163 241	30,000,000	12,000,000	30 18	
S.A.E. 3220, water quenched	87,000 165,000	60,000 115,000	60,000 143,000	36,000 83,000	58,000 72,000	187 331	30,000,000	12,000,000	33 16	Massive sections requiring deep casehardening
oil quenched	80,000 55,000	56,000 107,000	53,000 130,000	31,000 95,000	51,000 65,000	174 311	30,000,000	12,000,000	30 20	

TABLE 3.—PROPERTIES OF TYPICAL ALLOY STEELS.—(*Continued*)

Material, alloy and S.A.E. No.	Ultimate strength		Yield stress		Endurance limit in reversed bending	Brinell hardness number	Modulus of elasticity		Elongation in 2 in., %	Remarks and suggested uses
	Tension s_t	Shear s_s	Tension s_y	Shear s_y	s_{er}		Tension and compression E	Shear G		
S.A.E. 3240, oil quenched . .	110,000 200,000	73,000 130,000	87,000 180,000	52,000 105,000	84,000 90,000	229 388	30,000,000	12,000,000	23 14	Rollers, sprockets, gears
Chromium-molybdenum: S.A.E. 4140, oil quenched	120,000 190,000	90,000 150,000	100,000 165,000	55,000 91,000	60,000 95,000	240 380	30,000,000	12,000,000	25 12	
Chromium-vanadium: S.A.E. 6145, oil quenched . .	105,000 230,000	75,000 180,000	98,000 210,000	54,000 115,000	94,000 105,000	220 425	30,000,000	12,000,000	25 12	
Silicon-manganese: S.A.E. 9260, oil quenched . . .	158,000	120,000	100,000	60,000	62,000	240	30,000,000	12,000,000	16	Springs, gears
Stainless steel, 0.12 C, 18 0 Cr, 8 0 Ni	90,000 200,000	60,000 150,000	35,000 175,000	20,000 100,000	40,000 90,000	135 380	30,000,000	12,000,000	60 5	Hot-rolled Cold worked

The endurance limit for reversed shear may be taken as 55 per cent of the endurance limit in reversed bending

Tabulated values are minimum values to be expected with round bars up to $1\frac{1}{2}$ in. diameter. Smaller sections and careful heat-treatment will show higher values.

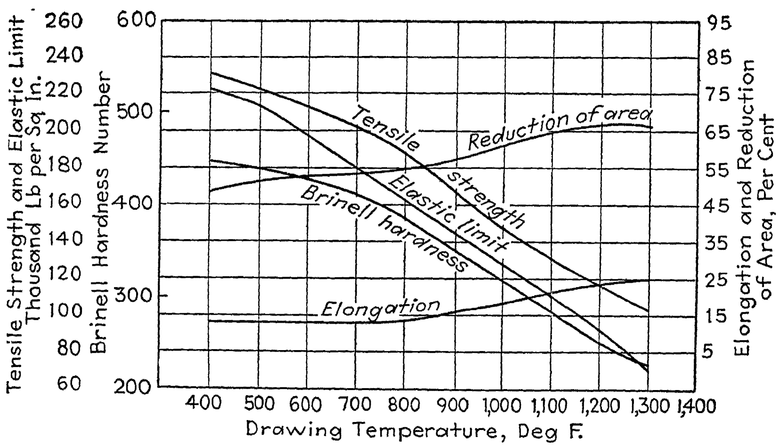
Upper figures: steel quenched and drawn to 1300 F.

Lower figures: steel quenched and drawn to 800 F.

Nickel increases the strength without sacrificing the ductility of the carbon steels. It also tends to retard the grain growth, which allows a larger range of heating and longer carburizing periods without damage due to coarse grain structure. Chromium is added principally to harden the steel and to raise the tensile strength by its action in forming carbides. Vanadium

NICKEL-CHROMIUM STEEL S.A.E. NO. 3330—OIL QUENCHED

Chemical composition		Approximate critical point		Suggested heat-treatment
Carbon	25– .35%	AC_1	1345 F	1. Heat to 1425–1475 F and quench in water.
Manganese	.30– .60	AC_2	1345 F	
Phosphorus	.04 Max	AC_3	1355 F	2. Draw at a temperature to give the desired hardness.
Sulphur	.04 Max			
Nickel	3 25–3.75			
Chromium	1 25–1 75			



If bars larger than $1\frac{1}{4}$ in. diameter are being heat-treated, reduce the tensile and elastic limit approximately in accordance with the following table:

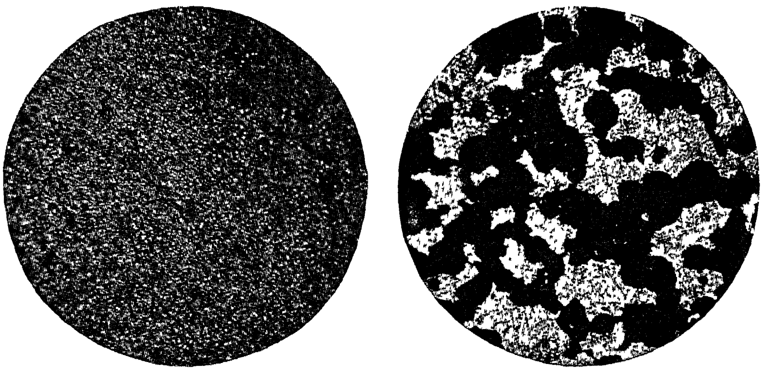
Size	$2\frac{1}{2}$	3	$3\frac{1}{2}$	4	$4\frac{1}{2}$	5	$5\frac{1}{2}$	6
Reduce by %	8.5	10 0	12 0	13.5	14 0	14 5	15.0	15 5

FIG. 2.—Effect of heat-treatment on the physical properties of steel.

is used to toughen and strengthen the steel, to reduce the grain size, and to act as a cleanser and degasifier. It has the desirable effect of increasing the life of tools, springs, and other members subjected to high temperatures. Molybdenum acts very much like chromium but is more powerful in its action. It also increases the depth of hardening after heat-treatment. Nickel-

molybdenum and nickel-chromium-molybdenum steels retain the good features of the nickel-chromium steels and in addition have better machining qualities. These steels respond very uniformly to heat-treatment and offer the highest strengths obtainable in commercial steels.

12. Heat-treatment. Heat-treating is the process of controlled heating and cooling of metals to change their structural arrangement and to insure certain desirable properties. The higher carbon steels and alloy steels are especially responsive



(a) S.A.E. 1095 Steel. Heated to 1430 F and cooled in the furnace. Fine grain, sorbitic structure.

(b) S.A.E. 1095 Steel. Heated at 1840 F for 10 minutes and quenched in oil. martensite and troostite structure.

FIG. 3.—Micrographs showing the effect of variation in heat-treatment. Enlargement about 100 diameters. (*Specimens from the University of Texas Laboratories.*)

to heat-treatment. Treatments recommended for the various alloys may be found in the S.A.E. Handbook and other engineering handbooks and will not be included here. The general effects of the common processes are described in the following paragraphs:

Annealing consists of heating the metal to a temperature slightly above the critical temperature and then cooling slowly, usually in the furnace, to produce an even grain structure, reduce the hardness, and increase the ductility. Annealing usually reduces the strength. Internal stresses and strain hardening due to previous treatment are also removed by this process.

Normalizing is a form of annealing (cooling in air, however) used to remove the effects of any previous heat-treatment and to produce a uniform grain structure before other heat-treatments are applied to develop particular properties in the metal.

Quenching, or rapid cooling from above the critical temperature by immersion in cold water or other cooling medium, is a hardening treatment. In steels this retains the carbon in the structure as iron carbide, which, being very hard, increases the hardness of the metal. The degree of hardness depends on the amount of carbon present, and on the rate of cooling, which can be varied by using such cooling mediums as ice water, cool water, oil, hot oil, molten lead, etc. Some alloy steels, notably those containing tungsten, harden when slowly cooled in air. The hardening treatment raises the strength of the metal and increases the wear resistance, but makes the metal brittle and of low ductility. Average strength and hardness relations are shown in Table 4.

TABLE 4.—RELATION BETWEEN TENSILE STRENGTH, BRINELL HARDNESS, AND ROCKWELL HARDNESS READINGS FOR STEEL

Brinell No.	Rockwell C No.	Tensile strength, psi	Brinell No.	Rockwell C No.	Tensile strength, psi	Brinell No.	Rockwell C No.	Tensile strength, psi	Brinell No.	Rockwell C No.	Tensile strength, psi
97		50,000	170	6	85,000	243	23.5	120,000	408	43	200,000
107		55,000	181	8.5	90,000	266	27	130,000	444	46	220,000
116		60,000	192	12	95,000	289	30.5	140,000	482	49.5	240,000
127		65,000	203	15.5	100,000	311	33	150,000	522	52.5	260,000
138		70,000	213	17.5	105,000	331	35	160,000	563	55.5	280,000
149		75,000	223	20	110,000	352	37	170,000	605	58	300,000
159	2	80,000	233	21.5	115,000	371	39	180,000	635	61	320,000

These values are approximately correct for carbon steels and low-carbon nickel, chrome-nickel, and nickel-chrome-molybdenum steels, such as S.A.E 2330, X-4130, and 4340.

Tempering, or drawing, consists of reheating the quenched metal to restore some of the ductility and reduce the brittleness. Increased toughness is obtained at the expense of high strength. The loss of hardness depends on the temperature and drawing time.

Casehardening, or carburizing, is a process of hardening the outer portion of the metal by prolonged heating free from contact with air while packed in carbon in the form of bone char, leather

scraps, or charcoal. The outer metal absorbs carbon, and, when the hot metal is quenched, this high-carbon steel hardens, whereas the low-carbon steel of the core remains soft and ductile. In gas carburizing, the metal is heated in an atmosphere of gas, controlled so that the metal absorbs carbon from the gas but will not be oxidized on the surface.

Cyaniding is casehardening with powdered potassium cyanide, or potassium ferrocyanide mixed with potassium bichromate, substituted for the carbon. For a very thin case, immersion in hot liquid cyanide is sufficient. Cyaniding produces a thin but very hard case in a very short time.

Nitriding is a surface hardening accomplished by heating certain steel alloys (Nitralloys) while immersed in ammonia fumes.

The principal treatments used with cast iron* are aging, to relieve castings of cooling and shrinkage stresses; baking, to remove brittleness caused by pickling in acid during the cleaning process; annealing, to reduce the hardness and permit machining; toughening, to increase the strength of white iron; quenching by rapid cooling, to obtain sufficient hardness to resist wear and indentation; drawing, to restore the strength of quenched castings; and carburizing, to increase wear and impact resistance.

13. Materials for High-temperature Service. Metal parts for steam engines, internal-combustion engines, valves, superheated steam equipment, oil stills, and similar service are stressed at temperatures ranging from 200 to 1800 F. The metals used for such parts must be specially selected from those materials which retain a large percentage of their strength at high temperatures and which do not creep excessively.

High-temperature or heat-resisting alloys are made in three principal groups. Alloys of the first group contain over 50 per

* COYLE, F. B., Heat Treatment Fundamentals of Plain and Alloy Cast Iron, *Metals and Alloys*, 1931.

WHITFIELD, R. W., Heat Treatment of Cast Iron, *Iron and Steel Ind.*, June, 1930.

WAHLS and HARTWELL, Some Phases of Heat Treatment of Cylinder and Alloy Irons, *Trans. Am. Foundrymen's Assoc.*, Vol. 2, March, 1931.

Report of Committee A-3 on Cast Iron, Appendix II, *Proc. Am. Soc. Testing Materials*, Vol. 29, Part II, 1929.

Oil-hardening and Air-hardening Cast Iron, *Foundry Trade J.*, Vol. 12, 1930.

"Nickel Cast Iron Data Sheets," International Nickel Company, Inc.

cent iron with additions of 10 to 30 per cent chromium and varying amounts of copper, nickel, tungsten, and silicon. Alloys of the second group contain up to 25 per cent iron, 50 to 60 per cent nickel and chromium, and small additions of manganese, tungsten, silicon, and molybdenum. Alloys of the third group contain from 0.5 to 6 per cent iron as an impurity, over 80 per cent nickel, considerable chromium, and small amounts of manganese and silicon. In this group may be placed alloys containing 25 to 30 per cent chromium and 70 to 75 per cent cobalt, which show yield stresses of over 40,000 psi at temperatures from 1200 to 1400 F. The alloys of the third group also have excellent resistance to corrosion at temperatures up to 1800 F.

The effect of elevated temperature on the strength of metals and the effect of creep is discussed more fully in Art. 82.

14. Copper. Commercial copper is a tough, ductile, and malleable metal containing less than 5 per cent of such impurities as tin, lead, nickel, bismuth, arsenic, and antimony. The strength is low compared to that of steel, and its properties depend largely on the mechanical treatment to which it has been subjected. Cold working makes it stronger, and somewhat brittle. Pure copper is not extensively used in machine design except for castings, condenser tubes, water pipe, and sheet-metal parts where resistance to corrosion by weather and water is important, or in places where its high heat conductivity is an advantage. Copper has a high conductivity for electricity, and about 60 per cent of all the copper produced is used for electrical work.

The alloys of copper form an important group of materials with a wide variety of properties. Some have high strengths, some are excellent bearing materials, some retain their strength at high temperatures, and others are valuable for their corrosion resistance. Three groups of copper alloys used in machine design are the brasses, the bronzes, and Monel metal. Copper is also used as an alloy in steel to increase its resistance to corrosion.

15. Copper-zinc Alloys. Brass is an alloy of copper and zinc, containing 45 to 90 per cent copper and small amounts of iron, lead, and tin as impurities. These alloys are highly resistant to corrosion; they machine easily and make good bearing materials.

Copper-zinc alloys may be divided into two broad groups: those containing over 64 per cent copper and consisting of a

TABLE 5.—PROPERTIES OF COPPER-ZINC ALLOYS (BRASS)

Material	Ultimate strength	Yield stress	Endurance limit s_{er}	Brinell hardness No.	Modulus of elasticity	Elongation in 2 in., %	Remarks and suggested uses
	Tension s_t	Tension s_y			Tension and compression E		
Commercial bronze							90 Cu, 10 Zn
Rolled, hard	65,000	63,000	18,000	107	15,000,000	18	Sheet rod wire tubes
Rolled, soft	35,000	11,000	12,000	52	15,000,000	56	
Forged, cold	40,000–65,000	25,000–61,000	12,000–16,000	62–102	15,000,000	55–20	
Red brass							85 Cu, 15 Zn
Rolled, hard	75,000	72,000	20,000	126	15,000,000	18	Sheet wire shapes tubes
Rolled, soft	37,000	14,000	14,000	54	15,000,000	55	
Forged, cold	42,000–62,000	22,000–54,000	14,000–18,000	63–120	15,000,000	47–20	
Low brass							80 Cu, 20 Zn
Rolled, hard	75,000	59,000	22,000	130	15,000,000	18	
Rolled, soft	44,000	12,000	15,000	56	15,000,000	65	
Forged, cold	47,000–80,000	20,000–65,000		63–133	15,000,000	30–15	
Spring brass							75 Cu, 25 Zn
Hard	84,000	64,000	21,000	107*	14,000,000	5	
Soft	45,000	17,000	17,000	57*	18,000,000	58	
Cartridge brass							70 Cu, 30 Zn
Rolled, hard	100,000	75,000	22,000	154	15,000,000	14	
Rolled, soft	48,000	30,000	17,000	70	15,000,000	55	
Deep-drawing brass							68 Cu, 32 Zn
Strip, hard	85,000	79,000	21,000	106*	15,000,000	3	
Strip, soft	45,000	11,000	17,000	13*	15,000,000	55	
Muntz metal							60 Cu, 40 Zn
Rolled, hard	80,000	66,000	25,000	151	15,000,000	20	
Rolled, soft	52,000	22,000	21,000	82	15,000,000	48	
Tobin bronze							60 Cu, 39.25 Zn, 0 75 Sn
Hard	63,000	35,000	21,000	165	15,000,000	35	
Soft	56,000	22,000		90	15,000,000	45	
Manganese bronze							58 Cu, 40 Zn
Hard	75,000	45,000	20,000	110	15,000,000	20	
Soft	60,000	30,000	16,000	90	15,000,000	30	

* Rockwell hardness F.

single metallurgical structure phase, alpha brass; and those containing 55 to 63 per cent copper in two phases, alpha-beta brass. The second group (containing high percentages of zinc) are of little commercial significance since the beta phase predominates, and the alloys are all brittle and difficult to work when cold. They can, however, be forged, rolled, and extruded when hot.

The alpha brasses (copper over 64 per cent) are very ductile at room temperatures and are readily cold-worked by rolling, upsetting, forming, stamping, deep drawing, or spinning. The higher the copper content the more the alloy hardens when cold-worked, those with over 85 per cent hardening virtually as much as pure copper. They can be rendered malleable after cold-working by annealing but cannot be hardened by heat-treatment. Brasses with over 80 per cent copper can be hot-forged, hot-rolled, or otherwise hot-worked without difficulty, provided the lead impurity is kept below 0.03 per cent.

Lead additions up to 3 per cent increase the machinability of the brass, and free-cutting alloys of 60 to 63 per cent copper with lead additions are used for automatic-screw-machine work. Additions of tin to the brasses improve their corrosion resistance. Some brasses are subject to season cracking, which is a corrosion effect indicated by short irregular cracks or long, straight cracks of considerable length. Since the cause is internal stress, these brasses should be stress-relieved by a low-temperature anneal. Season cracking is not usually found in alloys containing less than 15 per cent zinc.

Commercial bronze is a brass with 90 per cent copper, having excellent cold-working properties. It is readily forged, upset, drawn, or spun. It is used for hardware, screws, rivets, and similar parts.

Red brass (rich low brass) has excellent cold-working properties and is used for severe cold-drawing, stamping, and spinning work. In many uses it has better corrosion resistance than pure copper and is used for plumbing, in oil refineries, automobile radiators, and condenser tubes for inland use with brackish and slightly salty water but not for marine use. It is relatively immune to season cracking.

Low brass has properties very similar to red brass but may fail by season cracking. It may also fail by dezincing under certain corrosive action. This action causes localized pitting and con-

verts the material into a brittle and weakened state. The addition of small amounts of arsenic to the brass helps resist this action. It is used for bellows, flexible hose, and drawn and stamped parts.

High brasses, including cartridge brass, 70 per cent copper and 30 per cent zinc, deep-drawing brass, 68 per cent copper, and common high brass, 66 per cent copper, offer the optimum combination of strength and ductility, have excellent cold-working properties, and are readily cold-drawn, cold-forged, or spun. Impurities such as lead, iron, phosphorus, antimony, bismuth, nickel, chromium, and aluminum must be kept to the absolute minimum because of their adverse effects on toughness. These brasses are susceptible to season cracking; therefore, parts to be used under corrosive conditions should be stress-relieved by annealing at low temperatures. These brasses are used for pins, rivets, snap fasteners, radiators, lamp reflectors, electrical sockets, cartridge cases, and many other shapes which require deep drawing. They are available in sheet, plate, wire, bar, and tubes.

Spring brass has excellent cold-working properties and is fabricated by forging, rolling, drawing, and spinning. It is not usually hot-worked. It is used for springs when corrosion resistance is important and the loads are not excessive.

Muntz metal has good hot-working properties, but at room temperatures is somewhat harder than most commercial copper-zinc alloys. The ease with which it is hot-worked and its low first cost combine to make its use in sheet form very important. It is also used for salt-water corrosion resistance and has found considerable use as a sheathing for wooden ships, for marine condenser tubes, and heat-exchanger parts.

Tobin bronze (brass) has about the same strength as mild steel and is extensively used for shafts on small boats and for feed-pump shafts and valve trim where corrosive conditions exist.

Manganese bronze is really a brass, containing copper, zinc, and small amounts of lead, iron, and manganese. The manganese is added as an oxidizing agent, and the final product contains only a trace of the original manganese. It has high strength and ductility and good resistance to salt-water corrosion. It can be rolled, drawn, or cast and is used for ship propellers and heavy-duty bearings and gears.

16. Copper-tin Alloys. Bronze is an alloy of copper, tin, and a small amount of phosphorus. Bronzes with descriptive metallic names, such as aluminum bronze and manganese bronze, are not true bronzes and have no relation to the copper-tin alloys.

The bronzes are more costly than the brasses and hence are used only when the cheaper alloys do not prove to be satisfactory. They are used in the highest quality thermostatic bellows and other parts that require resistance to severe stretching together with good tensile and elastic strength.

Copper and tin alone do not readily alloy and small quantities of phosphorus are generally added, giving rise to the common trade name phosphor bronze. When tin is added to the molten copper, tin oxide forms, which is insoluble and, being practically the same specific gravity as the copper, cannot be readily removed with the slag. If tin oxide is not removed, it will make the cast ingots weak and brittle and unfit for fabrication. Phosphorus reduces the tin oxide to phosphorus pentoxide which can be removed by skimming.

The bronzes are particularly useful because of corrosion resistance, good fatigue properties, and useful spring characteristics, the latter being due to high elastic limit and creep resistance at room temperatures. As the tin increases, up to 8 per cent, the strength and ductility increase rapidly. More than 8 per cent tin increases the strength, but the ductility drops off.

Leaded phosphor bronze, containing 4 per cent lead, is free-cutting and is used for automatic-screw-machine stock.

17. Copper-aluminum Alloys. The usual aluminum bronzes are those containing copper and 5, 8, or 10 per cent of aluminum. They are golden yellow in color and have high corrosion resistance, high strength, good wear properties, and good fatigue properties. Aluminum-bronze bearing material is hard and is used chiefly where strength is important, *i.e.*, where high pressures are encountered. Aluminum bronzes are much used in aircraft for bushings, gears, bearings, valve guides, shock-absorber pistons, and similar parts. The material is sand-cast, centrifugal-cast, rolled, forged, and extruded and may be heat-treated.

18. Copper-silicon Alloys. These alloys appear under various trade names, such as Everdure and Duronze. These alloys in some cases have better corrosion resistance than copper. They have high strength, and are harder than most nonferrous alloys,

TABLE 6.—PROPERTIES OF COPPER-TIN ALLOYS (BRONZE)

Material	Ultimate strength	Yield stress	Endurance limit	Hardness Rockwell B	Modulus of elasticity		Elongation in 2 in., %	Remarks and suggested uses
	Tension s_t	Tension s_y	s_{er}		Tension and compression E	Shear G		
Wrought bronzes:								
Signal bronze								98.2 Cu, 1.8 Sn, .05 P
Hard	105,000							Wire
Soft	45,000							Wire
Phosphor bronze:								
Grade A								94.9 Cu, 5 Sn, 0.10 P
Hard	80,000	65,000		86			8	
Soft	50,000	20,000		28			50	
Grade B								92 9 Cu, 4 Sn, 0.10 P, 3 Pb
Hard	62,000	52,000		70			25	Leaded, free cutting
Soft	25,000							
Grade C								91.9 Cu, 8 Sn, 0.10 P
Hard	93,000	68,000		95			10	
Soft	60,000	25,000		50			60	
Grade D								89.9 Cu, 10 Sn, 0.10 P
Hard	100,000	73,000		98			12	
Soft	65,000	24,000		55			65	
Casting bronze:								
Tin bronze	40,000	18,000			10,000,000		20	88 Cu, 10 Sn, 2 Zn
Navy bronze	34,000	16,000			10,000,000		20	88 Cu, 6 Sn, 4 Zn, 1.5 Pb, 0 5 Ni
Leaded tin bronze	25,000	12,000			8,500,000		8	80 Cu, 10 Sn, 10 Pb
Leaded red brass	30,000	14,000					20	85 Cu, 5 Sn, 5 Zn, 5 Pb
High yellow brass	35,000	12,000					25	71 Cu, 1 Sn, 25 Zn, 3 Pb

TABLE 7.—PROPERTIES OF MISCELLANEOUS COPPER ALLOYS

Material	Ultimate strength			Yield stress		Endurance limit s_{er}*	Rockwell hardness*	Modulus of elasticity		Elongation in 2 in., %*	Remarks and suggested uses
	Tension s_t*	Compression s_c	Shear s_s	Tension s_y*	Compression s_y*			Tension and compression E	Shear G		
Aluminum bronze	55,000 92,000	....	....	22,000 65,000	...	10,000 20,000	35 B 92 B	17,000,000	6,500,000	65 7	95 Cu, 5 Al, sheet
Aluminum bronze	65,000 105,000	......	..	25,000 65,000	..	14,000 20,000	50 B 96 B	17,000,000	6,500,000	60 7	92 Cu, 8 Al, sheet
Aluminum bronze	65,000 80,000	...	.	25,000 50,000	..	14,000 18,000	...	17,000,000	6,500,000	65 30	92 Cu, 8 Al, rod
Aluminum bronze. Sand cast	70,000	124,000	..	32,000	22,000	..	131†	15,000,000	....	18	86.5 Cu, 10.2 Al, 3 3 Fe Bushings, gears, landing gear, valve guides, pump bodies
Centrifugal cast	77,000			35,000	...	.	137†	15,000,000		15	86 5 Cu, 10.2 Al, 3 3 Fe Bushings, gears, landing gear, valve guides, pump bodies
Forged	87,000		.	35,000		..	150†	15,500,000	.....	25	Shock absorber pistons, bearings, gears
Everdure:											
Tubes	40,000	...	..	15,000	.		55 B	..	....	50	96 Cu, 1 5 Si, 0 3 Mn
Sheet	65,000	.	....	50,000	....	.	75†	...		8	
Rod	40,000 70,000	...	..	15,000 55,000	..	.	55 F 80 B	...	.	50 15	
Duronze III			..	.....	....		....	....			91 Cu, 7 Al, 2 Si
Annealed	85,000										
Drawn	95,000										
Beryllium copper				.							2–2.25 Be, 0 25–0.50 Ni, + Cu
Rod, annealed	70,000		55,000	.		.	.	18,000,000			
Rod, hard-drawn	93,000	128,500	59,000		84,000		194†	17,000,000	6,900,000		
Rod, drawn and heat-treated	173,000	206,000	100,000		155,000	35,000	335†	18,400,000	7,100,000		
Sheet, annealed	70,000			31,000			65 B	18,000,000		45	
Sheet, No 4 hard	118,000			105,000			102 B	17,000,000		4 3	
Sheet, hard and heat-treated	193,000	..	..	138,000	.	..	114 B	18,400,000		2	
Cupronickel, sheet	48,000 74,000			20,000 65,000	..		79†			40 5	80 Cu, 20 Ni

* Upper figures represent annealed condition; lower figures, hard-drawn condition.
† Brinell hardness.

may be hot-forged or extruded, have lower coefficients of friction than copper or brass, and are free-machining but not so much so as highly leaded brass. They are used as rods, sheets, or forgings in chemical and hot water storage, machine-gun parts, compression fittings for oil and gasoline tubes, valve stems, universal joints, and power screws which mate with steel.

19. Copper-beryllium Alloys. These alloys contain copper, up to 2.75 per cent beryllium, and sometimes a small amount of nickel. The most common alloy is Anaconda Beryllium Copper, manufactured by the American Brass Company, containing 2 to 2.25 per cent beryllium, 0.25 to 0.50 per cent nickel, and the remainder copper. Alloys containing less than 1 per cent beryllium do not respond to heat-treatment; alloys containing 1 to 1.6 per cent are slightly heat-treatable; alloys containing 1.6 to 2.75 per cent respond readily.

Beryllium copper may be fully annealed by holding at 1475 F for 15 to 30 min and rapidly quenching. The mechanical properties may be raised by the precipitation treatment, consisting of holding at 480 to 580 F for a time determined by the properties desired. This alloy can be hot-worked by forging, hot-pressing, or extruding and can be cold-worked by rolling, drawing, or forming. Cold-working hardens it very rapidly. They can be welded with difficulty.

These alloys are expensive, but in the hardened or heat-treated conditions, the strength and elastic properties are far superior to any other nonferrous alloy. They are used for springs, diaphragms, bolts, precision bearings, and similar parts subjected to fatigue action under corrosive conditions. They are also used for small gears, valve sleeves, valve seats, etc., in internal-combustion engines, and for certain hand tools where their hardness and nonsparking qualities are an advantage.

20. Nickel-copper Alloys. Alloys containing less than 50 per cent nickel are called cupronickels or nickel silvers. The nickel silvers also contain zinc. These alloys are silver in appearance and tarnish very little under atmospheric conditions. They are malleable and can be worked without annealing.

Monel metals are the most important nickel-copper alloys, having high strength and toughness and excellent resistance to salt water, brine, caustic soda, sulphuric acid, hydrochloric acid, chlorinated solvents, and other corrosive agents. Monel is

readily attacked by moist chlorine and ferric, stannic, and mercuric salts in acid solution.

Monel metal contains 67 per cent nickel, 30 per cent copper, and small amounts of iron, manganese, silicon, and carbon and is a general-purpose material. It is used under corrosive temperatures up to 500 F, and if no sulphur is present, the temperature may be higher. Monel is slightly magnetic at room temperature but becomes nonmagnetic above 140 F. It is available in cast, rolled, and forged shapes. The strength may be increased by cold-working, and, although Monel may be annealed, it cannot be hardened by heat-treatment alone. It holds its strength at elevated temperatures better than bronzes and some steels and is used for turbine blades, high-temperature valve trim, and for springs subjected to temperatures up to 400 F. Unlike plain steel, it increases in strength, hardness, and fatigue strength as the temperature decreases to as low as −300 F, the elongation and impact strength remaining practically constant.

K-Monel contains 66 per cent nickel, 29 per cent copper, 2.75 per cent aluminum, and small quantities of iron, manganese, and carbon. The addition of the aluminum increases the strength and hardness above that of Monel and also makes it susceptible to hardening by heat-treatment. K-Monel has the same corrosion-resisting and high-temperature properties as Monel and is wholly nonmagnetic.

21. Nickel. Wrought nickel as used in the industries has mechanical properties similar to those of mild steel. It is one of the toughest of metallic materials, retains its strength to a good degree at elevated temperatures, and its ductility, toughness, and strength at sub-zero temperatures. It can be hot-forged and -rolled, cold-worked, cast, and welded. It has good corrosion resistance and is used as a lining for many types of containers.

Inconel contains 79.5 per cent nickel, 0.2 per cent copper, 13 per cent chromium, 6.5 per cent iron, and a small amount of silicon and manganese. It has the corrosion-resistant properties of Monel but has better resistance to sulphur at the higher temperatures. It retains its strength at extremely high temperature and can be used at 2100 F. Its creep properties are very good. It is nonmagnetic at all temperatures above −40 F.

Invar, an alloy of iron with 30 per cent nickel, has a very low coefficient of heat expansion, making it useful for measuring instruments.

TABLE 8.—PROPERTIES OF NICKEL ALLOYS

Material	Ultimate strength	Yield stress	Endurance limit	Brinell hardness	Modulus of elasticity		Elongation in 2 in., %
	Tension s_t	Tension s_y	s_{er}		Tension and compression E	Shear G	
Nickel:							
Annealed	60,000– 80,000	15,000– 30,000		90–120	30,000,000	11,000,000	50–35
As drawn	65,000–115,000	40,000– 90,000		125–230	30,000,000	11,000,000	35–15
Hot-rolled	65,000– 80,000	15,000– 30,000		90–120	30,000,000	11,000,000	50–35
Forged	65,000–105,000	20,000– 80,000		90–200	30,000,000	11,000,000	40–25
Monel.							
Annealed	70,000– 85,000	25,000– 40,000	30,000	110–140	26,000,000	9,500,000	50–35
As drawn	85,000–125,000	60,000–120,000		160–250	26,000,000	9,500,000	35–15
Hot-rolled	80,000– 95,000	40,000– 65,000		140–185	26,000,000	9,500,000	45–30
Forged	80,000–110,000	40,000– 85,000		140–225	26,000,000	9,500,000	40–20
K-Monel·							
Annealed	90,000–110,000	40,000– 60,000	37,500	140–180	26,000,000	9,500,000	45–35
Annealed, age-hardened	130,000–150,000	90,000–110,000		240–260	26,000,000	9,500,000	30–20
As drawn	100,000–125,000	70,000–100,000	42,000	175–250	26,000,000	9,500,000	35–15
Drawn, age-hardened	140,000–170,000	100,000–130,000	41,000	260–320	26,000,000	9,500,000	30–15
Hot-rolled	90,000–120,000	40,000– 85,000	41,000	140–225	26,000,000	9,500,000	45–30
Rolled, age-hardened	135,000–160,000	100,000–120,000	44,500	260–300	26,000,000	9,500,000	30–20
Forged	90,000–120,000	40,000– 90,000		140–240	26,000,000	9,500,000	40–25
Age-hardened	135,000–165,000	100,000–125,000		260–310	26,000,000	9,500,000	30–20
Inconel:							
Annealed	80,000–100,000	25,000– 50,000	30,000	120–170	31,000,000	11,000,000	50–35
As drawn	95,000–150,000	75,000–125,000	41,000	180–290	31,000,000	11,000,000	30–15
Hot-rolled	85,000–120,000	35,000– 90,000	38,500	140–210	31,000,000	11,000,000	45–30
Forged	85,000–120,000	35,000– 90,000		140–210	31,000,000	11,000,000	45–20

Yield stress is taken at 0 2 per cent offset
Endurance limit at 1,000,000,000 cycles.

22. Aluminum Alloys. Aluminum is one of the lightest metals used in machine construction. Commercial aluminum is soft and ductile, is about one-third as heavy as steel and about one-fourth as strong as structural steel. It can be strengthened by cold-working. At room temperatures, it has good resistance to the corrosive action of many chemicals.

Aluminum as commonly used is alloyed with copper, silicon, manganese, magnesium, iron, zinc, and nickel. These alloys retain the lightness of aluminum, machine better, are harder, and in the annealed or cast state have strengths up to twice that of commercial aluminum; certain of the alloys having strengths comparable to the structural steels. There are two general classes of alloys, those which can be hardened and strengthened

TABLE 9.—PROPERTIES OF ALUMINUM ALLOYS

Material, Aluminum Company of America Code No.	Ultimate strength, Tension s_t	Ultimate strength, Shear s_s	Yield strength, Tension s_y	Yield strength, Compression s_y	Endurance limit s_{er}	Brinell hardness, 500 kg on 10-mm ball	Elongation in 2 in., %	Remarks and suggested uses
Sand-casting alloys:								
43	19,000	14,000	9,000	9,000	6,500	40	6	High ductility and shock resistance
112	23,000	20,000	14,000	17,000	9,000	70	1.5	Easy machining, general purpose castings
122-T2	25,000	21,000	20,000	20,000	9,500	75	1	Pump cylinders and parts requiring good wear resistance
122-T61	36,000	29,000	30,000	43,000		100	1	
142-T2	27,000	21,000	18,000	18,000	6,500	75	1	Air-cooled engine heads and parts requiring good mechanical properties up to 600 F
142-T61	37,000	32,000	32,000	47,000	8,000	100	0.5	
195-T4	31,000	24,000	16,000	16,000	6,000	65	8.5	Heat-treated castings
212	22,000	20,000	14,000	14,000	8,000	65	2	Good casting qualities, general-purpose castings
Permanent mold castings.								
B-113	29,000	23,000	19,000	19,000		70	1.5	General-purpose castings
A-132-T4	38,000	29,000	30,000	30,000		100	1.5	Gas-engine pistons, has lowest coefficient of expansion of the aluminum alloys
Wrought alloys.								
2SO	13,000	9,500	5,000		5,000	23	45	Low-cost, easily fabricated alloys; most common non-structural alloy; used for cooking utensils and chemical equipment
2 SH	24,000	13,000	21,000		8,500	44	15	
3 SO	16,000	11,000	6,000		7,000	28	40	
3 SH	29,000	16,000	25,000		10,000	55	10	
14 ST	65,000	45,000	59,000		16,000	130	10	Best mechanical properties of the forging alloys
17 SO	26,000	18,000	10,000		11,000	45	22	Corrosion-resisting forgings; most popular heat-treatable alloy used for structural shapes
17 ST	62,000	36,000	40,000		15,000	100	22	
24 SO	26,000	18,000	10,000		12,000	42	22	Aeronautic structural shapes
24 SRT	70,000	42,000	55,000			116		
25 ST	55,000	35,000	30,000		15,000	100	16	Easy forging alloy
32 ST	52,000	38,000	40,000		14,000	115	5	High strength at elevated temperatures
52 SO	29,000	18,000	14,000		17,000	45	30	Higher strength, general-purpose alloy
52 SH	41,000	24,000	36,000		20,500	85	8	
53 ST	36,000	24,000	30,000		11,000	85	16	Forgings for severe corrosion resistance, as in dairies and breweries

Modulus of elasticity in tension, 10,300,000 psi, in shear 3,850,000 psi.
Yield stress taken at 0.2 per cent permanent deformation.
Endurance limit is based on 500,000,000 cycles of reversed stress.
Bearing strength is 1.8 times the tensile strength.

only by cold-working, and those which can be heat-treated to obtain the optimum mechanical properties.

The corrosion-resistant properties of aluminum are chiefly due to the formation of a tough adherent oxide on the surface, which resists further chemical action and which will replace itself if removed. The alloying elements added to increase the strength lower the corrosion resistance. Foods and beverages may be handled in aluminum containers. Sulphur and concentrated nitric and acetic acids do not attack these alloys readily. Hydrochloric acid and many alkalies dissolve the oxide coating, and corrosion becomes rapid. The resistance to corrosion may be increased by anodizing in a sulphuric or oxalic electrolytic bath. The aluminum is made the anode and collects a tough and resistant coating of oxide which is not easily broken off or injured by bending.

The aluminum alloys are designated by an alloy number. Wrought alloys are designated by the letter S following the alloy number. The letter O following the alloy number indicates that all effects of cold-working have been removed by full annealing. Those alloys which can be strengthened only by cold-working have their tempers designated by the symbols $\frac{1}{4}$H, $\frac{1}{2}$H, $\frac{3}{4}$H, and H, indicating the increase in strength by the corresponding spread between the fully annealed (O) condition and the fully hard (H) condition. Those alloys which harden at room temperature after quenching from the solution heat-treatment temperature have the symbol T following the alloy number. Those alloys which must be subjected to artificial aging or precipitation heat-treatment are designated by the symbol W. Alloys which have been strain-hardened after heat-treatment are designated by the symbol RT. Alloys which have properties produced by variations in the heat-treatment and cold-working are designated by the symbol T followed by a number, as T4. Casting alloys are designated by a number or by a number preceded by a letter to indicate a change from the original alloy to facilitate casting. Heat-treated cast alloys are designated by the symbol T and a number following the alloy number.

Aluminum-alloy castings made in sand molds, permanent metal molds, and die-casting machines are generally stronger than the poorer grades of cast iron. Better surface finish, closer dimensional tolerance, and better mechanical properties, together

with the savings in machining and finishing costs, make the use of permanent molds and die castings desirable when the quantity justifies the extra cost of equipment.

23. Heat-treatment of Aluminum Alloys. Aluminum alloys which have been hardened by cold-working may be softened by annealing. As in the case of steels, annealing consists of heating the part to a temperature depending upon the composition, holding at this temperature for a predetermined time, and cooling. The rate of cooling is not so important as it is in annealing steel, but too rapid cooling may cause distortion.

Heat-treatable alloys have their mechanical properties improved by the solution and precipitation processes. The solution treatment consists of holding the part at temperatures and time intervals determined by the alloy analysis and following with a rapid quench. The precipitation or aging treatment consists of holding the part at comparatively low temperature for a period of time determined by the alloy and temperature used. At room temperatures, aging time is about 4 days; at 315 to 325 F, about 18 hr; and at 345 to 355 F, about 8 hr.

24. Magnesium Alloys. Magnesium is a silvery white metal weighing, in the pure state, about 109 lb per cu ft. The usual alloys weigh slightly more, or approximately two-thirds the weight of the aluminum alloys. The common alloys contain from 4 to 12 per cent of aluminum and from 0.1 to 0.3 per cent of manganese; those with more than 6 per cent of aluminum can be heat-treated and aged to increase the yield strength. The alloys are resistant to atmospheric corrosion if kept dry, but when humidity is high, corrosion proceeds slowly with a powder forming on the surface. In very moist or salty atmosphere, or where rain is trapped on the part, surface roughening is pronounced. In general, all magnesium-alloy parts should be given a protective coating.

The magnesium alloys are resistant to attack by most alkalies and by some inorganic substances but are attacked by most acids, both strong and weak. Most aqueous salt solutions corrode these alloys rapidly, brines and chloride solutions being extremely corrosive. These alloys are resistant to pure chromic acid, pure concentrated hydrofluoric acid, alkali metal fluorides, chromates, and bichromates.

TABLE 10.—PROPERTIES OF MAGNESIUM ALLOYS (Mg, Al, Mn, Zn)

Material	Ultimate strength			Yield stress tension	Endurance limit	Brinell hardness	Modulus of elasticity		Elongation in 2 in., %	Remarks and suggested uses
	Tension s_t	Compression s_c	Shear s_s	s_y	s_{er}		Tension and compression E	Shear G		
Sand cast										
A, As cast	24,000	44,000	16,000	10,000	6,000	47	6,500,000		3	
Ht-Tr 1	32,000	44,000	16,000	10,000	6,000	47	6,500,000		8	
Ht-Tr 2	31,000	46,000	17,000	12,000	6,000	52	6,500,000		4	
B, As cast	19,000	45,000	15,000	15,000		69	6,500,000		0	Hard castings, pistons
Ht-Tr. 1	20,000	46,000	16,000	12,000		66	6,500,000		0.5	
Ht-Tr. 3	27,000	52,000	17,000	20,000		82	6,500,000		0	
G, As cast	21,000	47,000	16,000	12,000		52	6,500,000		1	Sand and permanent-mold castings
Ht-Tr. 1	30,000	48,000	18,000	11,000	8,000	49	6,500,000		6	
Ht-Tr 3	31,000	52,000	20,000	17,000	9,000	65	6,500,000		1	
C, As cast	23,000		18,000	14,000	10,000	62	6,500,000		1	Pressure-tight castings
Ht-Tr 1	39,000		20,000	14,000	10,000	61	6,500,000		10	Sand and permanent-mold castings
Ht-Tr 2	38,000		22,000	20,000	10,000	67	6,500,000		3	
Die cast:										
K, as cast	30,000			22,000		68	6,500,000		1	Thin-section die castings
R, as cast	33,000			20,000		66	6,500,000		3	General die castings
Extruded shapes:										
J as extruded	42,000		20,000	27,000	16,000	55	6,500,000		16	Good strength and weldability screw-machine stock
M as extruded	40,000			31,000	10,000	42	6,500,000		6	Best salt-water resistance
G as extruded	50,000		22,000	38,000	16,000	70	6,500,000		9	Highest strength and hardness
Forgings:										
J, pressed	41,000		21,000	25,000	17,000	56			9	General forgings, good ductility
M, pressed	33,000			19,000		43			6	Weldable forgings
L, hammered	37,000		16,000	22,000	10,500	51			6	

Stress values are average. Minimum values are 2,000 to 4,000 psi lower.
Yield stress taken at 0.2 per cent permanent set.
Ht-Tr 1 indicates the solution heat-treatment.
Ht-Tr 2 and Ht-Tr 3 indicate the solution treatment followed by the full aging treatment.
Endurance limit is for 500,000,000 reversals.

Alloy F* is used for forgings, sheet, plate, bars, extruded shapes, and similar items. Alloys E and A may be specified when maximum strength is required and when the forging is not intricate. Alloy M is used for moderately stressed parts where salt-water corrosion resistance is required.

Structural shapes such as channels, beams, angles, bars, rods, and tubes are usually made from alloy F which has good extrusion properties. Alloys E and A are more difficult to extrude, but are sometimes used. Alloy F is also used for sheet and plate in the hard rolled and annealed condition.

Castings made of alloy A, without heat-treatment, are used for moderate stresses and when cost is a deciding factor. Where higher strength, toughness, and shock resistance are required, sand castings of alloy A are used in the heat-treated condition. Alloy G, cast and heat-treated, has higher yield strength and hardness but lower toughness than alloy A. Its properties can be varied by using different heat-treatments. Alloy B has low elongation and toughness but, in the heat-treated condition, has the highest yield strength and hardness and is used where maximum hardness is required. Alloy T should be used only where thermal conductivity is most important. Alloy G is the best die-casting alloy, although alloy M is used when corrosion resistance to salt water and salt atmospheres is more important than mechanical properties.

Magnesium alloys can be welded by experienced operators using the oxyacetylene or oxyhydrogen processes. Electric-resistance welding and spot welding are also possible. The other welding processes are not now commercially practical. The alloys are readily machined, but since powdered magnesium is highly inflammable, dust must be removed rapidly and dull tools which might overheat the chips must be avoided.

The outstanding characteristic of the magnesium alloys is the light weight, and they are therefore used where weight reduction is a major advantage. Some of the uses are automobile and airplane parts, such as crankcases, parts which must be frequently lifted, such as handles and housings of portable tools, drills, grinders, vacuum cleaners, household equipment, foundry flasks, core boxes, match plates, and patterns. These alloys

* Symbols used here are the designations of the Dow Chemical Company, manufacturers of Dowmetal.

are also used for machine parts rotating or reciprocating at high speeds, such as parts of textile, packaging, folding, elevating, and conveying machinery.

25. Heat-treatment of Magnesium Alloys. Two types of heat-treatment are used for castings, the solution treatment and the aging or precipitation treatment. Solution treatment is the holding of the part for 16 to 18 hr at a temperature of about 770 F. This raises the tensile strength, ductility, and toughness. It does not alter the yield strength or hardness. Aging, or precipitation, consists of following the solution treatment by either 4 to 5 hr or 16 to 18 hr at about 35 F, which increases the yield strength and hardness but lowers the ductility and toughness.

TABLE 11.—PROPERTIES OF HAYNES STELLITE

Material	Ultimate strength			Brinell hardness	Remarks and suggested uses
	Tension s_t	Compression s_c	Tension at 1000 F s_t		
No. 1, welded	38,000	256,000	24,000	512	Not machinable; grind
No. 3, cast	38,000	311,000	25,000	600	
No. 6, cast	86,000	220,000	18,000	402	
No. 6, forged	134,000	259,000	23,400	. .	Can be rolled, forged, and punched at 1830 F
No. 12, welded	66,000	193,000	19,000	444	Hard-surfacing weld rod; not used for cutting tools

26. Stellite. Stellite is cobalt base metal alloyed with various proportions of chromium and tungsten with iron present only as an impurity. The most notable property of these alloys is high red hardness. They also have high abrasive resistance, are nontarnishing, corrosion-resistant, and nonmagnetic. The fact that at temperatures between 930 and 1560 F stellite is about 43 per cent harder (measured on the Rockwell C scale) than hardened high-speed steel and retains this hardness under prolonged heating makes it an excellent cutting tool for various machining operations. Stellite tools are suitable for all types of cutting operations on cast iron and malleable iron and for some, but not all, operations on steel. Because of the cost of stellite, the larger tools are made of steel with stellite tips welded on.

Coatings of stellite, applied by welding, are used for surface protection against wear and abrasion on such items as oil-well bits, cement-mill grinding rings, and plowshares and for some bearing surfaces where lubrication is impossible or unreliable The coefficient of friction on dry metals varies from 0.15 to 0.24 with an average of 0.18.

27. Tungsten Carbide. Tungsten carbide is an extremely hard but brittle material. In the granular form it is applied by welding to form an abrasion-resistant surface. In this form it is widely used on the cutting edges of oil-well drag bits, roller-type rock bits, scrapers, excavating shovels, ditch diggers, etc. When ground to a fine powder, mixed with powdered cobalt, pressed into form under high pressures, and sintered to form a solid mass, it is used for the tips of high-speed cutting tools. For some types of cutting tools it is combined with tantalum carbide or titanium carbide.

28. Powder Metallurgy. Certain materials that cannot be alloyed by melting can be formed into useful products by mixing in powdered form, compressing under high pressure, and bonding by sintering. Fine powders of most metals can be obtained by mechanical grinding or crushing, by electrolytic depositing, by chemical precipitation, by reducing oxide powders, or by condensing vapors of the metals. After the powders are obtained, they are intimately mixed and compressed in hard steel dies under pressures up to 100 tons per sq in., depending on the materials. The compressed mass is usually weak mechanically and has a density of about 0.8 of that of the solid material. To increase the strength, the compressed powder is sintered by heating in a neutral or reducing atmosphere to a temperature where the grains will grow together to form a strong solid mass. In some cases a bonding powder is used in the mixture. This powder is melted at the sintering temperature and is absorbed by diffusion to bind the mass together.

By varying the treatment, the porosity of the product can be varied from 3 to 30 per cent by volume. By including powders of materials that may be volatilized by heat, a material full of interconnecting pores is obtained that can absorb and retain oil or other liquid and serve as self-lubricating bearings. Graphite may be incorporated in the powder mixture to act as a lubricating material.

The largest use of these materials is probably for bearings, although they are used for small gears, cams, electric-motor brushes, and sintered cutting tool tips such as tungsten carbide and titanium carbide.

29. Nonmetallic Materials. Intensive chemical research has created a large number of nonmetallic materials that are rapidly being recognized as raw materials in machine design. These synthetic materials are produced with a wide range of properties and many of them have no resemblance to the raw materials from which they are produced. The selection of the proper material for a particular part involves the consideration of mechanical strength, molding and fabrication properties, electrical properties, chemical resistance, heat resistance, dimensional stability, optical properties, or merely decorative properties.

30. Plastics. Many of these synthetic materials are molded into their final form by the application of pressure, either with or without heat, and are therefore grouped under the general term of plastics. Plastics are also fabricated by casting, rolling, extruding, laminating, and machining. There are two general types of plastics: thermosetting plastics, which take permanent shape when placed in a suitable mold and subjected to high pressure and temperature; and thermoplastics, which can be softened by increasing the temperature and molded or pressed into the required shape. The thermosetting plastics when formed by combined heat and pressure are permanently changed in their chemical structure and converted into entirely new substances that cannot be again softened by heat and are extremely resistant to chemical reactions. On the other hand, thermoplastics are not chemically changed on heating; they may be heated and reshaped many times and are subject to chemical attack by any chemical that would attack them before the initial forming. Raw materials for fabrication are available in liquid, powder, solid, and laminated forms. The properties of a few plastics are given in Table 12.

31. Plastic Fillers. To reduce costs and to produce special properties, fillers are added to the raw materials. Fibrous fillers which become entangled in the plastic generally increase the impact strength but lower the tensile strength. When the fibres are aligned, the tensile strength will be greatest in the direction of alignment.

TABLE 12.—PROPERTIES OF PLASTICS

Material	Specific gravity	Tensile strength, s_t	Compressive strength, s_c	Shear strength, s_s	Flexural strength, s_b	Elongation, in 2 in., %	Impact value, ft-lb Izod	Brinell hardness, 500 kg on 10-mm ball	Modulus of elasticity in tension	Coefficient of linear expansion per °F	Maximum operating temperature, °F
Phenolics:											
General purpose	1.40	6,500–8,500	....	.	8,800–13,000	....	0.13–0.20	20–30	880,000–980,000	....	300
Shock-resistant	1.37	6,300–7,100	....	......	8,000–11,000	....	1.90–2.70	.	1,000,000	..	240
Heat-resistant	1.65	4,600–5,500	...	..	8,000	....	0.13–0.16	..	....	....	400
Transparent	1.27	8,000	...	.	16,000	....	0.20	35–40		..	225
Laminated, paper base	1.36	12,000	30,000	10,000	21,000	....	0.6–7.6	40	1,500,000	0.00001	255
fabric base	1.38	10,000	40,000	9,000	20,000	....	1.4–15	38	1,000,000	0.00001	230
Urea-formaldehyde	1.50	5,500–13,000	20,000–30,000		10,000–15,000	....	0.28–0.32	48–50	1,550,000–1,650,000	0.00001–0.00002	175
Cellulose nitrate	1.50	6,000–9,000	7,500–30,000	....	....	4–20	.	10–20	200,000–400,000	0.000065–0.000090	140
Cellulose acetate	1.30	2,800–9,900	7,500–30,000		3,700–18,000	5–40	0.7–4.2	28–105	200,000–350,000	0.000061–0.000091	140–250
Vinyl acetate	1.35	8,000–10,000	....	...	10,000–13,000	1–5	0.30–0.60	12–15	350,000–400,000	0.00004	125
Polystyrene	1.07	5,500–6,500	11,500–13,500	..	6,500–7,500	.	0.20–0.35		400,000–600,000	0.000035–0.000045	150
Vulcanized fibre	1.30	5,000	20,000	4,500	11,000	.	..	8	980,000	0.0000045	110
Borosilicate glass	2.37	9,800	17,900	..	..	...		...	10,400,000		

The principal filler materials are paper pulp, cotton fibers, or macerated fabric for increased shock resistance; chopped canvas and cordage for heavy impact resistance; asbestos for heat resistance, low coefficient of heat expansion, low moisture absorption, and dimensional stability; powdered mica for low moisture absorption, dimensional stability, and low dielectric loss; graphite, with or without wood flour, for bearings and parts requiring fair strength and moisture resistance; carbon for electric conductivity; barytes for chemical resistance and surface hardness; iron for permeability cores; diatomaceous silica for water and chemical resistance; powdered aluminum, brass, and other flaked material for coloring and decorative affects.

32. Molded Plastics. Thermosetting plastics are generally formed by the compression molding process in which raw material in the form of powder is placed in a die and heated while pressure is applied. The continued heat and pressure complete the chemical reaction, or cure, the time required depending upon the type of plastic and the size.

In cold molding, the raw material is compressed to the desired shape in the mold and then removed and heated in ovens for hardening and curing.

In injection molding, used for thermoplastics, the raw material is heated in a cylinder under pressure. It is then injected into a mold which is kept cooled. After the injected material has cooled and hardened, it is removed from the mold in its final form.

Transfer molding is used to produce some intricate shapes of thermoplastics. The raw material is compressed in a cylinder until it becomes fluid. It is then forced through an orifice into the mold where heat and pressure are maintained until the cure is completed.

Some thermoplastics are molded into shape by extrusion, or forcing through properly shaped dies, to form continuous strips which harden by cooling after leaving the die.

33. Cast Plastics. Casting is generally used only for small quantities and where press capacities limit the size of molded parts. Liquid raw plastics must be used since the solid and powdered types cannot be liquefied by the application of heat without pressure. Plain mold casting, centrifugal casting, and dip molding are all used. Molds of latex, plaster, or lead are

used, lead molds being the most common because of low initial cost and the advantage of remelting the molds for repeated use. One-piece and split molds are made by dipping steel patterns in molten lead, the adhering lead being stripped off when cold. The lead molds are filled with liquid plastic and oven-baked from 4 to 10 days. The completed casting cannot be liquefied again.

Some plastics are cast in blocks, partially baked to the consistency of rubber, cooled, sliced into sheets, stacked with wax-paper separators, and baked again while weighted to ensure flatness.

34. Laminated Plastics. Laminated plastics are sheets of resin-impregnated fillers, bonded together by heat and pressure to form a solid body. This material has the physical properties of the filler combined with the chemical properties of the plastic. Rag paper, cloth, canvas, asbestos paper, asbestos fabric, and sometimes woven glass are used for fillers. Laminated materials are made in sheet, rod, and tubular forms.

35. Thermosetting Plastics. When carbolic acid, or phenol, is combined with formaldehyde under definite controlled conditions in the presence of an acid catalyst, a new resinlike material is formed. Furfuraldehyde combines with phenol in the same way. These phenol resins may be softened by moderate heat and are soluble in certain solvents like alcohol. Further heating will cause the material to become permanently hard and very resistant to chemical reactions. These phenol plastics are the most useful general-utility plastics and are produced in the molded, cast, and laminated forms. They are sold under such trade names as Bakelite, Durez, Durite, and Textolite.

Urea is chemically produced from ammonia and carbon dioxide gas. Urea combines with formaldehyde in the presence of an alkaline catalyst to form a resin molding base. Pure wood cellulose, wood flour, and cotton flock are used as fillers. Since the urea plastics are largely used where color effects are important, the filler most used is pure wood cellulose, which gives a clear product desirable for uniform blending of the coloring pigments. These plastics are fairly resistant to heat; they resist the action of weak alkalies but not acids; and they are good electrical insulators but crack on continued exposure to moisture. Since they are tasteless and odorless, they are used for food containers. Urea resins are available as waterproof adhesives

for plywoods. The urea plastics are produced in the molded and laminated forms and are sold under such trade names as Bakelite Urea, Beetle, and Plaskon.

36. Thermoplastics. Cellulose nitrate, or celluloid, was the original plastic, but because of its inflammability, its use was limited. Cellulose nitrate is made by treating wood cellulose, cotton linters, or tissue from pure cotton rags, with sulphuric and nitric acids. This material combined with camphor forms a pliable mass that hardens on drying and may be formed by the application of pressure and heat. These plastics are fabricated by molding, blowing, extrusion, pressing, or machining. Edges of finished parts may be softened by treating with acetone so that parts may be pressed together to form a perfect cemented or welded whole. Cellulose nitrate plastics are available in sheets, rods, tubes, and coils of paper like material. They are sold under such trade names as Celluloid, Nitron, and Pyralin.

Cellulose acetate is made by treating wood cellulose or cotton linters with acetic acid and adding one of several plasticizers. It is similar to cellulose nitrate but is slower burning. This material molds rapidly, has good dielectric properties and good shock resistance, machines readily, becomes brittle at low temperatures, is slightly affected by weak acids, is nonresistant to strong acids, and has poor resistance to water. Cellulose acetate plastics are used for insulation, lacquers, films, rayons, and jewelry, and are sold under such trade names as Bakelite AC, Fibestos, Nixonite, and Tenite I.

Vinyl acetate is made by passing acetylene gas through acetic acid. Vinyl chloride is made by passing acetylene gas through hydrogen chloride. Combinations and derivatives of these resins are used as adhesives for metals, mica, glass, plywood, and porcelain. Some forms are used for gaskets, oil seals, wire insulation, and coatings for good containers. An important application is extruded tubing for flexible oil and gas lines where the temperature is below 175 F.

Styrene, or vinyl benzene, made from ethylene and benzene, is a clear liquid. When polymerized, it forms the clear plastic polystyrene. Polystyrene is crystal-clear, highly transparent, odorless, and tasteless and has high resistance to water and alcohol, high dielectric strength and low power factor, and dimensional stability.

37. Vulcanized Fibre. Vulcanized fibre is made by soaking thin absorbent paper, which is made from hydrolized cotton fibre, in zinc chloride solution. This paper is wound on large heated rolls, the heat causing the fibres to become gelatinous and adhere to each other. The sheets are cut from the rolls and the zinc chloride removed by soaking in water. The sheets are then straightened under heated presses. The resulting material is strong, hard, tough, pliable, and bonelike and may be bent and formed when softened by immersion in either hot or cold water. Vulcanized fibre is available in sheets, rods, and tubes and is used in making gears, cams, rollers, screws, brake shoes, clutch linings, bushings, handling trays, and electrical insulation.

38. The Design of Plastic Parts. The use of plastics has certain limitations, just as do the metals. Selection of the material, as far as its physical and chemical properties are concerned, is similar to the selection of a proper metal. After selecting the material, the method of forming must be considered as it relates to design limitations. Cost of tooling and production, as compared to similar parts made of other materials, must be studied before finally adopting a part made from plastics.

Cast plastics would be considered for relatively small numbers of parts, or for very large parts, since the initial tooling costs would be less than for pressure molding. On the other hand, cast parts may require more machining, which would partially offset the advantage of low initial tooling cost. Castings are made in single straight-draw molds, cored molds, and split molds. A minimum taper of 0.0015 in. per in. is required to permit withdrawal from the mold. Walls of parts should be as nearly uniform in thickness as possible and should have a minimum thickness of $\frac{3}{16}$ in. ($\frac{5}{32}$ in. on small parts), or thicker walls as required by strength considerations. All corners should be rounded or filleted to facilitate casting and to reduce stress concentrations. Large flat surfaces should be mottled to avoid rejections because of color imperfections. In the larger castings, shrinkage must be carefully considered since it amounts to about 1 per cent.

Molding plastics, of which there are hundreds available, have their own molding and design problems, but the following general items apply to all. The part must be designed to permit easy removal from the mold, otherwise tooling and production

costs will be excessive. The minimum wall thickness consistent with strength should be used, with ribs provided to stiffen the part, prevent warping, and act as feeders for the material flow. The minimum wall thickness for the average small part is $\frac{3}{32}$ in., but walls up to $\frac{3}{8}$ in. are used for large parts. All corners should be rounded or filleted, except at the mold parting line. Good practice allows about 3 deg draft in the direction of draw from the mold. Tolerances of ± 0.010 in. per in. are practical in the direction of molding pressure, and ± 0.005 in. per in. is possible. Perpendicular to the direction of pressure, tolerances of ± 0.005 in. per in. are practical and ± 0.002 is possible.

Inserts of metal, fibres, paper, or premolded plastics may be used to strengthen the part or to facilitate assembly. In general inserts should be positioned vertically in the mold and should be surrounded by plastic at least as thick as the insert diameter. Inserts may be anchored by grooving or simply by knurling the contact surface. Either male or female threads of not more than 32 threads per inch can be accurately molded. Since this involves unscrewing the part from the mold or removing a threaded mandrel, it is often cheaper to machine the threads.

39. Glass. Although glass has an extremely high intermolecular strength in both tension and compression, it is usually thought of as a weak brittle material. This weakness in practice is due to internal stresses and defects and to unavoidable surface flaws. The greatly improved mechanical properties of the newer glass compositions justify their consideration in design problems. Glass is used for all parts of centrifugal pumps for acids, piping for chemical processes, pipe fittings, heat exchangers, bearings for heavy shafts in pickling vats, and linings for tanks and fittings. Glass parts can be molded by heat and pressure or by heating and blowing. They can be finished and cut by grinding and can be machined with carbide tools, but the surface will be too rough for ordinary uses. The tensile strength ranges from 6,000 to 12,000 psi with small surface scratches reducing the strength about 50 per cent. Compressive strength is over 100,000 psi. The modulus of elasticity varies from 6,500,000 to 10,000,000 psi. Permissible working stresses range from 500 to 1,000 psi.

40. Rubber. Raw, or crude, rubber is obtained by the coagulation of the latex or milky fluid obtained by tapping certain

tropical trees and shrubs. Raw rubber is used in the pure state for rubber cements, surgical tape, and similar uses. Most rubber used in industry is vulcanized by adding sulphur and heating. Carbon black, zinc oxide, glue, clays, and other materials are added to increase the strength and abrasion resistance. Barytes, whiting, silica, soapstone, and other materials are added to toughen and otherwise modify the product. Oil, tar, and pitch are added to soften and facilitate handling during manufacturing. Soft, semihard, and hard, or vulcanized, rubbers are made by varying the vulcanizing time and the proportion of sulphur and other ingredients. In industry, hard rubber, or vulcanite, is used for electric insulation, switch handles, bearings, etc. Semihard rubbers are used for rubber belting, flexible tubing, hose, springs, vibration controls, and many other devices.

TABLE 13.—PHYSICAL PROPERTIES OF RUBBER AND RUBBER-LIKE MATERIALS

Material	Form	Specific gravity	Compressive strength s_c	Tensile strength s_t	Transverse strength s_b	Hardness, Shore durometer	Maximum temperature for use, °F	Effect of heat	Coefficient linear expansion $\times 10^{-6}$ 32–140 F
Duprene		1.27–3.00		200–4,000		15–95	300	Stiffens slightly	
Koroseal	Hard	1.3–1.4		2,000–9,000		80–100	212	Softens	
Koroseal	Soft	1.2–1.3		500–2,500		30–80	190	Softens	
Plioform	Plastic	1.06	8,500–11,000	4,000–5,000	7,000–9,000		160–248	Softens	
Rubber	Hard	1.12–2	2,000–15,000	1,000–10,000	9,000–15,000	50*–80*	130–160	Softens	35
Rubber	Soft	0.97–1.25		525–600			150–200	Softens	36
Rubber	Linings	0.98–1.35					190	Softens	

* Scleroscope.

41. Synthetic Rubber. Engineers have been using various types of synthetic rubbers for many years. Neoprene, the first used in this country, and the Buna rubbers are the most like natural rubber. The cost of the synthetics has limited their use, except when compounded for particular properties such as resist-

ance to oils, heat, cold, and oxidation. The properties of a few of the hundreds of compounded synthetic rubbers are given in Table 13. Some synthetics, like Thiokol, are thermoplastic, whereas others, like Neoprene and the Buna rubbers, are thermosetting. Both types can be compounded for use in molded products.

42. Carbon. Although carbon has long been used in electrical applications, it has not been generally used in mechanical installations. Low friction losses, low wear rates when operating against metals, and freedom from swelling and warping have led to its use in chemical handling equipment. Pump rotors, vanes, and gears of carbon have been used with good results to replace similar parts of bronze and laminated plastics. Clutch plates and rings of metal and cork have been replaced by carbon parts. Carbon throw-out bearings for automobile clutches are common. The properties* of carbon are shown in Table 14.

TABLE 14.—PROPERTIES OF CARBON

Strength, Psi:	
Compression	2,000–25,000
Tension	400–3,500
Shear	3,200 (max)
Impact	0 10–1.2, Izod ft-lb
Maximum operating temperature	850 F
Hardness	40–135 scleroscope
Modulus of elasticity	1,330,000–2,420,000 psi
Coefficient of friction:	
Carbon on hard steel, unlubricated	0.18–0 30
well lubricated	0 04–0 08
Carbon on glass, unlubricated	0 17

Graphite, a form of carbon, has long been used in bronze bearings as a lubricant, but now bearings made entirely of carbon are being used with operating pressures as high as 1,000 psi at low speeds.

* KORFMANN, F. W., Carbon Offers Solution to Materials Problems, *Machine Design*, May, 1942.

CHAPTER III

STRESSES IN ELEMENTARY MACHINE MEMBERS

The three stresses, tensile, compressive, and shear, determined by the fundamental stress equations in Chap. I are called simple or direct stresses. As indicated, the stresses due to bending forces are, in fact, simple tensile and compressive stresses; but shear stresses are also present, as will be shown later. The principal stress considered in torsion members is shear, but tension stresses are also present. In most machine members, the external loading is such that the designer must deal with combinations of these direct stresses and with certain not-so-apparent induced stresses. The stresses resulting from combinations of external loadings, each of which produces an apparent direct stress, are called combined stresses.

43. Flexure and Bending Stress. Stresses due to direct tension, compression, and shear are indicated by Eq. (1), page 3, but when the loads are applied in such a manner that the apparent stresses are not direct tension, compression, or shear, the equation is not directly applicable. A beam deforms so that the fibers on the convex side are elongated, *i.e.*, in tension, while the fibers on the concave side are shortened, *i.e.*, in compression. Between these extreme fibers there is a neutral surface which is neither elongated nor compressed, and is not stressed. This surface, in a straight beam, passes through the center of area of the cross section of the beam, if the beam is made of homogeneous material. The stress in the beam is proportional to the distance of the fiber from the neutral surface, provided that no fiber is stressed beyond the proportional limit. Texts on mechanics show that the stress due to bending or flexure is given by the equation

$$s_b = \frac{Mc}{I} \tag{5}$$

where s_b = bending stress, psi.
M = bending moment, lb-in.
c = distance from the neutral surface to the fiber where the tress is s_b, in.
I = rectangular moment of inertia of the cross-sectional area about the neutral axis, in.^{4}

This equation does not hold for stresses above the proportional limit, but for purposes of comparison, the bending moment causing rupture may be used in this equation to determine an apparent ultimate strength called the modulus of rupture.

Deflection in beams is, in general, expressed by the equation

$$y = K\frac{FL^3}{EI} \qquad (6)$$

where y = deflection at any point, in.
K = a constant determined by the type of loading and the point at which the deflection is measured.
F = applied load, lb.
L = length, in.
E = modulus of elasticity, psi.
I = rectangular moment of inertia, in.^{4}

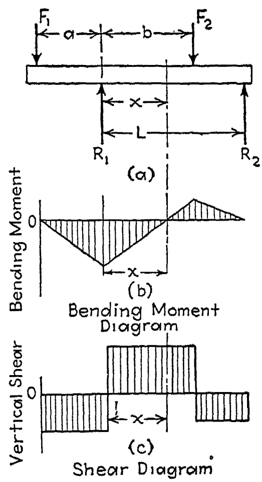


FIG. 4.—Diagrams showing the variation of the bending moment and vertical shear throughout the length of a simple beam.

Values of K for many forms of beams may be found in handbooks or determined from deflection theories as given in texts on structural mechanics.

44. Shear in Beams. Vertical shear in a beam is the algebraic sum of all the forces (components of the loads and reactions), normal to the neutral surface, acting between one end of the beam and the section at which the shear is to be found. The total vertical shear divided by the cross-sectional area of the beam is the average shear at the section but not the maximum shearing stress. Texts on structural mechanics show that the shearing stress varies from zero at the outer fibers to a maximum at the neutral surface, and that the value of this stress is given by the equation

$$s_h = \left(\frac{V}{bI}\right) Ay \tag{7}$$

where s_h = shear stress at distance a from the neutral surface, psi.
V = total vertical shear on the section, lb.
b = width of the beam at distance a from the neutral surface, in.
I = rectangular moment of inertia of the cross-sectional area of the beam, in.^{4}
A = cross-sectional area included from a to the outer surface of the beam, sq in.
y = distance from the neutral surface to the center of gravity of the area A, in.

45. Shear in Torsion Members. A body subjected to the action of two equal and opposite couples acting in parallel planes is said to be in torsion, and the fibers of the body are subjected to shearing stresses that vary in magnitude from zero at the centroidal axis to a maximum at the outer surface. The torsional shearing stress in a member of circular cross section is given by the equation

$$s_s = \frac{Tc}{J} \tag{8}$$

where s_s = shearing stress, psi.
T = torque, lb-in.
J = polar moment of inertia of the cross-sectional area of the body about the axis of rotation, in.^{4}
c = distance from the axis of rotation, or neutral axis, to the stressed fiber, in.

The apparent stress at failure, determined by substituting the torque that causes failure in this equation, is called the modulus of rupture in torsion.

The deformation or angular twist of the body is given by the equation

$$\theta = \frac{TL}{JG} \tag{9}$$

where θ = angular twist in a length L, radians.
T = torque, lb-in.
L = axial length, in.
G = modulus of rigidity, psi.

The equations for shearing stress and angular twist are strictly true only for bodies of circular cross section. The equations may be used for noncircular sections by using a modified value of the polar moment of inertia, suitable values of which are given in mechanical engineering handbooks.

46. Combined Tension and Bending. In Fig. 5 a beam is subjected to an axial load F_t, a bending load F_b, and the reactions R_1 and R_2. At the section BC there is a tension stress s_t, uniformly distributed over the section, equal to F_t/A and represented in magnitude by the length CD.

The load F_b causes a bending stress on the section BC, which varies from zero at the neutral surface to a maximum in tension

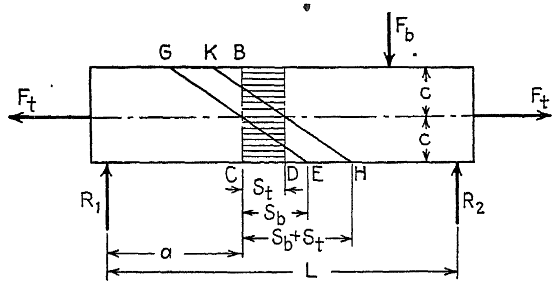


FIG. 5.

at the lower surface, and a maximum in compression at the upper surface. The magnitude of the stress is Mc/I and is represented by the distance between the lines BC and EG.

On the lower surface both stresses are tension and in their effect are added together. On the upper surface the bending stress is compression. The combined stresses at the outer surfaces are given by the equation

$$s = \frac{F_t}{A} \pm \frac{Mc}{I} \tag{10}$$

the plus sign indicating tension and the negative sign indicating compression. The variation of the combined stress across the section is indicated by the line HK.

47. Eccentric Loading. A short member in tension with the external forces acting at a distance e from the center line is shown in Fig. 6. Without changing the conditions of equilibrium, we can add the forces F_1 and F_2, equal and opposite to each other and acting along the center line or neutral axis. If we make F_1 and

F_2 equal to F, then the original force has been replaced by an equal axial force and a couple of magnitude Fe. The couple produces a bending moment in the member, and we have the same condition as in the beam discussed in the preceding article. We then have for the stress in the member

$$s = \frac{F}{A} \pm \frac{Mc}{I} = \frac{F}{A} \pm \frac{Fec}{Ak^2} = \frac{F}{A}\left(1 \pm \frac{ec}{k^2}\right) \tag{11}$$

where k = radius of gyration of the section about the Y axis and c is c_t when the plus sign is used and c_c when the minus sign is used.

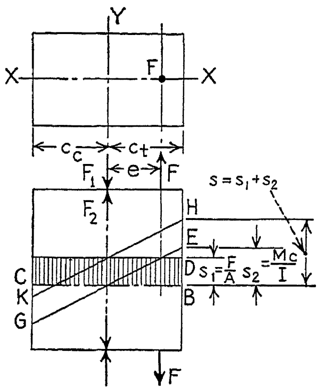


FIG. 6.

If the point of application of the load is not on one of the principal axes of the section, we have eccentricity in two directions. This case is illustrated in Fig. 7. The stress at any corner of this section is given by the equation

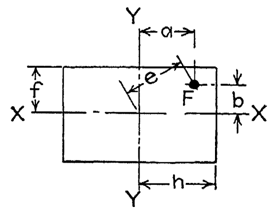


FIG. 7.

$$\begin{aligned} s &= \frac{F}{A} \pm \frac{Fah}{I_y} \pm \frac{Fbf}{I_x} \\ &= \frac{F}{A}\left(1 \pm \frac{ah}{k_y^2} \pm \frac{bf}{k_x^2}\right) \end{aligned} \tag{12}$$

where I_x and I_y = moments of inertia of the section about the X and Y axes, in.^{4}

k_x and k_y = radii of gyration of the section about the X and Y axes, in.

48. Columns. When a body in compression has a length more than ten or twelve times its least dimension perpendicular to the load line, simple compression ceases to be the direct cause of failure. The failure occurs as a result of lateral deflec-

tion, which depends on the modulus of elasticity of the material and the slenderness ratio of the member, as well as the crushing strength of the material. A compression member of this type is called a column or strut. The ratio of the length of the column to the least radius of gyration of the cross section is the slenderness ratio, which ranges from 45 to 130 in columns generally used in machine design. When the slenderness ratio is less than 30 the effect of lateral deformation is negligible, and the member may be considered to be in simple compression.

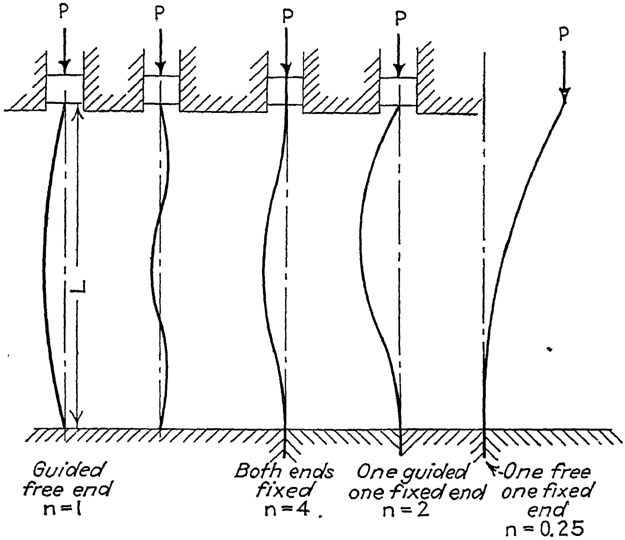


FIG. 8.—Types of columns and the curves assumed by the column axes.

The load-carrying capacity is dependent on the condition of restraint of the column ends. The different types of columns and the approximate curves assumed by the neutral axes when under loads near the failing load are shown in Fig. 8.

49. Euler's Formula. A slender column has a certain critical load at which lateral deflection will occur, increasing without additional load until the column ruptures. In 1757 Euler developed the following rational equation for this critical load.

$$F_{cr} = \frac{n\pi^2 EI}{L^2} = \frac{n\pi^2 EAk^2}{L^2} = \frac{n\pi^2 EA}{\left(\frac{L}{k}\right)^2} \tag{13}$$

where F_{cr} = critical load that will cause rupture, lb.
E = modulus of elasticity, psi.
I = moment of inertia of the cross section, in.^{4}
L = length of the column, in.
A = area of cross section, sq in.
k = radius of gyration of cross section, in.
n = constant depending on the condition of restraint of the column ends:
= 0.25 for one fixed end and one free end;
= 1 for both ends free, *i e.*, pivoted but restrained to axial movement;
= 2 for one fixed end and one end free but guided;
= 4 for both ends fixed.

This equation indicates that as L/k approaches zero, the critical load will approach infinity, and, likewise, the fiber stress will approach infinity. The stress, which does not appear in the equation, should never exceed the yield stress. If both terms of the equation are divided by the factor of safety, $FS = F_{cr}/F = s_y/s$, the equation for the working load is

$$F = \frac{F_{cr}}{FS} = \frac{s}{s_y}\left(\frac{n\pi^2 EA}{\left(\frac{L}{k}\right)^2}\right), \qquad \text{and} \qquad s = \frac{F}{A}\left(\frac{s_y\left(\frac{L}{k}\right)^2}{n\pi^2 E}\right) \tag{14}$$

where s is the permissible stress and s_y is the yield stress in compression.

Experimental evidence indicates that Euler's equation should never be used for steel columns when L/k is less than 150, the critical load indicated being dangerously above the critical load as determined by actual tests on columns.

50. The Rankine Formula. About 1860 Rankine proposed a formula that in modified form is one of the most common formulas used by machine designers. This formula for the stress on the concave side of the column for any load below the critical is

$$s = \frac{F}{A}\left[1 + C\left(\frac{L}{k}\right)^2\right] \tag{15}$$

When s_y is substituted for s, the load F becomes the critical load F_{cr}.

When the load is eccentric, *i.e.*, when it is applied at a distance e from the neutral axis, the formula becomes

$$s = \frac{F}{A}\left[1 + C\left(\frac{L}{k}\right)^2 + \frac{ce}{k^2}\right] \tag{16}$$

where c is the distance from the neutral axis to the extreme fiber, and C is a constant determined experimentally and depending on the material. On account of the great variation in the strength and stiffness of the materials used in machine design, the values of C generally given in texts on strength of materials are not satisfactory. In 1873 Ritter proposed that the constant be given the value

$$C = \frac{s_y}{n\pi^2 E}$$

where s_y is the yield stress in compression and the other symbols have the same meaning as before.

Substituting Ritter's constant in Rankine's formula and solving for the permissible load, we have

$$F = \frac{sA}{1 + \dfrac{s_y}{n\pi^2 E}\left(\dfrac{L}{k}\right)^2} \tag{17}$$

51. The Parabolic Column Formula. The J. B. Johnson parabolic formula is an empirical formula extensively used by aeronautical designers and is now being adopted by machine designers. The critical load as determined by this formula is

$$F_{cr} = As_y\left[1 - \frac{s_y}{4n\pi^2 E}\left(\frac{L}{k}\right)^2\right] \tag{18}$$

where s_y equals the stress at the yield point, psi, and the other symbols have the same meaning as before. This is the equation of a parabola with its vertex at s_y when L/k equals zero, and tangent to Euler's curve when F/A equals $s_y/2$. The value of L/k at the tangent point is the highest slenderness ratio for which the parabolic formula should be used, and the lowest for which Euler's formula should be used. When the value of s_y (outside of the bracket) is made equal to the working stress s_d, this equation gives the permissible load F that may be applied to the column.

This formula, within its range of applications, agrees more closely with experimental results than do the other formulas, and hence is recommended as the correct formula for machine designers.

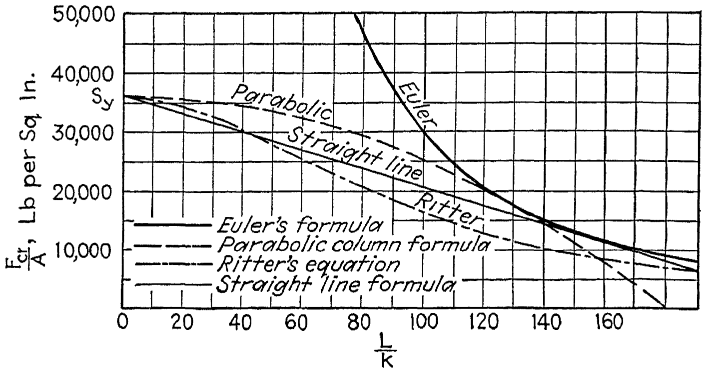


FIG. 9.—Comparison of the critical loads F_{cr}/A as given by the common column formulas.

$s_y = 36{,}000$ psi.
$E = 30{,}000{,}000$ psi.
$n = 1.$

When the load is applied with an eccentricity, e, the formula becomes

$$s = \frac{F}{A}\left[\frac{1}{1 - \frac{s_y}{4n\pi^2 E}\left(\frac{L}{k}\right)^2} + \frac{ec}{k^2}\right] \tag{19}$$

52. Straight-line Column Formulas. Some machine designers have adopted the practice of civil engineers in using straight-line formulas, all of which are empirical in nature. These formulas are easy to use and are satisfactory when used in fields where they have been thoroughly tested by experience. They do not permit simple allowances for differences in steel, new design materials, load eccentricities, and situations peculiar to the design of machinery.

The straight-line formula is the equation of a line which passes through s_y when L/k is zero and is tangent to Euler's curve. It has the form

$$\frac{F_{cr}}{A} = s_y - C\frac{L}{k} \tag{20}$$

where

$$C = \frac{2s_y}{3\pi}\left(\frac{s_y}{3nE}\right)^{\frac{1}{2}} = \text{a constant} \tag{21}$$

Assuming n equal to unity, as is generally the case in machine design, the following straight-line equations are obtained:

For structural steel with L/k less than 130

$$\frac{F_{cr}}{A} = 36{,}000 - 154\frac{L}{k} \tag{22}$$

For good gray cast iron

$$\frac{F_{cr}}{A} = 24{,}000 - 107\frac{L}{k} \tag{23}$$

For aluminum alloys

$$\frac{F_{cr}}{A} = 23{,}200 - 135\frac{L}{k} \quad \text{for 2S-H and} \quad \frac{L}{k} < 115 \tag{24}$$

$$= 28{,}100 - 104\frac{L}{k} \quad \text{for 3S-H and} \quad \frac{L}{k} < 104 \tag{25}$$

$$= 43{,}800 - 350\frac{L}{k} \quad \text{for 17S-T and} \quad \frac{L}{k} < 83 \tag{26}$$

$$= 44{,}000 - 475\frac{L}{k} \quad \text{for 24S-T and} \quad \frac{L}{k} < 75 \tag{27}$$

$$= 36{,}000 - 334\frac{L}{k} \quad \text{for 52S-H and} \quad \frac{L}{k} < 85 \tag{28}$$

All these straight-line formulas are for the critical loads, and a factor of safety must be used to determine the permissible working loads, the factor depending upon the material and the type of loading.

53. Working Stresses for Columns. Failure of a column is determined by the load that produces the critical lateral deformation; not by the ultimate strength of the material. In all the column formulas, except Euler's, the critical load corresponds to a stress equal to the yield stress; hence, the factor of safety may be taken as F_{cr}/F or s_y/s. For steady loading the factor of safety is 2.25 to 2.5 for ductile materials; and 3 to 3.5 for brittle materials. Modifications of the working stress for other conditions of loading are discussed in Chap. IV.

The column formulas that have been discussed apply to shapes that are not subject to local failure. For example, a cylindrical tube of 3 in. outside diameter and $2\frac{31}{32}$ in. inside diameter has about the same cross-sectional area as a cylindrical tube of 1 in. outside diameter and $\frac{7}{8}$ in. inside diameter, and has a radius

of gyration many times as large. The column formulas will indicate that the 3-in. tube will support the larger load, but this is not true. Under comparatively light loads the walls of the thin cylinder will buckle locally, causing failure of the whole. For this reason, hollow steel or aluminum alloy tubes having a ratio of t/d less than 0.03 should never be used as columns. Experimental work on local failures of other shapes is now under way, but results are not conclusive enough to warrant their inclusion here.

54. Induced Stress. Consider the body in Fig. 10 to have unit thickness and to be subjected to external forces F_t producing a direct tension stress s_t on the cross-sectional area. Imagine the body to be cut by a plane CB at any angle θ. If the left

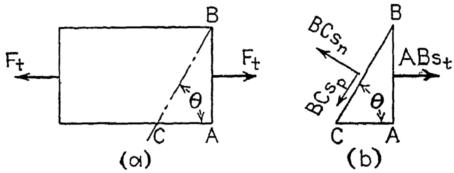


FIG. 10.

part of the body is removed, forces acting normal and parallel to the plane BC must be supplied to hold the right portion in equilibrium. Resolving all forces acting on the right portion into components acting parallel to the plane BC, we have

$$ABs_t \cos \theta - BCs_p = 0$$

from which

$$s_p = (s_t \cos \theta) \frac{AB}{BC} = s_t \cos \theta \sin \theta = s_t \frac{\sin 2\theta}{2} \tag{29}$$

The stress s_p is a shearing stress along the plane BC and will be maximum when $\sin 2\theta$ is maximum, *i.e.*, when θ is 45 deg. We then have for the maximum shear induced in a body by direct tension loading

$$s_{s\,\max} = \frac{s_t}{2} \tag{30}$$

When there are three mutually perpendicular tension forces acting on the body, a similar analysis will show that

$$s_{s\,\max} = \frac{s_1 - s_2}{2} \tag{31}$$

where s_1 and s_2 are the largest and smallest direct tensile stresses. A compressive stress must be considered to be a negative tensile stress.

55. Combined Shear and Tension. Consider the body shown in Fig. 11 with the forces F_t and F_s acting as shown to produce direct tension and shear stresses. Since the body is in equilibrium, shearing forces must also act on the surfaces BC and DA. Taking moments about any point in the body, we have

$$s_s \times AB \times BC = s_2 \times BC \times AB$$

and

$$s_s = s_2$$

showing that the unit shear stresses on the surfaces AB and BC are equal in magnitude.

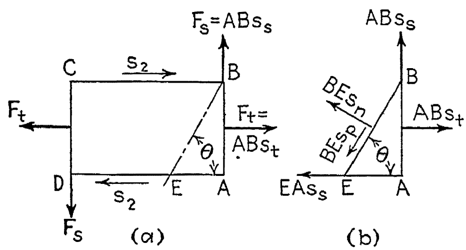


FIG. 11.

If an imaginary plane BE cuts away the left part of the body, forces $s_n \times BE$ and $s_p \times BE$ must be supplied to maintain equilibrium. Resolving all forces acting on the right portion into components parallel to the plane BE, we have

$$ABs_t \cos\theta + ABs_s \sin\theta - AEs_s \cos\theta - BEs_p = 0$$

Dividing through by BE and transposing, we have

$$\begin{aligned} s_p &= s_t \sin\theta \cos\theta + s_s \sin^2\theta - s_s \cos^2\theta \\ &= \tfrac{1}{2}s_t \sin 2\theta - s_s \cos 2\theta \end{aligned} \tag{32}$$

Differentiating with respect to θ and equating to zero, we find that s_p is maximum when

$$\tan 2\theta = -\frac{s_t}{2s_s}$$

$$\sin 2\theta = \frac{s_t}{\sqrt{s_t^2 + 4s_s^2}}$$

and

$$\cos 2\theta = \frac{-2s_s}{\sqrt{s_t^2 + 4s_s^2}}$$

Substituting these values in Eq. (32), and noting that s_p is a shearing stress on the plane BE, we find the maximum shear induced by the external tension and shearing loads to be

$$s_{s\,\max} = \tfrac{1}{2}\sqrt{s_t^2 + 4s_s^2} \tag{33}$$

In a similar manner, we can resolve the forces in Fig. 11 perpendicular to the plane BE and show that the maximum normal stress in the body is

$$s_{n\,\max} = \tfrac{1}{2}(s_t + \sqrt{s_t^2 + 4s_s^2}) \tag{34}$$

56. Theories of Failure. It is evident from the preceding discussions that external loads, which apparently produce simple direct stresses, actually cause induced stresses normal and parallel to all planes passing through the body. These induced stresses may be much larger than the direct stresses and must be considered in discussions of failure.

Since no machine member should be stressed beyond the elastic limit of the material, designers consider failure to occur at the stress that produces a permanent set, or a plastic yielding. Engineers differ as to what really causes failure, and there are different theories advanced, several of which will be discussed at this time.

57. Maximum-normal-stress Theory. In the normal-stress theory it is held that any machine member fails when the stress normal to any plane reaches the yield stress of the material as indicated by a pure tension test. In the case of external loads producing direct tension or compression stresses s_1, s_2, and s_3, acting normal to each other, failure would occur when any one reached the yield stress, the action of the other two stresses having no effect on failure.

In the case of external loads producing both tension and shear, or compression and shear, failure would occur when the maximum normal stress as computed from Eq. (34) reached the yield stress in pure tension.

58. Maximum-shear Theory. This theory, called the Guest or Guest-Hancock theory, holds that failure occurs when the

direct shear or the maximum induced shear, computed by Eqs. (31) and (33), reaches a value equal to one-half the yield stress in a pure tension test. This theory is accepted by the majority of engineers as being applicable to the failure of ductile materials such as iron, soft steel, brass, and aluminum alloys.

59. Maximum-strain Theory. This theory, presented by Saint-Venant, holds that elastic failure occurs when the deformation normal to any plane in the body reaches a value equal to the strain at the yield stress in a pure tension test. It is well known that when a material is elongated or shortened by axial forces, the material also deforms in the lateral direction. The ratio of the unit lateral strain to the unit axial strain is called Poisson's ratio, average values of which are given in Table 15.

TABLE 15.—VALUES OF POISSON'S RATIO m

Material	Poisson's Ratio
Aluminum, cast	0 330
Wrought	0 330
Brass, cast, 66% Cu, 34% Zn	0 350
Bronze, cast, 85% Cu, 7.2% Zn, 6 4% Sn	0.358
Cast iron	0 260
Copper, pure	0 337
Phosphor bronze, cast, 92.5% Cu, 7.0% Sn, 0.5% Ph	0 380
Steel, soft	0.300
1% C	0 287
Cast	0 280
Tin, cast, pure	0 330
Wrought iron	0 280
Zinc	0 210
Nickel	0 310

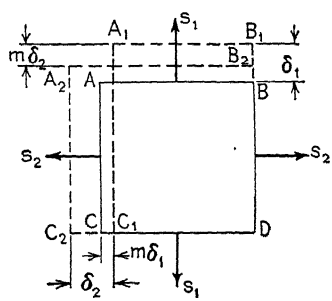


FIG. 12.

In Fig. 12 is shown a body subjected to external forces producing the direct stresses s_1 and s_2 acting normal to each other. Under the action of the stress s_1 alone, the body will assume the shape $A_1B_1DC_1$ elongating an amount δ_1 and contracting in the lateral direction by an amount $m\delta_1$, where m is Poisson's ratio. The application of the stress s_2 will produce an elongation δ_2 parallel to s_2 and a contraction in the direction of

s_1 equal to $m\delta_2$. The net strain in the directions of s_1 and s_2 will then be

$$\delta_1' = \delta_1 - m\delta_2$$

and

$$\delta_2' = \delta_2 - m\delta_1$$

Since stress is unit strain times the modulus of elasticity, we have

$$s_1' = E\delta_1' = E(\delta_1 - m\delta_2) = s_1 - ms_2 \qquad (35)$$
$$s_2' = s_2 - ms_1$$

The stresses s_1' and s_2' are called equivalent stresses, and, according to the maximum-strain theory, when either becomes equal to the yield stress in pure tension or compression, the material will fail. It is evident that a tension stress s_2 will allow an increase in the direct tension s_1 before failure will occur, whereas a compressive stress will cause failure at a lower stress in the direction of s_1.

When there are three direct and mutually normal stresses acting on the body, the equivalent stresses become

$$s_1' = s_1 - ms_2 - ms_3, \text{ etc.} \qquad (36)$$

Applying the same line of reasoning to the case of external forces producing both direct tension and shear, Eq. (34) becomes

$$s_{t\,\max}' = \tfrac{1}{2}[(1 - m)s_t + (1 + m)\sqrt{s_t^2 + 4s_s^2}] \qquad (37)$$

60. Strain-energy Theory. When a body is stressed, the product of the average stress applied during elongation times the corresponding elongation represents energy absorbed by the body. The amount of this strain energy absorbed per unit volume is sometimes used to determine the stress limit at which failure occurs. This theory is useful in certain analytical work but is not used to any great extent in general machine design.

61. Choice of the Theory of Failure. An examination of the fracture of tension specimens loaded to destruction shows that ductile materials begin to fail along lines at angles of approximately 45 deg with the load axis. This indicates a shear failure. At the yield stress, the yielding of the material by shear is also indicated by the appearance of Lüders lines, showing slip on an oblique plane. Brittle materials, on the other hand, rupture

on planes normal to the load axis when subjected to pure tension, indicating that the maximum normal stress determines failure. In compression, brittle materials also fail by shear on oblique planes, indicating that the shear strength of brittle materials is greater than the tensile strength but less than the compressive strength.

The maximum-normal-stress theory is now generally accepted for brittle* materials, and the maximum-shear theory for ductile materials. Experimental evidence does not show that the maximum-strain theory consistently determines the failure condition, but it does give very accurate results for certain load combinations and is widely used in the design of thick cylinders for hydraulic presses and in the design of large-caliber guns.

62. Temperature Stresses. Most substances expand or increase in volume as their temperature is raised. If the body is free to expand without hindrance, no stress will be set up; but if the tendency to expand is resisted by adjoining bodies or machine members, stresses will be set up. These temperature stresses are of great importance in the design of large internal-combustion engines, steam machinery, furnaces, and similar structures.

The linear expansion due to a temperature change is

$$e = c(t_2 - t_1)L \tag{38}$$

where e = total increase in length, in.
c = coefficient of linear expansion, in. per in. per °F.
$t_2 - t_1$ = change in temperature, °F.
L = original length, in.

If the member is held so that it cannot expand, the effect is the same as though a compressive force is applied of sufficient magnitude to produce a compression of e in. Then the stress set up by the temperature change is

$$s = E\frac{e}{L} = cE(t_2 - t_1) \tag{39}$$

If two materials having different coefficients of expansion are rigidly fastened together throughout their length, and then

* A material may be assumed to be brittle if its total elongation at rupture is less than 5 per cent in a specimen 2 in. long.

subjected to a temperature increase, they will tend to expand different amounts; however, they must expand equally, and the material having the higher coefficient of expansion will be subjected to compression stresses, whereas the other material will be in tension. The composite part will assume a curvature, the amount of which will depend upon the temperature change. This principle is used in the manufacture of metallic thermometers and in the design of temperature-control apparatus.

63. Rubber in Compression. When under load, rubber acts like an incompressible fluid which flows from regions of high

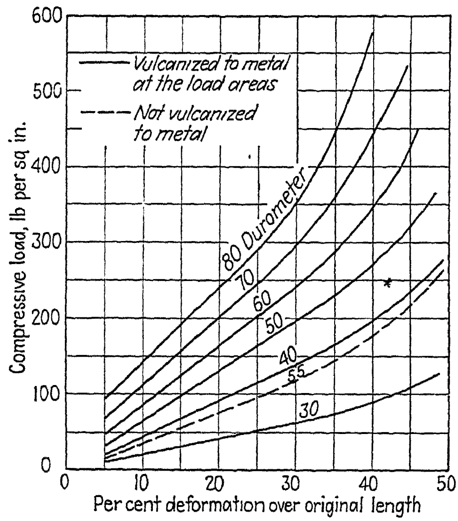


FIG. 13.—Load-deformation curves for a 1-in. cube of rubber.

pressure to regions of lower pressure wherever available. Adhesion to contacting surfaces and internal stresses resist this flow and make it necessary to consider the type and area of the contact surface and the geometrical proportions of the rubber member when determining deformations.

Deformation does not increase in direct ratio to the applied load, and the relationship differs with the various rubber compounds. The load-deformation curves in Fig. 13 show that below 30 per cent deformation, the deformation is almost a straight-line function of the load, and that beyond this deformation the stiffness increases rapidly, contrary to the action of most

metals. The modulus of elasticity for metals is the stress which would produce an elongation of 100 per cent if the stress-strain curve remained straight. With rubber, the modulus is taken as the stress-strain ratio at 25 per cent deformation, based on the unloaded dimensions of the part. Since the deflection varies with the shape, proportions, and durometer hardness, the basic values of E and G must be referred to some definite basis, usually a 1-in. cube of 55 durometer hardness. An experimentally determined form factor applied to the basic values give the values of the modulus to be used for any particular design problem. Deformation of rubber parts takes place with the total volume remaining constant, so that Poisson's ratio is practically 0.5, and this value used in Eq. (4) shows that the modulus of elasticity E is three times the modulus of rigidity G.

As noted, the load-deformation characteristics of rubber compression members are greatly affected by shape. Considering rectangular shapes only, the following have been found to be approximately true. The percentage deformation will increase directly as the height or length of the compression test specimen because the rubber has a greater chance to bulge at the middle, permitting the almost incompressible rubber to have an increased relative deformation in the direction of the load. Experiments indicate that for a constant load area, the percentage deformation increases as the $\frac{2}{3}$ power of the length, *i.e.*, as $L^{\frac{2}{3}}$. When the load area is increased, relative shape remaining constant, the percentage deformation decreases inversely as the square root of the area, *i.e.*, as $1/A^{\frac{1}{2}}$. A long narrow strip will deform more than a square section of the same area, thickness, and hardness. Calling K_R the form factor or ratio of the long side to the short side, the percentage deformation varies as $K_R^{\frac{2}{3}}$. The values of E and G increase with the durometer hardness approximately as shown in Fig. 14.

The load deformation curve for a 1-in. cube of 55 durometer rubber is shown in Fig. 13. Using this as a standard of comparison and considering the relationships given above, the deformation y of any rectangular shape may be found from the following equation with an error of less than 5 per cent:

$$y = \frac{(LK_R)^{\frac{2}{3}}}{A^{\frac{1}{2}}E}(E_{55}y_{55}) \qquad \text{for rectangular members} \qquad (40)$$

where y = deformation, in.
L = length, in.
E = modulus of elasticity, psi.
E_{55} = modulus of elasticity of 55 durometer rubber, psi.
y_{55} = deflection of a 1-in. cube of 55 durometer rubber, in.

Rectangular slabs with holes punched in them in the direction of the load will deflect much faster than solid slabs having the

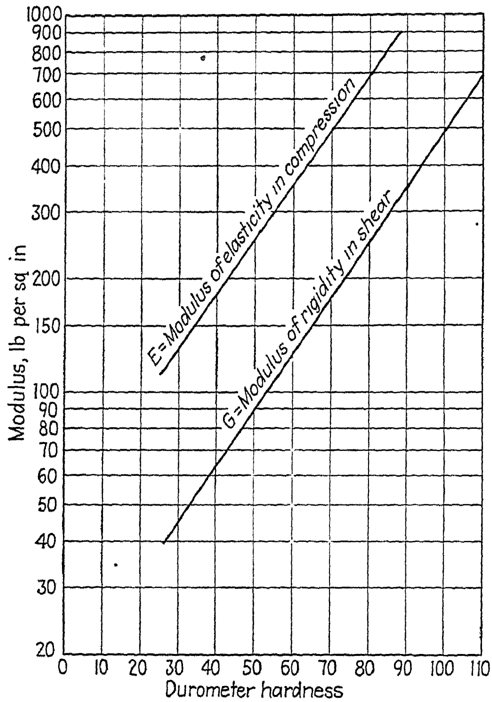


FIG. 14.—Variation of modulus of elasticity and rigidity of rubber.

same load area because of the greater area available for lateral expansion. Form factors for each application must be determined since insufficient data are available to establish general form factors for Eq. (40).

The percentage deformation of cylindrical members in compression is expressed by the equation

$$y = \frac{K_c}{A^{\frac{1}{2}}E} (E_{55}y_{55}) \qquad \text{for cylindrical members} \qquad (41)$$

where K_c is the form factor corresponding to the ratio of the loaded circular areas to the total surface area. This factor is given in Fig. 15.

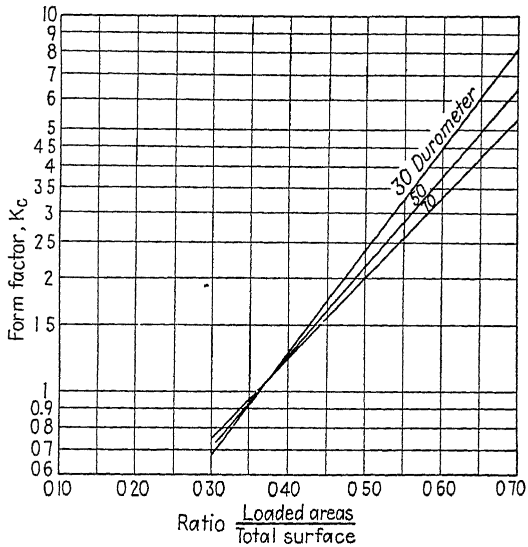


FIG. 15.—Form factors for cylindrical rubber compression members. For use in Eq. (41).

64. Rubber in Shear. From the definition of the modulus of rigidity G, it is evident that

$$G = \frac{\text{stress}}{\text{strain}} = \frac{F}{A\theta} \quad \text{or} \quad \theta = \frac{F}{AG} \quad \text{in radians} \tag{42}$$

where F = applied load, lb.
A = cross-sectional area parallel to the applied load, sq in.
G = modulus of rigidity from Fig. 14, psi.
θ = angular deformation, radians.

When the shear loads act parallel to each other, as in Fig. 16, the linear deflection can be approximated very closely by the following equation when the angular deformation is less than 30 deg.

$$y = t\theta = \frac{Ft}{AG} \quad \text{approximately} \tag{43}$$

where y is the linear deflection and t is the distance between the shearing forces, both in inches.

Referring to Fig. 17, it is noted that with rubber bonded between concentric cylinders, the shear areas and the shear stresses vary from the outer to the inner surfaces of the rubber. In this case the deformation can be determined from the equation

$$y = \frac{F}{2\pi G}\left(\frac{D_o - D_i}{L_o D_i - L_i D_o}\right) \log_e \frac{L_o D_i}{L_i D_o} \tag{44}$$

65. Rubber in Torsional Shear. Rubber vulcanized between outer and inner sleeves is frequently used as a torsional spring

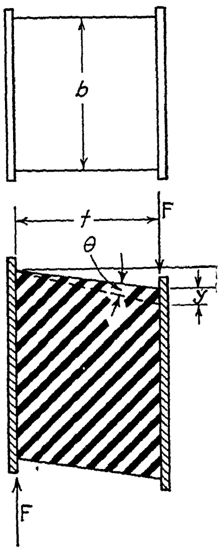


FIG. 16.—Rectangular rubber shear mounting.

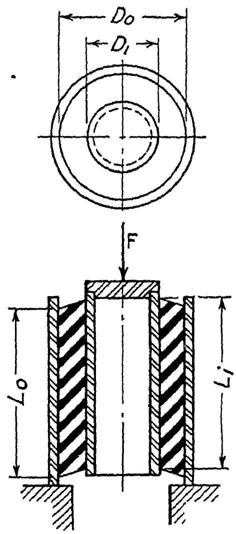


FIG. 17.—Cylindrical rubber shear member.

or as a vibration damper, as illustrated in Fig. 18. The angular deformation of these rubber bushings is expressed by the equation

$$\theta = \frac{T}{\pi L G}\left(\frac{1}{D_o^2} - \frac{1}{D_i^2}\right) \tag{45}$$

where θ = angular deformation, radians.
T = applied torque, lb-in.
L = effective length of bushing, in.
G = modulus of rigidity, psi.
D_o = outside diameter, in.
D_i = inside diameter, in.

In installations of this nature, the applied load will cause the center of the inner cylinder to be deflected radially by an amount approximated by the equation

$$y = \frac{s_c}{12E} \tag{46}$$

where y = radial deflection, in.

s_c = compressive stress on projected area of inner cylinder, psi.

E = modulus of elasticity in compression, psi.

Equation (46) applies to rubber bushings having a wall thickness less than $D_i/3$ and a length greater than $4D_i$.

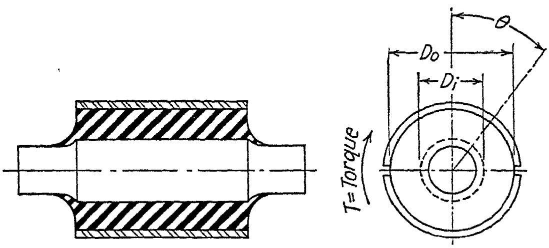


FIG 18.—Torsion bushing.

66. Design Stress for Rubber Members. Compression members with load areas vulcanized to metal are designed with stresses from 75 to 600 psi, depending upon the particular composition used.

In parallel shear spring units with the rubber bonded to metal plates, 25 psi maximum for continuous loading and 35 psi for variable cyclic loading is considered satisfactory. For torsional shear, 50 psi for continuous loading and 70 psi for wide variations of torque are satisfactory. Durometer values from 30 to 60 are used in shear.

67. Miscellaneous Properties of Rubber. When rubber cools from the vulcanizing temperature, it shrinks about $\frac{3}{16}$ to $\frac{1}{4}$ in. per ft, and when vulcanized to metal shells, the inner and outer shells should both be split to permit this shrinkage to take place freely.

High temperatures and oil rapidly deteriorate rubber, and 150 F is usually considered to be the maximum operating temperature, although 200 F is sometimes permissible. Temperatures as low as -10 F are permissible. The stiffness varies with the

temperature and the change in deformation due to temperature may be estimated by applying the factors from Fig. 19 to the deformations obtained from Eqs. (40) and (41).

A change in dimensions occurs with the first few successive applications of load after vulcanizing. Rubber, after vulcanizing, also undergoes a change in deformation properties called creep.

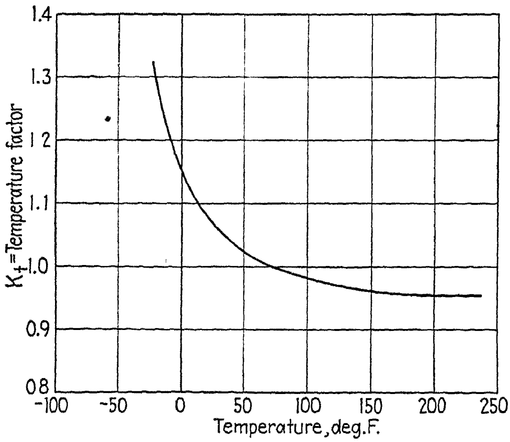


FIG. 19.—Temperature correction factor. Multiply deformation from Eqs. (40) and (41) by this factor to obtain deformation at any given temperature.

The creep depends upon the magnitude and time of application of previous load applications and is usually high for the first few load applications and decreases with subsequent loading. These and other characteristics of rubber are not well enough established to justify definite figures to be used in design, and the rubber manufacturers should be consulted before adopting new designs.

CHAPTER IV

DESIGN STRESS AND FACTOR OF SAFETY

68. General Discussion. The selection of the design stress to be used in computing the size of a machine member is one of the most important problems to be met by the designer. At the same time, it is one of the most vague and difficult problems. The experienced designer arrives at the design stress by an analysis of the service conditions that must be met, by an evaluation of the strength of the material under service conditions, and by deciding on the margin of safety that he desires between service conditions and actual failure.

Service conditions include such factors as the magnitude of the loads applied; the method of load application, whether steady, variable, or subject to impact; the type of stress: tension, compression, shear, or combined; and the temperature at which the member is to operate. Other factors such as appearance, the use of standard parts, ease of assembly, ease of repair, and the wearing qualities must also be considered; and in many cases these may be so important that the problem of strength may be given only minor consideration.

Tensile and fatigue tests are the ones from which failure can be most readily predicted. Other tests provide information that can be used to supplement these. Failure may mean actual rupture, a sudden increase in the strain without an increase in stress, a stress producing a permanent strain, or a stress exceeding the proportional limit.

For ductile materials, the yield stress is usually considered to be the criterion of failure. When there is no well-defined yield point, the stress corresponding to a permanent elongation of 0.2 per cent is accepted as the yield stress. For brittle materials, the breaking stress (ultimate strength) is the criterion of failure. Ductile materials subjected to fluctuating stress behave like brittle materials so far as failure is concerned, and the endurance limit is the criterion of failure.

Lack of stiffness or rigidity may be considered to constitute failure, in which case stress may be relatively unimportant. For example, if the deformation of a steel member is to be limited, it may not be advisable to use the high-strength alloy steels, since the modulus of elasticity, which is the measure of rigidity, is practically the same for all steels. In order to use advantageously the high strength of the alloy steels, higher unit stresses must be used, and the deformation will increase in direct proportion to the stress. If, when the member is proportioned to give the permissible deformation, the stress is within the limits permitted with the lower strength steels, then the plain carbon steels should be used unless alloy steels are required by other considerations.

69. Design and Working Stress. The design stress is the stress value which is used in the mathematical determination of the required size of the machine member. It may be considered to be the stress that the designer hopes will not be exceeded under operating conditions. When the properties of the material are definitely known and when the stress can be accurately determined, this stress may be as high as 80 per cent of the yield stress, but 50 per cent is the usual value for nonshock and nonfatigue conditions. The working stress, as distinguished from the design stress, is the stress actually occurring under operating conditions.

The stress equations developed in texts on mechanics and strengths of materials are directly applicable only when the forces on the machine member can be accurately determined and analyzed. In service, however, many machine members are subjected to load combinations that do not permit accurate stress determinations, and more or less satisfactory approximations must be used. The designer bases his computations on the principal or predominating stress and makes allowances for minor stresses by an intelligent modification of the permissible or design stress.

When the stress varies across the section, the maximum value should be the one controlling the design, except in cases of high local stress in ductile material under static load conditions. The high local stress, if reaching the elastic limit, will cause local plastic flow of the material at the section with a consequent readjustment of the stresses without yielding or failure of the entire member. Because of this readjustment, high local stresses

are not serious under static loading, and the mean stress may be used to control the design. Note that this statement does not apply to shock or fatigue loading conditions. Under these conditions, the existence of high local stresses requires serious consideration and careful design.

For ductile materials, it is generally accepted that failure occurs according to the maximum-shear theory, *i.e.*, when the maximum shear stress is one-half the yield stress in tension. Hence, for cases of combined stress, we have from Eq. (33)

$$2s_{s\,\max} = \sqrt{s_t^2 + 4s_s^2} \leqq s_d \tag{47}$$

where s_d = design stress, or tensile stress which is not to be exceeded, psi.

s_t = apparent direct tensile stress, psi.

s_s = apparent direct shear stress, psi.

$s_{s\,\max}$ = maximum combined shear stress due to the combined effect of s_t and s_s, psi.

For brittle materials, the maximum-normal-stress theory applies, and, for cases of combined stress, we have from Eq. (34)

$$s_{t\,\max} = \tfrac{1}{2}(s_t + \sqrt{s_t^2 + 4s_s^2}) \leqq s_d \tag{48}$$

70. Factor of Safety. When machines were operated at low speeds, their design required only a knowledge of simple statics and kinematics. Designers developed a highly perfected judgment of static stresses, and the use of an apparent factor of safety to cover the ignorance of the real stresses was not serious. The increasing use of high speeds and the increasing size of individual machines have introduced dynamic problems which cannot be handled in such a simple manner and the determination of the design stress is becoming more dependent on critical analysis of the type of loading and the stress distribution.

The conventional method of determining the design stress to be used in computations is to divide the ultimate strength of the material by a factor of safety. Designating this factor by the symbol *FS*, we have

$$\text{Design stress} = s_d = \frac{s_u}{FS}$$

For convenience in arriving at the proper factor of safety, it may be considered to be the product of several independently deter-

mined factors. Thus

$$FS = a \times b \times c \tag{49}$$

and

$$s_d = \frac{s_u}{a \times b \times c} = \frac{0.8 s_y}{b \times c} = \frac{0.8 u s_y}{b}$$

where a = elastic ratio.
b = shock or service factor.
c = real margin of safety.
u = utilization factor applied to yield stress = $1/c$.

Each of these factors will be discussed in the following articles. The factor 0.8 is used with s_y since the elastic limit is slightly lower than the yield stress and to allow for uncertainty in the value of the yield stress.

71. Factor *a*. The Elastic Ratio. The factor a is the ratio of the ultimate strength to the elastic limit (usually taken as the yield stress), when static loads are considered, and is used to reduce the design stress below the stress at which permanent set would occur. When the load is variable and applied in repeating cycles, fatigue effects must be considered and the factor becomes the ratio of the ultimate strength to the endurance limit. The determination of the endurance limit is discussed in Art. 74.

72. Factor *b*. The Shock Factor. The factor b is the ratio of the stress produced by a shock or suddenly applied load to the stress produced by the same load when gradually applied. Texts on mechanics show that the stress produced by an impact load is

$$s_i = \frac{F}{A}\left(1 + \sqrt{1 + 2\frac{h}{y}}\right) = s\left(1 + \sqrt{1 + 2\frac{h}{y}}\right) \tag{50}$$

where F = applied load, lb.
A = cross-sectional area, sq in.
y = deformation produced by load F under static conditions, in.
h = height of free fall to produce the velocity of impact, in.
s_i = stress produced, psi.
s = static stress, psi.

Since s is the static stress, the factor b for Eq. (49) is

$$b = \left(1 + \sqrt{1 + 2\frac{h}{y}}\right)$$

For torsional loads h and y must be expressed as the impact travel and angular deformation in radians.

When the load is steady or gradually applied, the factor b will be unity. When the load is applied suddenly without impact or initial velocity, h becomes zero and the factor b will be 2, indicating that suddenly applied loads are twice as severe as the same loads under static conditions. Equation (50), however, is based on the assumption that the supporting members are absolutely rigid, which is never true in an operating machine. Since the supporting members deform and absorb some of the shock energy, the stresses are less than indicated by the equation. The actual magnitude of impact stresses can be accurately determined only in very simple cases such as longitudinal impact on rounded-end bars, impact of elastic spheres, and similar arrangements. It is therefore necessary to rely on experimental and service data in modifying the value of the factor b as computed. When impact causes a series of oscillations about a mean value that represents the mean stress, the factor b may be estimated by comparing the maximum oscillation with the mean value or static strain. Wheel loads moving on tracks are examples of this type of loading. Average values of the factor b are 1.25 to 1.50 for light shock; 2 to 3 for very heavy shock; and in a few extreme cases as high as 5.

73. Factor *c*. The Real Margin of Safety. The factor c is the real margin of safety which the designer uses to allow for unreliable material, such as dirty or seamed steel, castings subject to blowholes, and shrinkage stresses; the probable effect of unknown, unforeseen, or accidental overloads; temperature effects; the probable accuracy of the mathematical analysis; stresses encountered during machining, assembly, shipping, or intentional overloading during acceptance tests; the chances of failure leading to injury, loss of life, costly shutdowns, and expensive repairs. Average values of the factor c are 1.5 to 2 for ductile materials of uniform structure, such as iron and steel, 2.5 to 3 for cast iron and other brittle materials, and 3 to 4 for timber.

74. The Endurance Limit. It has long been known that stresses far below the ultimate strength of the material, as determined by a single-load application, will cause rupture if repeated a sufficient number of times. This so-called fatigue

failure is caused, not by crystallizing of the material as was formerly believed, but by the progressive growth across the section of a minute crack formed in the material at some point of highly localized stress. The point of localized stress may be at a

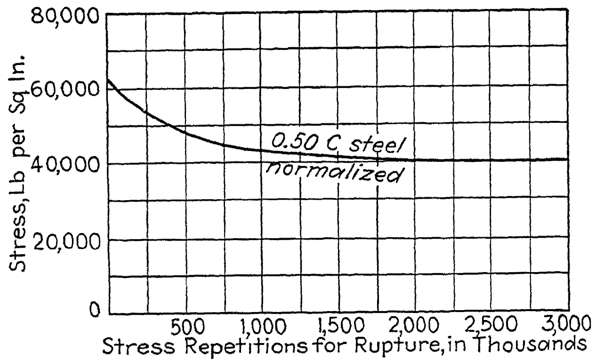


FIG. 20.—Typical curve showing relation between stress and number of repetitions to cause rupture.

small flaw in the material or at some scratch or discontinuity of the surface.

The relation between the applied stress and the number of stress repetitions required to produce rupture is represented by a curve similar to that shown in Fig. 20. This curve replotted to

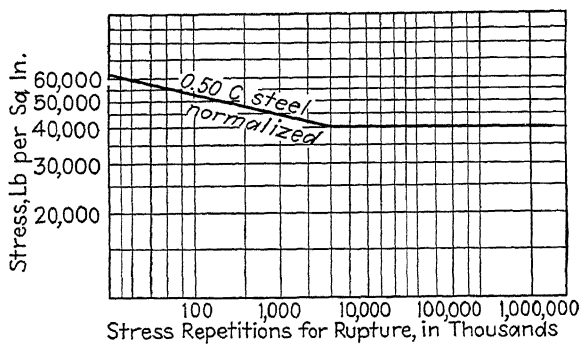


FIG. 21.—Typical logarithmic plot of stress-repetition curve.

logarithmic scales gives a straight-line relation between stress and stress repetitions, as shown in Fig. 21. If the stress is reduced very slightly below that corresponding to the break in the curve, it is found that the material shows no signs of failure after 100,000,000, or more, repetitions. The stress

corresponding to the break in the curve is called the endurance limit, and it is generally assumed that when the applied stress is below this value, the material will withstand an indefinite number of stress repetitions without failure. This idea is not strictly true, as will be shown later. The number of stress repetitions required to determine the endurance limit is about 5,000,000 for wrought ferrous materials, about 10,000,000 for cast ferrous materials, about 2,000,000 for wood, less than 1,000,000 for magnesium alloys, and up to 500,000,000 for some aluminum alloys.

The endurance limit does not seem to be altered by the rate of stress repetition below 5,000 cycles per min; higher speeds increase the endurance limit slightly. The endurance limit may be raised slightly by cold-working the surface, as by cold-rolling, burnishing, or shot blasting, and by nitriding, case-hardening, or by any method for producing a thin layer of high-strength material on the outer surface.

Tabular values for the endurance limit usually refer to complete reversals of stress in bending. In reversed axial loading, the limit is about 70 per cent of that in reversed bending. When the endurance limit has not been definitely determined, it may be assumed to be $0.50s_u$ for ferrous materials, and from 0.25 to $0.35s_u$ for nonferrous materials.

75. Endurance Limit for Cast Iron. The fatigue properties of cast iron are not so well known as those of steels. Cast iron, unlike steel, is many times stronger in compression than in tension under steady stress. A similar condition exists when cast iron is subjected to various combinations of tension and compression in repeated stress cycles, tests made at the University of Illinois indicating that the endurance limit in completely reversed bending is approximately 35 per cent of the ultimate strength in tension, and 10 per cent of the ultimate strength in compression. When the stress cycle is entirely compressive, the endurance limit is about four times the endurance limit when the stress cycle is entirely tensile.

Professor Kommers suggests that when tension is numerically the maximum stress in any stress cycle, the maximum rupture stress may be found by the formula

$$s_e = \frac{3s_{er}}{2 - \frac{s_{min}}{s_{max}}} \tag{51}$$

and when compression is numerically the greater

$$s_e = \frac{6s_{er}}{1 - \dfrac{5s_{min}}{s_{max}}} \tag{52}$$

where s_e = endurance limit, psi.

s_{er} = endurance limit for completely reversed stress, psi.

s_{max} = maximum stress imposed, psi.

s_{min} = minimum stress imposed (compressive stress being considered as a negative tensile stress), psi.

76. Endurance Limit in Torsion. The endurance limit in reversed torsion is about 55 per cent of that in reversed bending. When the torsional stress is not completely reversed, the endurance limit is given by the formula

$$s_{es} = \frac{2s_{ers}}{1 - \dfrac{s_{min}}{s_{max}}} \tag{53}$$

where s_{ers} is the endurance limit in completely reversed torsion.

77. Design Stress for Variable and Repeated Loads. Since the endurance limit is the real ultimate strength in members subjected to high numbers of stress repetitions, it should replace the ultimate strength or yield stress in determining the design stress. Equation (49) then becomes

$$s_d = \frac{s_e}{b \times c \times f \times K} \tag{54}$$

where s_e is the endurance limit for the particular range of stress variation as determined in the accompanying articles, b and c have the same meaning as in Eq. (49), f is the surface-finish factor, and K is the stress concentration factor. The factors f and K are discussed in succeeding articles. In no case should the value of s_e used in this equation be higher than the yield stress of the material under static load conditions.

When the stress is not completely reversed but varies from a maximum in tension (or bending) to a lower value in tension, to zero stress, or to a compressive stress, the endurance limit will be higher than that obtained with complete stress reversals. Experimental evidence indicates that the endurance limit of wrought ferrous materials for any stress variation may be found

from the formula

$$s_e = \frac{3s_{er}}{2 - \dfrac{s_{\min}}{s_{\max}}} \tag{55}$$

where the symbols have the same meaning as in Eqs. (51) and (52).

Although it is generally assumed that a member will never fail if stressed below the endurance limit as found by the break in the curve in Fig. 21, more recent research indicates that beyond this point the endurance limit decreases according to the relation

$$s'_e = s_{er}\left(\frac{N_{er}}{N'_e}\right)^{0.139} \tag{56}$$

where s'_e = stress that will cause rupture at N'_e stress repetitions, psi.

s_{er} = usual value of the endurance limit in reversed stress, psi.

N_{er} = number of repetitions corresponding to the break in the curve or the stress s_{er}.

For wrought ferrous materials, N_{er} may safely be taken as 5,000,000 Using this value and substituting s'_e from Eq. (56) for s_{er} in Eq. (55), we have

$$s_e = \frac{25.6s_{er}}{(N'_e)^{0.139}\left(2 - \dfrac{s_{\min}}{s_{\max}}\right)} \tag{57}$$

For stress repetitions up to 500,000,000, the endurance limit may be determined from Eq. (55); for greater life Eq. (57) should be used.

78. Factor *f*. Surface Finish. Under static or steady load conditions, the surface finish has little effect on the strength, and the factor *f* in Eq. (54) may be taken as unity.

Fatigue tests are usually made on specimens with highly polished surfaces to eliminate the effect of surface defects and obtain results which may be directly compared with other fatigue data. Surface imperfections, such as mill scale, file and tool marks, notches, and grooves, act as stress raisers and greatly reduce the apparent endurance strength. The curves in Fig. 22 indicate values of the factor *f* for various surface conditions.

79. Factor K. Stress Concentrations. In machine members which are subjected to dynamic loads of a cyclic nature, local stresses and fatigue effects become important and any discontinuities, such as abrupt changes in size, holes, notches, keyways, and small fillets acting as stress raisers, must be considered in determining the factor of safety and the design stress. Thus the

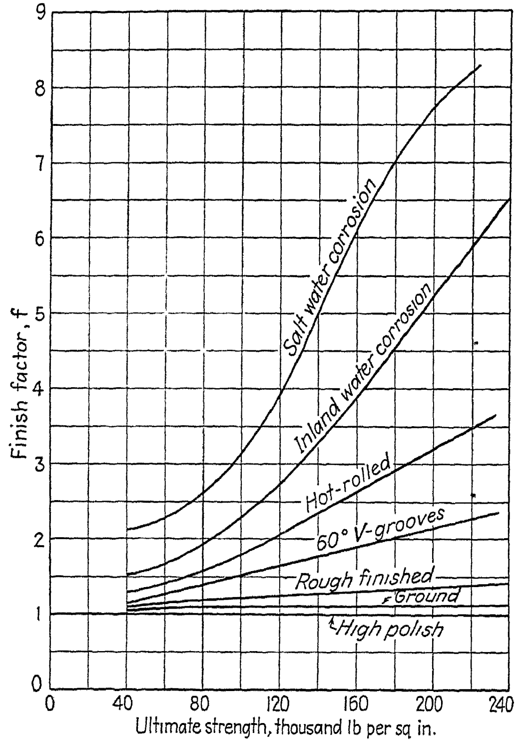


FIG. 22.—Finish factor f.

factor K in Eq. (54) must be estimated. Mathematical analysis based on the theory of elasticity, and experimental analysis by means of photoelastic studies, soap films, brittle varnish, coatings, etc., have, in many cases, indicated the magnitude of these stress concentrations, and values of the stress concentration factor K derived from these analyses are given in the following paragraphs.

A circular hole in a flat plate subjected to tensile or compressive stresses will cause a stress distribution similar to that shown

in Fig. 23. If the plate is assumed to have an infinite width, the stress at any distance x from the center of the hole is given by the equation

$$s = \frac{s_a}{2}\left[2 + \left(\frac{d}{2x}\right)^2 + 3\left(\frac{d}{2x}\right)^4\right] \qquad (58)$$

$$s = Ks_a$$

$$K = \frac{1}{2}\left[2 + \left(\frac{d}{2x}\right)^2 + 3\left(\frac{d}{2x}\right)^4\right]$$

where s_a = average stress in the net section, psi, and K is the stress-concentration factor. At the points e and f on the boundary of the hole, x is equal to $d/2$ and the maximum stress is

$$s_{max} = 3s_a \qquad \text{or} \qquad K = 3 \qquad (59)$$

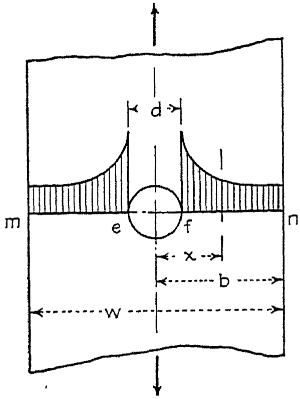


FIG. 23.

Although this equation is based on a plate of infinite width, it may be used with finite widths with an error of less than 6 per cent when the ratio d/w is less than $\frac{1}{4}$. Messers Wahl

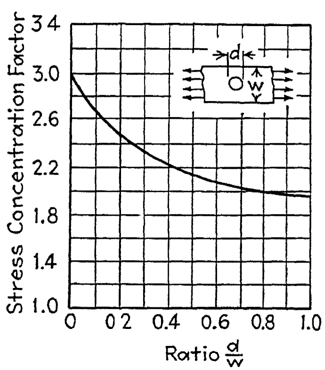


FIG. 24.—Stress-concentration factors for flat bars with holes.

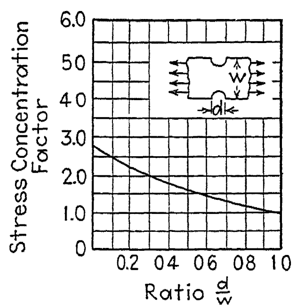


FIG. 25.—Stress-concentration factors for flat bars with semicircular notches.

and Beeuwkes* report a series of photoelastic studies of flat plates having central holes and circular edge notches. Figures 24 and 25 give the stress-concentration factors resulting from these tests.

* WAHL, A. M., and BEEUWKES, R., Stress Concentrations Produced by Holes and Notches, *Prod. Eng.*, March, 1934, p. 92.

When a plate in tension contains an elliptical hole having one of its axes parallel to the tensile load, the stress at the end of the axis that is perpendicular to the load is expressed by the equation

$$s = s_a\left(1 + 2\frac{a}{b}\right), \qquad \text{or} \qquad K = \left(1 + 2\frac{a}{b}\right) \tag{60}$$

where s_a = average stress, psi.
a = axis perpendicular to the load.
b = axis parallel to the load.

When the ratio a/b is large, approximating a crack, the maximum stress becomes very large causing the crack to spread. If small holes are drilled at the end of the crack, the stress concentration will be reduced and spreading of the crack may be prevented.

The case of a circular hole near the edge of a plate is shown in Fig. 26.

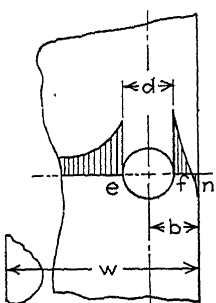


FIG. 26.

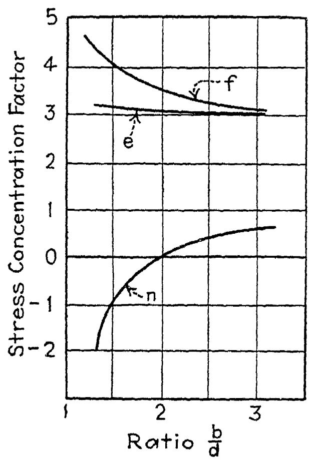


FIG. 27.—Stress-concentration factors for Eq. (61).

The stresses at the points e, f, and n are given by the equation

$$s = Ks_a = K\frac{F}{(w - d)t} \tag{61}$$

where the concentration factor K is taken from the curves in Fig. 27, which are plotted from the results of studies by G. B. Jeffery.*

A uniform pressure p_i may be applied to the boundary of the hole by fluid pressure, by force fits, or by hot rivets when being driven. In this case the tangential stress at f on the boundary

* JEFFERY, G. B., *Phil. Trans., Roy. Soc. London*, Vol 221, 1921

will be

$$s_f = p_i\left(\frac{4b^2 + d^2}{4b^2 - d^2}\right) \quad \text{or} \quad K = \left(\frac{4b^2 + d^2}{4b^2 - d^2}\right) \tag{62}$$

and the stress at n will be

$$s_n = p_i\left(\frac{4d^2}{4b^2 - d^2}\right) \quad \text{or} \quad K = \left(\frac{4d^2}{4b^2 - d^2}\right)$$

When a plate having a small hole is subjected to two stresses in normal planes, as shown in Fig. 28, the stresses at the points e, f, h, and g are

$$s_e = s_f = s_{a2}\left(3 - \frac{s_{a1}}{s_{a2}}\right) \quad \text{or} \quad K = \left(3 - \frac{s_{a1}}{s_{a2}}\right) \tag{63}$$

and

$$s_g = s_h = s_{a1}\left(3 - \frac{s_{a2}}{s_{a1}}\right) \quad \text{or} \quad K = \left(3 - \frac{s_{a2}}{s_{a1}}\right)$$

where s_{a1} and s_{a2} are the average stresses in the net section normal to s_1 and s_2. When either stress is compressive, use $-s_{a1}$ or $-s_{a2}$.

If s_{a1} and s_{a2} are equal in magnitude and one is compressive, the factor K becomes 4. If s_{a2} is the compressive stress, the resultant stresses at e and f will be compressive, and the resultant stresses at g and h will be tensile.

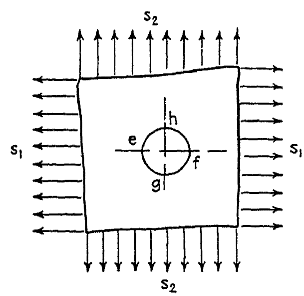


FIG. 28.

Stress-concentration factors for flat tension members with fillets are shown in Fig. 29.

Stress-concentration factors for rectangular filleted members in pure bending are shown in Fig. 30.

Stress-concentration factors for filleted cylindrical members subjected to torsion are shown in Fig. 31.

Stress-concentration factors for members having transverse grooves and notches, and subjected to tension, compression, or flexure are shown in Fig. 32.

A series of notches such as formed by screw threads is not so severe as a single notch. Tests at the University of Illinois, with bolts subjected to repeated tension, indicate that the stress-

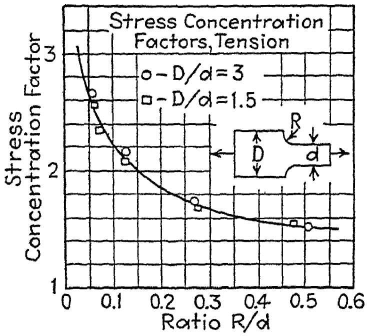


FIG. 29.—Stress-concentration factors for fillets in tension (*Timoshenko, Theory of Elasticity.*)

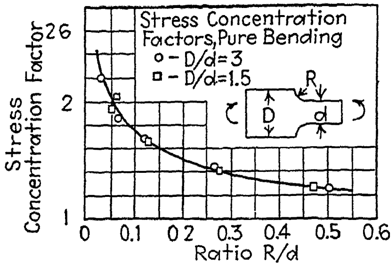


FIG. 30.—Stress-concentration factors for fillets in bending. (*Timoshenko, Theory of Elasticity.*)

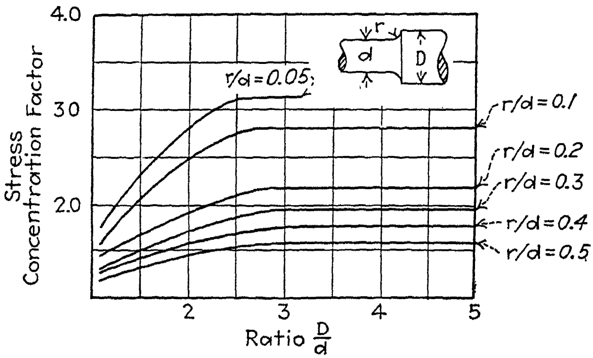


FIG. 31.—Stress-concentration factors for fillets in torsion.

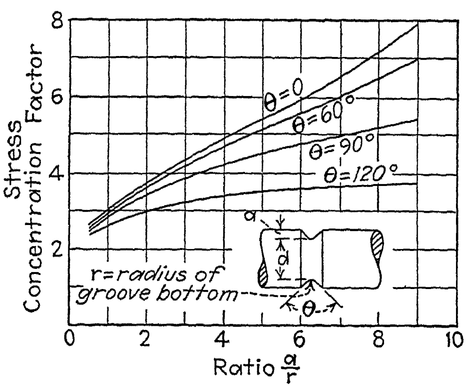


FIG. 32.—Stress-concentration factors for notches in tension, compression, and bending.

concentration factor is about 4. An interesting discussion of effects of this kind is given by Dr. R. V. Baud in an article entitled Avoiding Stress Concentration by Using Less Metal, published in *Product Engineering*, May, 1934.

In all the cases mentioned the stress-concentration factors have been independent of the material. This is true for static loads producing stress within the elastic limit. Tests made to determine the fatigue stress-concentration factor indicate that the material must be considered.* Fatigue stress-concentration factors have not been determined with sufficient reliability to warrant their inclusion here. Experimental work indicates, however, that in every case the effective stresses are smaller than those determined by the use of the theoretical stress factors discussed in the preceding paragraphs. Hence, when exact information is not available, the use of these factors will give safe designs.

80. Graphical Determination of the Endurance Limit. The term endurance limit is commonly applied to the highest stress to which the material may be subjected an infinite number of times without rupture. As has already been shown, this stress depends upon a number of conditions, one of which is the stress range. It is often convenient to use a diagram as a substitute for Eq. (55) in determining the endurance limit.

This endurance diagram, shown in Fig. 33, can be constructed for any material when the ultimate strength, yield stress, and endurance limit in reversed stress are known. Analysis and experimental work indicate that, if a variable stress is superimposed upon a steady stress, the plotted results will determine a maximum- and a minimum-stress line between which safe operation can be maintained. These lines are EU and BU in Fig. 33 and are probably slightly curved, but may be assumed to be straight without appreciable error. The stress corresponding to point U is the ultimate strength, and the points E and B correspond to the endurance limit for complete stress reversals. Any point on OU, as C, represents a steady or mean stress s_m, and CD and CH represent the variable stress s_v, which may be combined with s_m. The lines EU, BU, and OU indicate stress combinations that will ultimately cause rupture, but in no case

* PETERSON, R. E., Stress-concentration Phenomena in the Fatigue of Metals, *Trans. A.S.M.E.*, Vol. 55, 1933.

should the maximum stress exceed the yield stress. Hence the figure *EFNKB* limits the possible stress combinations. This figure forms the endurance diagram for flexural stresses. Direct

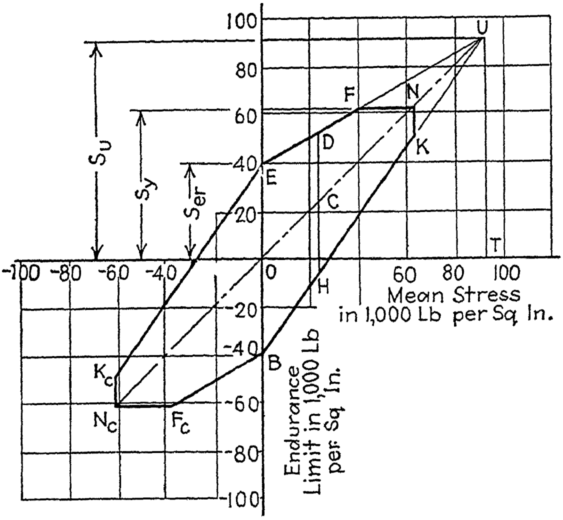


FIG. 33.—Endurance diagram for S.A.E. 1045 steel.

tensile stresses may be taken as 80 per cent of those shown in the diagram, and the shearing stresses as 55 per cent.

For certain purposes this diagram may be simplified as in Fig. 34, by moving the point *U* to the horizontal axis. Points *F* and *N* will take the positions indicated, and *CD* will represent the variable stress that, combined with the steady stress *OC*, will just cause failure. For simplicity, the straight line *EN* can replace the line *EFN* as the stress limit line. Introducing the stress utility factor *u*, to provide the desired margin of safety, we obtain the line *VS*, or working-stress limit line, and *CH* represents the variable stress that may be superimposed on a steady stress *OC*.

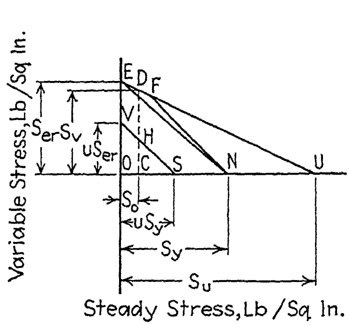


FIG. 34.

Example 1. To illustrate this method of checking working stresses, consider a coiled valve spring. When the valve is open the stress in the spring

is 35,000 psi, and when closed the stress is 25,000 psi. The spring material has a yield stress of 80,000 psi in torsion, and an endurance limit of 45,000 psi in reversed torsion. For this service, a utilization factor of 0.5 will be suitable, and the permissible stresses are

$$s = 0.5 \times 80{,}000 = 40{,}000 \text{ psi, in torsion}$$

and

$$s = 0.5 \times 45{,}000 = 22{,}500 \text{ psi, in reversed torsion}$$

These values are used to construct the line *VS* in Fig. 35. Lay off *OC* equal to 30,000 psi to represent the average or mean stress on the spring. Then *CD*, measured to scale, represents the permissible variable stress. The permissible variable stress scales 5,625 psi, and since the variable stress on the spring is only 5,000 psi, the imposed stresses are safe.

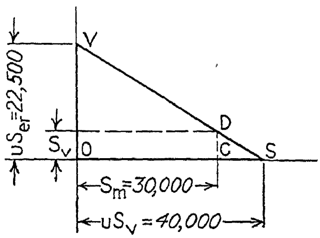


FIG. 35.

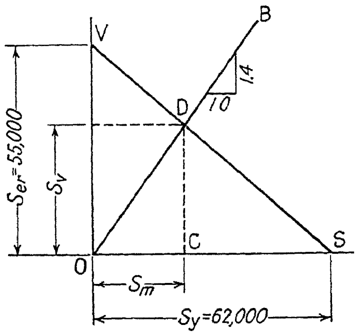


FIG. 36.

Example 2. To illustrate the use of the diagram in determining permissible design stresses, consider a piston rod for a reciprocating pump. The rod will be subjected to a total load having a cyclic variation from 30,000 lb in compression to 5,000 lb in tension. The expected life is 10 years of operation at 30 strokes per min, 24 hr per day, and 300 days per year. Determine the rod size using S.A.E. 2340 steel. The expected life is equivalent to 12,960,000 stress repetitions and therefore fatigue effects must be considered.

Construct Fig. 36 from the data of Table 3. We do not know the mean stress, but the stress range will be the same as the load range. The mean load is $-12{,}500$ lb and the variable load is 17,500 lb. Hence

$$\frac{F_v}{F_m} = \frac{s_v}{s_m} = \frac{17{,}500}{12{,}500} = 1.40$$

Through point *O* in the figure draw the line *OB* having the slope 1.40 and intersecting *VS* at *D*. The point *D* indicates the mean and variable stresses which can be used with the stress ratio 1.40. Scaling the stresses

from the diagram, we have

$$s_m = OC = 24{,}400$$
$$s_v = CD = 34{,}150$$
$$s_e = s_m + s_v = 58{,}550$$

Assuming that the impact factor is 1.25, the real margin of safety 1.50, the surface-finish factor 1.10, and the stress-concentration factor 2.25, the design stress will be

$$s_d = \frac{s_e}{b \times c \times f \times K} = \frac{58{,}550}{1.25 \times 1.5 \times 1.1 \times 2.25} = 12{,}620 \text{ psi}$$

The required rod area is 30,000/12,620 = 2.376 sq in., corresponding to a diameter of 1.74 in., or in round numbers $1\frac{3}{4}$ in., neglecting any column effects on the rod.

21. Photoelastic Analysis.* Study of stress distribution in complex forms, where the shape and discontinuities make direct mathematical analysis impossible, can be made by several methods, one example of which is photoelasticity. Polarized light, when passed through a loaded scale model of a machine member made of certain tran parent plastics, will produce color bands which are indicative of the stress in the model. These color bands may be studied directly, projected on a screen, or photographed for later analysis. In black and white photography, the stress bands appear as alternating dark and light stripes, or bands.

The simple cantilever beam shown in Fig. 37 illustrates the application of photoelasticity to a simple case of stress concentration. The photoelastic picture of the supported end of the beam shows, by the concentration of stress bands, that there is a region of high stress at the fillet joining the beam and its support. The stress variation along the upper surface of the beam, determined by photoelastic analysis, is shown by the solid line and

* The material in this article is briefed from W. M. Murray, Seeing Stresses with Photo-elasticity, *Metals Progress*, February, 1941, p. 195.

For further reference, see:

FILLON, L. N. G., "Photo-elasticity for Engineers," The Macmillan Company.

FROCHT, M. M., "Photo-elasticity," John Wiley & Sons, Inc.

ORTON, R. E., Photo-elastic Analysis in Commercial Practice, *Machine Design*, March, 1940.

HETENYI, M., Photo-elastic Stress Analysis Made in Three Dimensions, *Machine Design*, December, 1938

the stress from the usual beam formula, $s = Mc/I$, is shown by the broken line. The beam formula indicates a maximum stress of 2,700 psi at the fillet, but the photoelastic method indicates a stress of 4,350 psi. The stress concentration factor at the fillet is therefore 1.61, and this factor will apply at the fillet of all other geometrically similar beams, *i.e.*, beams having the same ratio of fillet radius, beam depth, and width.

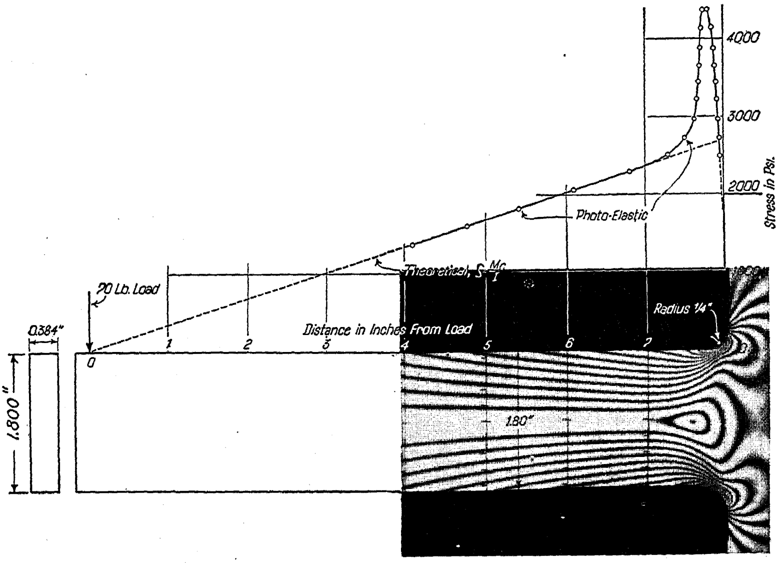


FIG. 37.

Each dark band, or fringe, in the photoelastic picture represents a line of constant difference in the principal stresses,* *i.e.*, a constant shear stress. The model material is calibrated by loading a small tensile specimen. If the specimen is examined under polarized monochromatic light, it will pass through cyclic changes from light to dark and back again as the tensile stress is increased. Each change from dark to dark is called "one order of interference" and corresponds to the change from one dark line to the next

* In Art. 54, it was shown that if a plane is passed through a body stressed in two directions, there will be a resultant stress acting on this plane composed of normal, s_n, and shear, s_p, components and that the normal, or tensile, stresses will vary from a maximum to a minimum as the angle of the plane changes. The maximum and minimum stresses will be at angles 90 deg apart and are called the principal stresses. The maximum shear stress in the body is equal to one-half the difference in the principal stresses and acts along a plane 45 deg from the planes of the principal stresses.

on a loaded model of the same material. By plotting tensile load on the specimen against orders of interference, as in Fig 38, a calibration of the material is obtained The order of interference is directly proportional to the material thickness, and a fringe constant must be determined The fringe constant is the change in the difference of the principal stresses for one order of interference in material of unit thickness In the tensile test, the difference of the principal stresses is the direct tension stress, hence

$$C_f = \text{fringe constant} = \frac{\text{load}}{\text{area}} \times \frac{\text{thickness}}{\text{order of interference}} \tag{64}$$

and

$$s_{\text{diff}} = C_f \frac{\text{order of interference}}{\text{thickness}} \tag{65}$$

In the model, locate a line or point where the difference in the principal stresses is zero. In the beam in Fig. 37 the zero point is the dot near the

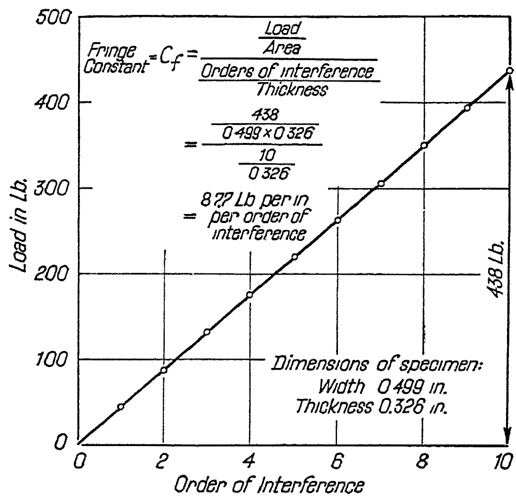


FIG. 38.

center of the beam at the support. Count the dark lines from this point to the point being investigated on the beam A point on the upper surface, about 2.6 in. from the support, is on the eighth fringe line. The fringe constant for this material is 87.7 (see Fig. 38), and the difference of principal stresses on the eighth fringe line is

$$s_{\text{diff}} = 87.7 \frac{8}{0.384} = 1{,}830 \text{ psi}$$

Since one of the principal stresses is zero at the boundary, or outer surface, of the beam, this value represents the tensile stress in the outer fiber of the beam at a point 2.6 in. from the support.

From the beam formula

$$s = \frac{Mc}{I} = \frac{70(8 - 2.6)6}{0.384 \times 1.800^2} = 1{,}823 \text{ psi}$$

which compares with the value of 1,830 psi found by the photoelastic method.

82. Effect of High Temperature. The selection of the proper design stress for members subjected to high temperatures is dependent upon two properties of the material. First, the strength of the material is modified by temperature, and, if the temperature is maintained for a sufficient period of time, structural changes that will still further affect the strength and other properties will take place. The effect of temperature

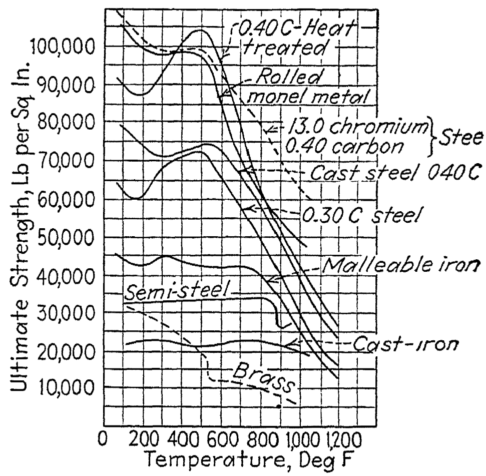


FIG. 39.—Effect of temperature on the ultimate strength of typical materials.

on the strength of typical materials is shown in Fig. 39. Results of this kind must be used with care, since they are usually taken from tests made shortly after the material has reached the desired temperature, and before structural changes due to prolonged exposure to heat have taken place.

The second item that must be considered is the change in size at high temperatures and the gradual deformation, or creep, that accompanies stress at high temperatures. *Creep* is the term used in referring to the continuous increase in the strain, or deformation, of any material subjected to stress. An examina-

tion of the curves in Fig. 40 shows that the rate of creep varies with the stress, temperature, and time. The rapid initial deformation produces a strain hardening of the material that tends to decrease the creep rate. The effect of continued high temperature on the structure of the metal is to temper it and increase its ductility and thus the creep rate. When the tempering effect predominates, a transition point is reached, beyond which the creep increases very rapidly. At low temperatures and stresses, the curves apparently indicate that the creep finally ceases, but at the higher temperatures and stresses, the transition point is very pronounced. After a sufficient length of time has elapsed, there will probably be a transition point for any combination of temperature and stress.

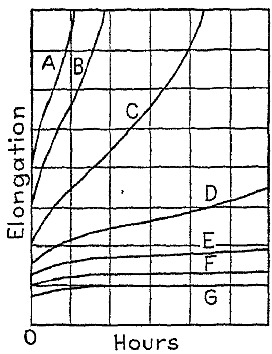


FIG. 40.—Typical creep curves Tests made at constant temperature with the stress decreasing from A to G.

Careful consideration of creep data leads to the conclusion that extrapolation beyond the time limit of the test data must be carried out with extreme caution. The designer, however, must select design stresses that will keep the creep or deformation within prescribed limits during the life of the machine, which may be 10 to 25 years or more. The presentation and development of creep-estimating methods are beyond the scope of this text, and the student is referred to current literature for discussions of this rapidly developing subject.*

* McVETTY, P. G., Factors Affecting Choice of Working Stresses for High-temperature Service, *Trans. A.S.M E.*, APM-55-13, 1933.

McVETTY, P. G., Working Stresses for High-temperature Service, *Mech. Eng.*, March, 1934.

The Effect of Temperature upon the Properties of Materials, *Trans. A.S.M.E.*, Vol. 46, 1924; *Proc. A S.T.M* , Vol. 24, part II, 1924.

McVETTY, P. G , Creep of Metals at Elevated Temperatures, *Mech. Eng.*, Vol. 53, pp. 197–200, 1931.

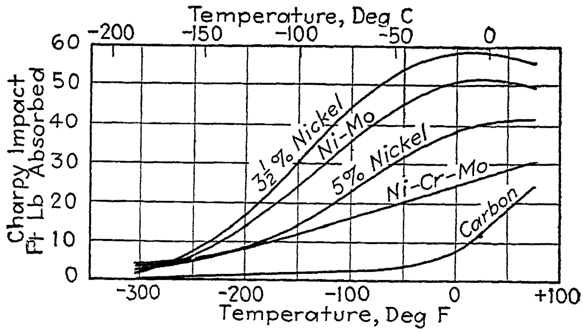
NADAI, A., The Creep of Metals, *Trans. A.S.M.E.*, APM-55-16, 1933.

Report of Committee A-10, Safe Working Stresses at Elevated Temperatures, *Proc. A.S.T.M.*, Vol. 30, part I, 1930.

BAILEY, R W., and ROBERTS, A. M., Testing of Materials for Service in High-temperature Steam Plant, *Proc. I.M.E.*, Vol. 122, 1932.

MACCULLOUGH, G. H., Applications of Creep Tests, *A.S M.E.*, released

83. Effect of Low Temperatures. Machinery that is to operate in cold climates, and machinery to be used in refrigerating service and in oil dewaxing plants, must show satisfactory service at the lower temperatures encountered. The strength and elasticity of steels are not affected in an adverse manner, but embrittlement may become a major consideration in the selection of the proper material. Hence, the impact properties at low



Steel type	Approximate steel composition	Approximate Brinell hardness as annealed
Carbon steel	S.A.E. 1030	140–155
3½% nickel steel	S.A.E. 2315	
5% nickel steel	S.A.E. 2512	
Nickel-molybdenum steel	0.12% C, 3½% Ni, 0.25% Mo	
Nickel-chromium-molybdenum steel	0.30% C, 3½% Ni, 0.9% Cr, 0.25% Mo	250

FIG. 41.—Charpy impact resistance of annealed carbon- and nickel-alloy steels at low temperatures. (*From Nickel Steel Topics, October*, 1933. *International Nickel Co.*)

temperatures must be given careful consideration when selecting the design stress.

As the temperature is decreased, plain carbon and low-carbon steels reach a condition where the loss of impact resistance becomes very rapid, and then the steel becomes very brittle. Most carbon steels reach this condition within atmospheric ranges, at times only slightly below room temperatures, and thus may become dangerously brittle at service temperatures.

for publication December, 1932.

EVERATT, F. L., Strength of Materials Subjected to Shear at High Temperatures, *Trans. A.S.M.E.*, APM-53-10, Vol 53, 1931.

DE BAUFRE, W. L., Stresses in Boiler Tubes Subjected to High Rates of Heat Absorption, Annual Meeting *A.S.M.E.*, New York, December, 1932.

Nickel in steel acts to offset this condition by raising the impact resistance at room temperatures, by lowering the temperature at which rapid loss of impact resistance occurs, and by lowering the rate of loss in this region. For extreme low temperatures, around −300 F, such as are encountered in liquid-air machinery, steels containing 42 per cent or more of nickel are used. These steels retain their impact resistance unimpaired to temperatures of −310 F, and probably much lower.

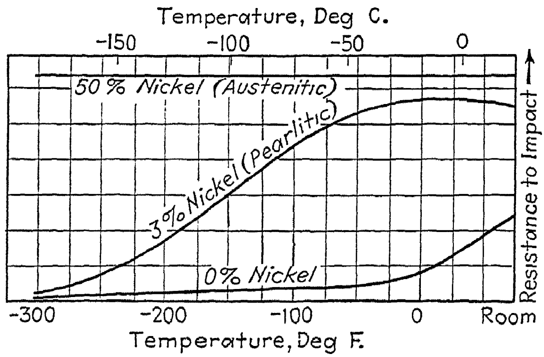


FIG. 42.—Chart illustrative of the effects of high and low nickel contents upon the low temperature properties of steel. (*From Nickel Steel Topics, October,* 1933. *International Nickel Co.*)

84. Allowance for Corrosion and Wear. Some materials deteriorate with age, others are subject to corrosion by gases and liquids, and others are subject to wear and abrasion under operating conditions. In the case of castings, the shifting of cores or poor molding procedure may result in variations in the thickness of vital parts of the casting. The designer must recognize these conditions and allow for them in size computations. The ordinary factor of safety or the selected design stress does not provide for these items, and an additional amount of metal must be provided, over and above the computed size of the machine member. The allowance varies from $\frac{1}{8}$ in. on small castings, to $\frac{3}{8}$ in. on large castings.

CHAPTER V

RIVETED JOINTS

85. Uses of Riveted Joints. In the assembly of the individual members to form a complete machine or structure, some form of fastening such as riveting, welding, bolting, or keying must be used. Riveting makes a permanent joint that can not be disassembled without destruction of the rivets. Tanks, pressure vessels, bridges, and building structures are commonly built of steel plates and rolled shapes riveted together.

Riveted joints may be roughly divided into three classes. In the first class, strength and rigidity are the chief requirements. Coal bunkers, low-pressure liquid containers, bins for bulk materials, and ship hulls are included in this group. Liquid containers and ship hulls must also be liquid-tight, but since the pressures to be dealt with are low, it is not particularly difficult to make the joints tight. In the second class, strength and rigidity are also required, but in addition the joint must be proportioned so that leakage will not occur. Boiler drums, high-pressure liquid containers, and gas tanks are included in this class. In the third class, strength and rigidity are the only requirements, leakage not entering into the problem at all. Machine frames, building structures, and bridges are included in this class.

86. Rivets. Rivets are made of wrought iron or soft steel for most uses, but where corrosive resistance or light weight is a requirement, rivets of copper or aluminum alloys are used. In making up the joint, the rivet is inserted in a punched or drilled hole and the second head formed by a die or rivet set, pressure being exerted by hand hammers, air hammers, or air- or hydraulic-pressure machines, the particular type of machine to be used depending upon the rivet size and the class of work desired. For the best class of work, the rivets are heated to a white heat before they are inserted in the holes and headed. Hot-driven rivets shrink on cooling, and, since the tensile stress due to this cooling can not be determined, these rivets are seldom

used as tension members. In some classes of work, the rivets are accurately sized, fitted in the holes, inserted cold, and the heads formed by peening or spinning the cold metal.

The forms of heads generally used are shown in Fig. 43. The round or button, the pan, and the cone heads are the most commonly used. Countersunk heads are not so strong as the other forms and should not be used unless required by small clearances and where smooth outer surfaces are necessary.

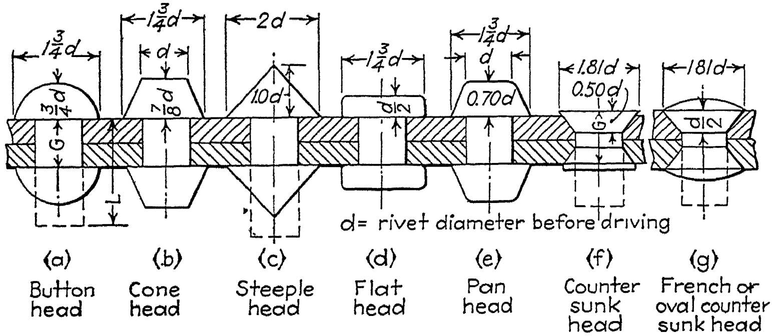


FIG. 43.—Proportions of rivet heads in common use.

Boiler and structural rivets are furnished commercially in diameters increasing by $\frac{1}{16}$ in. from $\frac{3}{8}$ in. to $1\frac{5}{8}$ in., the even $\frac{1}{8}$ in. sizes being the ones commonly carried in stock. In special cases, such as large water-, oil-, and gas-storage tanks, rivets up to 4 in. in diameter are used. The holes for the rivets should be $\frac{1}{16}$ in. larger than the rivets.

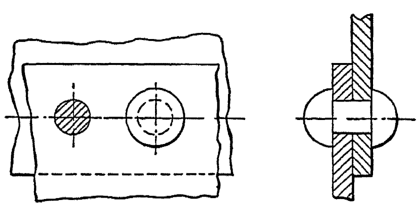


FIG. 44.—Single-riveted lap joint.

87. Types of Joints. When the two plates are simply laid over each other at the joint and riveted together as in Fig. 44, they form a *lap* joint. The tension forces acting on the plates, not being in the same plane, create a bending moment and thereby produce bending stresses in the plates and tension in the

rivets When the plates are placed end to end and connected by cover plates, as in Fig. 45, they form a *butt* joint. On account

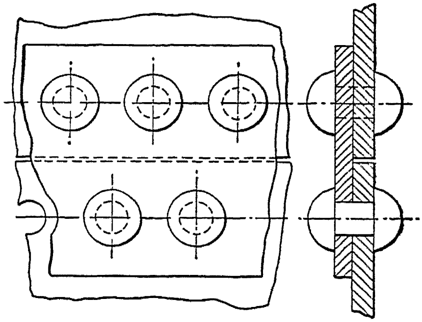


FIG. 45.—Single-riveted butt joint with single strap.

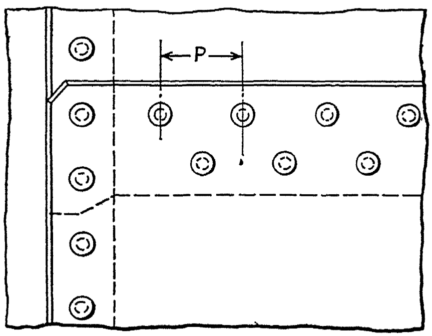


FIG. 46.—Lap joint, longitudinal or circumferential, double-riveted. (*A.S.M.E. Boiler Code.*)

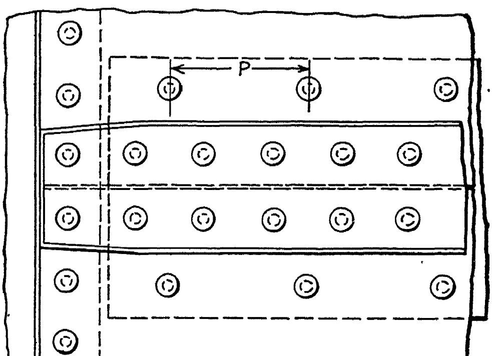


FIG. 47.—Butt and double strap joint, double-riveted. (*A.S.M.E. Boiler Code.*)

of the bending stresses, lap joints and single-cover-plate butt joints should not be used for high-pressure service. Various

types of joints used in boiler and tank design are illustrated in Figs. 45 to 51.

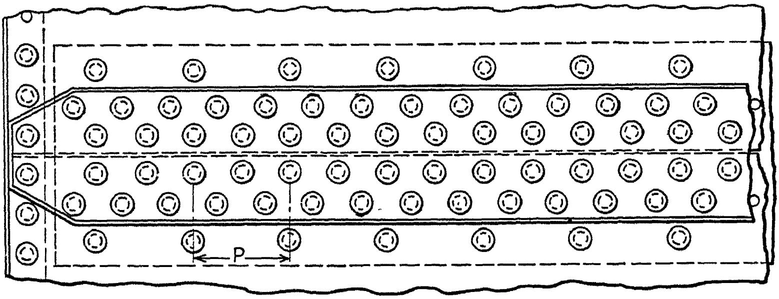


FIG. 48.—Triple-riveted butt joint, joining single-riveted lap joint. (*A.S.M.E. Boiler Code.*)

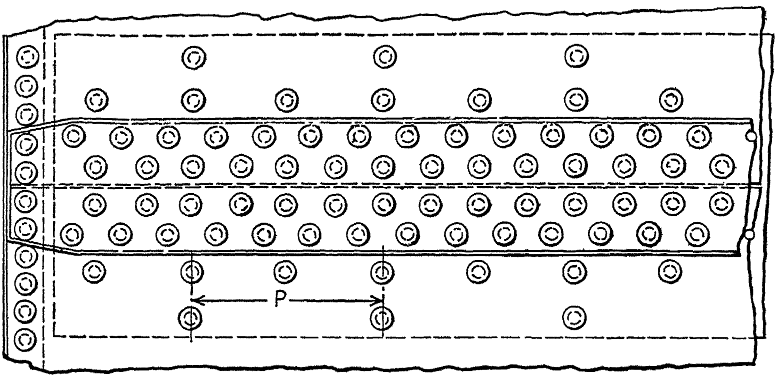


FIG. 49.—Quadruple-riveted butt and double strap joint. (*A.S.M.E. Boiler Code*)

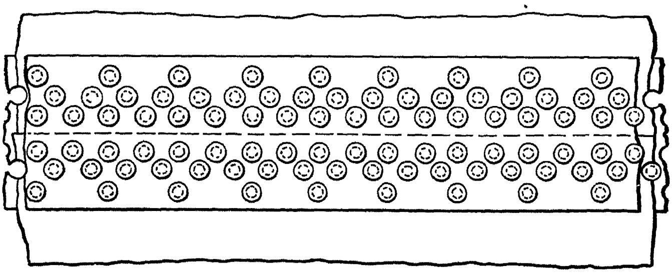


FIG. 50.—Triple-riveted butt and double strap joint with straps of equal width. (*A.S.M.E. Boiler Code.*)

88. Assumptions in the Conventional Design of Riveted Joints. Riveted joints fail in a number of different ways, as

shown in Figs. 52 to 56, which illustrate failures by tension, shear, and crushing in various parts of the joint. The actual

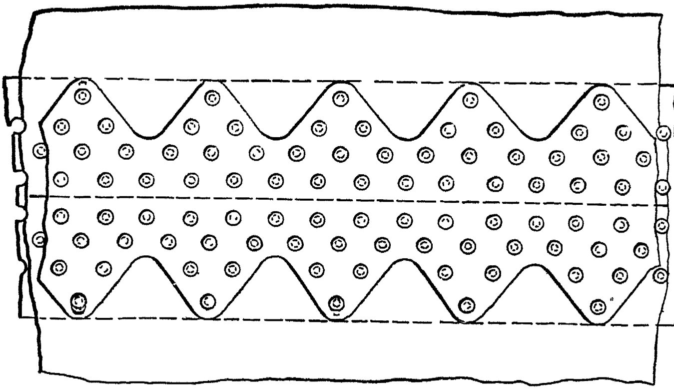


FIG. 51.—Quadruple-riveted butt and double strap joint of the saw-tooth type. (*A.S.M.E. Boiler Code.*)

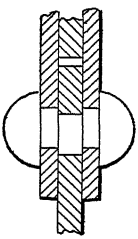


FIG. 52.

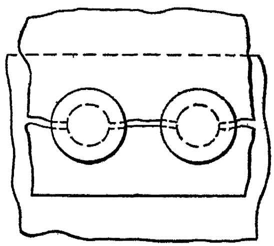


FIG. 53.

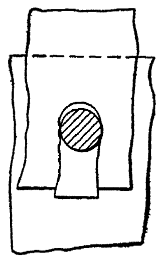


FIG. 54.

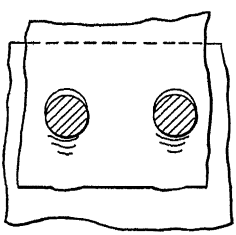


FIG. 55.

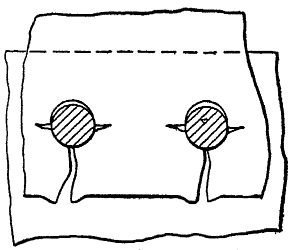


FIG. 56.

stresses set up in a riveted joint are complicated and not subject to exact calculation. The usual methods employed in the design

of such joints are based on the apparent direct stresses, with the unknown stresses provided for in the factor of safety. Common assumptions may be stated as follows:

a. The load is equally distributed among all rivets.

b. There is no bending stress in the rivets.

c. The tensile stress is equally distributed over the section of metal between the rivets.

d. The crushing pressure is equally distributed over the projected area of the rivets.

e. In a rivet subjected to double shear, the shear is equally distributed between the two areas in shear.

f. The holes into which the rivets are driven do not weaken the member if it is in compression.

g. After driving, the rivets completely fill the holes.

h. Friction between adjacent surfaces does not affect the strength of the joint.

89. Notation Used with Riveted Joints. The following symbols will be used in the discussion of riveted joints:

F = total load carried by any repeating group of rivets.

F_t, F_s, F_c = total load that may be carried in tension, shear, or crushing by a repeating group, lb.

s_t, s_s, s_c = unit stress in tension, shear, or crushing, psi.

t = main-plate thickness, in.

t_c = cover-plate thickness, in.

d = rivet-hole diameter, in.

p = pitch or center distance of rivet holes, in. In joints of more than one row of rivets, the pitch is measured in the outer row. Subscripts refer to the row, beginning with the inner row.

p_b = back pitch or distance between rows of rivets, in.

p_c = pitch on calking edge or outer row of cover plate, in.

n = number of rivet areas in any repeating group. When used with a subscript n refers to the row indicated by the subscript.

a = edge distance or distance from plate edge to center of nearest rivet, in.

e = joint efficiency. When used with the subscript t, s, or c it refers to the efficiency in tension, shear, or crushing only.

90. Efficiency of Riveted Joints. Plates joined by riveting are never so strong as the original plates, since metal is removed from the plates to allow insertion of the rivets. Since the joints are designed on the basis of maximum allowable working stresses, the efficiency of the joint may be defined as the ratio of the load that will produce the allowable stress in any part of the joint, to the load that will produce the allowable tension stress in the unpunched plate.

The ideal joint would be one in which the allowable stresses in tension, shear, and crushing would all be produced by the same load. This joint is not practical, since it is advantageous to use standard rivet diameters and symmetrical grouping of the

TABLE 16 —EFFICIENCY OF COMMERCIAL BOILER JOINTS

Type of joint	% efficiency	Maximum efficiency
Lap joints:		
Single riveted	45–60	63.3
Double riveted	63–70	77.5
Triple riveted	72–80	86.6
Butt joints:		
Single riveted	55–60	63.3
Double riveted	70–83	86.6
Triple riveted	80–90	95.0
Quadruple riveted	85–94	98.1

Maximum efficiencies are for ideal equistrength joints with $s_t = 11{,}000$, $s_s = 8{,}800$, and $s_c = 19{,}000$ psi.

rivets. In practice, no joint is equally strong in every possible method of failure, and the efficiency is always less than that of the ideal joint. The average efficiency obtained in commercial boiler joints is shown in Table 16, the larger value in each case being that obtained with the thinner plates.

91. Rivet Diameters. In boilers and high-pressure vessels, the rivet holes are always drilled or reamed with the plates and cover plates bolted in position, after which they are separated and all fins and rough edges removed. When the plates are reassembled and the rivets driven at high temperature by machines exerting enormous pressures (about 80 tons per sq. in. of rivet area), the rivets expand and completely fill the holes.

Hence, in all strength calculations, the diameter of the hole is used and not the diameter of the rivet. This is the practice in boiler design, but does not apply to the calculation of structural joints where the holes are punched and not drilled or reamed.

For an ideal or maximum-strength joint, the rivets should be equally strong in shear and crushing. For this condition

$$\frac{\pi d^2}{4} s_s = dts_c$$

and

$$d = 1.273t \frac{s_c}{s_s} \tag{66}$$

for rivets in single shear. For rivets in double shear,

$$d = 0.637t \frac{s_c}{s_s} \tag{67}$$

The design stresses commonly used in pressure vessels are s_t equal to 11,000, s_s equal to 8,800, and s_c equal to 19,000 psi. Substituting these values in the equations,

$$d = 2.75t \qquad \text{for single shear} \tag{68}$$

and

$$d = 1.375t \qquad \text{for double shear}$$

Any rivet smaller than this will reach the maximum allowable stress in shear before the maximum in crushing is reached. The diameter used in most pressure-vessel joints is smaller than the value just indicated because of the high pressures required to drive the rivets in the heavier plates, and because the calking requirements make short pitches and smaller rivets desirable. A common rule is to make the rivet-hole diameter from $1.2\sqrt{t}$ to $1.4\sqrt{t}$ for rivets in single or double shear. This rule gives diameters as shown in Table 18.

92. Pitch of Rivets. The theoretical pitch of the rivets in the outer row of the joint may be found by equating the strength of the rivets (in a repeating group) in shear to the tensile strength of the plate between the rivets of the outer row. Hence

$$(p - d)ts_t = n \frac{\pi d^2}{4} s_s$$

and

$$p = \frac{n\pi d^2}{4t} \frac{s_s}{s_t} + d \tag{69}$$

Uneven plates, scale on the plates, fins formed in punching the rivet holes, warping of the plates by the heating during riveting, and other causes prevent the plates from coming into close contact to make a leakproof joint. The joint must be calked by forcing the edge of the cover plate against the main plate by means of a flat-nosed tool, followed by a round-nosed tool, as illustrated in Fig. 57.

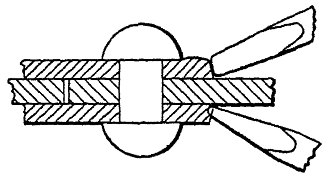


FIG. 57.

Wide spacing of the rivets along the calking edge renders it difficult to insure tightness, and the effectiveness of the calking therefore limits the permissible pitch, which is usually less than that given by Eq. (69). A common rule is that the pitch on the calking edge should not exceed eight times the thickness of the cover plate. This rule is safe for low pressures; but since it does not decrease the pitch with increasing pressure, it should be used only as a guide to the maximum available pitch. In the "Design of Steam Boilers and Pressure Vessels," by Haven and Swett, the following formula is developed for the permissible pitch along the calking edge of the outside cover plate

$$p_c - d = 21.38 \sqrt[4]{\frac{t_c^3}{P}} \tag{70}$$

where P = fluid pressure, psi.

To obtain effective calking, the outer cover plate is commonly made narrower than the inner one, as shown in Figs. 47, 48, and 49, thus permitting a short calking pitch while retaining the higher efficiency of the multiple-row joints.

The back pitch in chain riveting and the diagonal pitch in staggered riveting must be great enough to prevent rupture of the plate between two rows of rivets. The A.S.M.E. Boiler Code gives the following rules for determining the back pitch. For longitudinal joints, the distance between any two adjacent rows of rivets, measured at right angles to the direction of the joint, shall have the following minimum values:

a. If p/d is 4 or less, the minimum value is $1\frac{3}{4}d$.

b. If p/d is over 4, the minimum value is

$$1\tfrac{3}{4}d + 0.1(p - d) \tag{71}$$

where p is the pitch in the outer row, where a rivet in the inner row comes midway between two rivets in the outer row; or p is the pitch in the outer row less the pitch in the inner row where two rivets in the inner row come between two rivets in the outer row.

The outer row of rivets in any joint must be placed to allow clearance for riveting. The dies used to form the rivet heads usually have a diameter equal to twice the rivet diameter plus $\frac{3}{4}$ in.

93. Design Stresses. Complete specifications for the various types of steel used in pressure vessels are given in the A.S.M.E.

TABLE 17.—VALUES OF WORKING STRESS AT ELEVATED TEMPERATURES

Maximum temperature, °F	Minimum of the specified range of tensile strength of the material, psi				
	45,000	50,000	55,000	60,000	75,000
0–700	9,000	10,000	11,000	12,000	15,000
750	8,220	9,110	10,000	11,200	13,000
800	6,550	7,330	8,000	9,000	10,200
850	5,440	6,050	6,750	7,400	8,300
900	4,330	4,830	5,500	5,600	6,000
950	3,200	3,600	4,000	4,000	4,000

From A S.M.E. Boiler Code

Boiler Code.* The steel must have a minimum strength of 55,000 psi in tension, and 95,000 psi in crushing. The shearing strength of steel rivets must be 44,000 psi, and that of iron rivets 38,000 psi.

Design stresses for pressure vessels are based on a factor of safety of 5. Design stresses for various grades of boiler steel and different operating temperatures are given in Table 17.

94. Procedure in Designing Joints. In general, the procedure in designing the joint of a pressure vessel is as follows:

a. Determine the thickness of the plate required for a seamless shell, using Eq. (362) page 425.

b. Select the type of joint and a suitable efficiency (see Tables 20 and 16). Divide the thickness found in (*a*) by the efficiency to

* Rules for the Construction of Steam Boilers, American Society of Mechanical Engineers.

find the required plate thickness. From Table 18 select the cover-plate thickness.

TABLE 18.—MINIMUM COVER-PLATE THICKNESS AND RECOMMENDED RIVET HOLE DIAMETERS

Thickness of shell plate	Thickness of cover plate	Diameter of rivet hole	Thickness of shell plate	Thickness of cover plate	Diameter of rivet hole
$\frac{1}{4}$	$\frac{1}{4}$	$\frac{11}{16}$	$\frac{17}{32}$	$\frac{7}{16}$	$\frac{15}{16}$
$\frac{9}{32}$	$\frac{1}{4}$	$\frac{11}{16}$	$\frac{9}{16}$	$\frac{7}{16}$	$1\frac{1}{16}$
$\frac{5}{16}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{5}{8}$	$\frac{1}{2}$	$1\frac{1}{16}$
$\frac{11}{32}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{1}{2}$	$1\frac{3}{16}$
$\frac{3}{8}$	$\frac{5}{16}$	$\frac{13}{16}$	$\frac{7}{8}$	$\frac{5}{8}$	$1\frac{5}{16}$
$\frac{13}{32}$	$\frac{5}{16}$	$\frac{13}{16}$	1	$\frac{11}{16}$	$1\frac{7}{16}$
$\frac{7}{16}$	$\frac{3}{8}$	$\frac{15}{16}$	$1\frac{1}{8}$	$\frac{3}{4}$	$1\frac{7}{16}$
$\frac{15}{32}$	$\frac{3}{8}$	$\frac{15}{16}$	$1\frac{1}{4}$	$\frac{7}{8}$	$1\frac{7}{16}$
$\frac{1}{2}$	$\frac{7}{16}$	$\frac{15}{16}$	$1\frac{1}{2}$	1	$1\frac{9}{16}$

For shell plates over $1\frac{1}{2}$ in thick the cover-plate thickness should be at least $\frac{2}{3}t$. Cover-plate thickness from A.S.M.E. Boiler Code.

c. Determine $(p_c - d)$ for effective calking. From this value determine the pitch in the outer row. Equate the strength of the plate in tension in the outer row to the strength of all rivets in a repeating group in shear. Solve for the hole diameter.

d. Using the value of d just determined and the known value of $(p_c - d)$, find the pitch.

e. Calculate the efficiency of the joint, and check this with the efficiency assumed in part (*b*). If the final efficiency is too low, examine the values of F_t, F_s, F_c, etc., and modify the joint to raise the efficiency.

f. Determine the back pitch and the edge distance.

95. Some Practical Considerations. Tests indicate that when the rivets are placed at least $1\frac{1}{2}$ diameters from the plate edge there is no danger of the plate shearing or tearing in front of the rivet. Insuring a leakproof joint by calking, will be difficult when this distance is too large. The A.S.M.E. Boiler Code requires that the edge distance must be not less than $1\frac{1}{2}d$ or more than $1\frac{3}{4}d$.

In order to insure a reasonable rigidity, and to allow for corrosion and unknown handling stresses, certain minimum thicknesses must be maintained as indicated in Table 19. When

the plates are supported by stays, the minimum thickness is $\frac{5}{16}$ in. The minimum thickness of the cover plate to be used in double-strap joints is given in Table 18. For main-plate thicknesses in excess of $1\frac{1}{2}$ in. the cover plates must not be less than two-thirds of the main-plate thickness.

TABLE 19.—MINIMUM THICKNESS OF BOILER PLATES

Shell plates		Tube sheets of fire-tube boilers	
Diam. of shell, in.	Minimum thickness after flanging, in.	Diam. of tube sheet, in.	Minimum thickness, in.
36 and under	$\frac{1}{4}$	42 and under	$\frac{3}{8}$
36–54	$\frac{5}{16}$	42–54	$\frac{7}{16}$
54–72	$\frac{3}{8}$	54–72	$\frac{1}{2}$
72 and over	$\frac{1}{2}$	72 and over	$\frac{9}{16}$

From A.S M E Boiler Code.

The longitudinal joints of shells in excess of 36 in. diameter must be of the double-strap butt construction. Smaller shells may be lap-welded if the pressure does not exceed 100 psi. The strength of circumferential joints, when the head is not stayed by tubes or through braces, must be at least 50 per cent that of longitudinal joint.

TABLE 20.—SUGGESTED TYPES OF JOINTS

Diam. of shell, in.	Thickness of shell, in.	Type of joint
24–72	$\frac{1}{4}$–$\frac{1}{2}$	Double riveted
36–84	$\frac{5}{16}$–1	Triple riveted
60–108	$\frac{3}{8}$–$1\frac{1}{4}$	Quadruple riveted

When the joint is subjected to shock such as is encountered in hydraulic work, the factor of safety should be increased above the usual value of 5.

Example. Design of a Typical Boiler Joint. A boiler is to be designed for a steam pressure of 350 psi. The inside diameter of the largest course of the drum is 54 in. The completed joint is shown in Fig. 58.

The working stresses to be used are s_t equal to 11,000, s_s equal to 8,800, and s_c equal to 19,000 psi.

Following the procedure outlined in the preceding paragraph,

a. $t = \frac{PD}{2s_te} = \frac{350 \times 54}{2 \times 11,000 \times e} = \frac{0.859}{e}$ in.

b. From Table 20 a triple-riveted double-strap butt joint is found to be suitable, and from Table 16 the efficiency of this joint may be taken as 85 per cent.

$$t = \frac{0.859}{0.85} = 1.011, \text{ say } 1\tfrac{1}{16} \text{ in.}$$

Table 18 gives the cover-plate thickness as $\frac{3}{4}$ in.

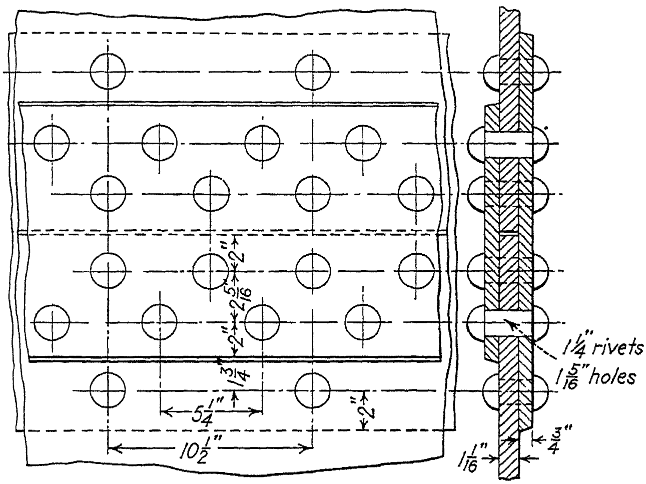


FIG. 58.

c. From Eq. (70)

$$p_c - d = 21.38 \sqrt[4]{\frac{t_c^3}{P}} = 21.38 \sqrt[4]{\frac{0.75^3}{350}} = 3.98 \text{ in.}$$

and

$$p_c = d + 3.98$$

In the triple-riveted joint selected, the pitch in the outer row is twice that along the calking edge, and

$$p = 2p_c = 2d + 7.96$$

A repeating group of rivets will be equal in length to the pitch in the outer row. The tensile strength of the plate between rivets in the outer row is

$$\begin{aligned} F_t &= (p - d)ts_t = (2d + 7.96 - d)1.0625 \times 11,000 \\ &= (d + 7.96)11,688 \end{aligned}$$

There are four rivets in double shear and one in single shear in each repeating group. Then the total shearing strength of the rivets is

$$F_s = 9\frac{\pi d^2}{4}s_s = \frac{9\pi d^2}{4}8,800 = 62,190d^2$$

Equate F_t and F_s and solve for d.

$$(d + 7.96)11,688 = 62,190d^2$$
$$d = 1\ 31, \text{ say } 1\tfrac{5}{16} \text{ in.}$$

The rivet diameter will be $\frac{1}{16}$ in less, or $1\frac{1}{4}$ in.

d. $p_c = d + 3\ 98 = 1.3125 + 3\ 98 = 5\ 2925$ in., say $5\frac{1}{4}$ in and

$$p = 2 \times 5\ 25 = 10\tfrac{1}{2} \text{ in.}$$

e. The actual efficiency of the joint can now be computed from the strengths of the joint in all the different modes of failure.

The strength of the unpunched plate for one repeating group is

$$F = pts_t = 10.5 \times 1.0625 \times 11,000 = 122,720 \text{ lb}$$

The joint may fail by tension in the plate between the rivets in the outer row; then

$$F_t = (p - d)ts_t = (10.5 - 1.3125)1.0625 \times 11,000 = 107,380 \text{ lb}$$

The joint may fail by shearing all rivets; then

$$F_s = \frac{9\pi d^2}{4}s_s = \frac{9\pi \times 1.3125^2}{4} \times 8,800 = 107,160 \text{ lb}$$

The joint may fail by crushing all rivets; then

$$F_c = (4dt + dt_c)s_c = (4 \times 1.0625 + 0.75)1.3125 \times 19,000$$
$$= 124,690 \text{ lb}$$

Since the last two equations show that the joint is stronger in crushing than in shear, it is useless to check any other type of failure involving crushing of the rivets.

The joint may fail by tension in the plate between the rivets in the second row and shearing the rivet in the outer row; then

$$F_t + F_s = (p - 2d)ts_t + \frac{\pi d^2}{4}s_s$$
$$= (10.5 - 2 \times 1.3125)1.0625 \times 11,000 + \frac{\pi 1.3125^2}{4} \times 8,800$$
$$= 103,950 \text{ lb}$$

Then the efficiency of the joint as designed is

$$e = \frac{103,950}{122,720} = 0.847 = 84.7 \text{ per cent}$$

which is only slightly lower than the efficiency assumed and may be considered to be satisfactory.

f The design is now completed by determining the back pitches and the distance from the rivet holes to the plate edges.

In this joint there is a rivet in the inner row between each two rivets in the second row and p/d equals 4. Hence the back pitch will be

$$p_b = 1\tfrac{3}{4}d = 1\tfrac{3}{4} \times 1.3125 = 2.297 \text{ or } 2\tfrac{5}{16} \text{ in.}$$

The rivets in the third or outer row must be placed to provide clearance between the rivet die and the edge of the cover plate. This distance must be

$$\frac{2 \times 1.25 + 0.75}{2} = 1.625, \text{ say } 1\tfrac{3}{4} \text{ in.}$$

The edge distance must be greater than $1\tfrac{1}{2}d$, *i e.*, 1.969, and less than $1\tfrac{3}{4}d$, or 2.297. Hence 2 in. is satisfactory.

96. Tank and Structural Joints. Ordinary tanks, coal bunkers, and similar structures where leakage is of minor importance, may have proportions approaching the theoretical proportions of equal strengths in all methods of failure. The thicker plates, however, usually require excessive rivet sizes, and for practical reasons the rivet diameter is made approximately $1.2\sqrt{t}$. The joints may be designed substantially as outlined in the preceding article, except that the pitch may be determined by equating the strength of a repeating group in tension to the strength of the rivets in shear, instead of by consideration of the calking requirements.

TABLE 21 —PERMISSIBLE STRESSES FOR STRUCTURAL DESIGN

Type of Stress	Permissible Unit Stress
Tension	20,000
Compression	20,000
Shear on power-driven rivets	15,000
Shear on hand-driven rivets	10,000
Crushing on power-driven rivets:	
In single shear	32,000
In double shear	40,000
Crushing on hand-driven rivets:	
In single shear	16,000
In double shear	20,000

For the design of riveted connections in bridges, building structures, and machine frames, the reader is referred to texts on structural design and to the handbooks of the major steel-manufacturing companies.

The permissible working stresses for this class of riveting are somewhat higher than those used in pressure-vessel design. The working stresses commonly used are given in Table 21.

97. Eccentric Loads.* In general, the line of application of the load should pass through the center of gravity of the rivet areas. When it is necessary that the load line must be displaced from this position, the direct shear on each rivet will be supplemented by a secondary shear caused by the tendency of the force to twist the joint about the center of gravity. The effect of eccentric loading can best be shown by an illustrative problem.

Example. Assume the arrangement shown in Fig. 59. All rivets are the same size.

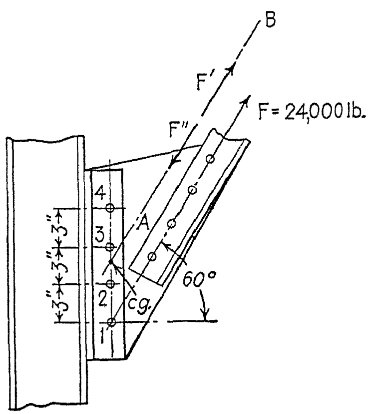


FIG. 59.

Solution. The load F should be applied along the line AB through the center of gravity of the rivets 1, 2, 3, and 4. The loads F' and F'', equal to F, can be added to the system without changing the condition of equilibrium.

The force F' produces a direct shear on each rivet of 24,000/4 or 6,000 lb acting parallel to F.

The couple F-F'' produces a twisting moment about the center of gravity of 24,000 (4.5 sin 30) or 54,000 lb-in. This moment is resisted by the combined moments of the forces acting on the rivets 1, 2, 3, and 4 Hence

$$54{,}000 = F_1'' \times 4\tfrac{1}{2} + F_2'' \times 1\tfrac{1}{2} + F_3'' \times 1\tfrac{1}{2} + F_4'' \times 4\tfrac{1}{2}$$

If the angles and gusset plate are considered to be rigid, the forces F_1'', F_2'', F_3'', and F_4'' will be proportional to their distances from the center of

* The discussion of eccentric loads on welded joints in Art. 103 and on bolts in Art. 113 may be applied to riveted joints.

gravity, and

$$F_1'' = F_4''$$

and

$$F_2'' = F_3'' = \frac{1.5}{4.5} F_1''$$

Substituting these values in the moment equation and solving,

$$F_1'' = F_4'' = 5{,}400 \text{ lb}$$

and

$$F_2'' = F_3'' = 1{,}800 \text{ lb}$$

These secondary shear forces acting normal to the lines joining the rivets and the center of gravity must be added to the direct shear forces by vector addition. Hence the actual load on rivet 1 is

$$\begin{aligned} F_1 &= \sqrt{6{,}000^2 + 5{,}400^2 + 2 \times 6{,}000 \times 5{,}400 \times \cos 60} \\ &= 9{,}880 \text{ lb} \end{aligned}$$

Similarly,

$$\begin{aligned} F_2 &= 7{,}070 \text{ lb} \\ F_3 &= 5{,}330 \text{ lb} \\ F_4 &= 5{,}720 \text{ lb} \end{aligned}$$

It should be noted that the largest actual force acting on one of these rivets is nearly twice the apparent direct shear force.

CHAPTER VI

WELDED JOINTS

98. General Discussion. Welding is the art of joining metals by pressure, after heating to a plastic or semimolten state, or of joining the metals by fusion alone. In the past, many designers

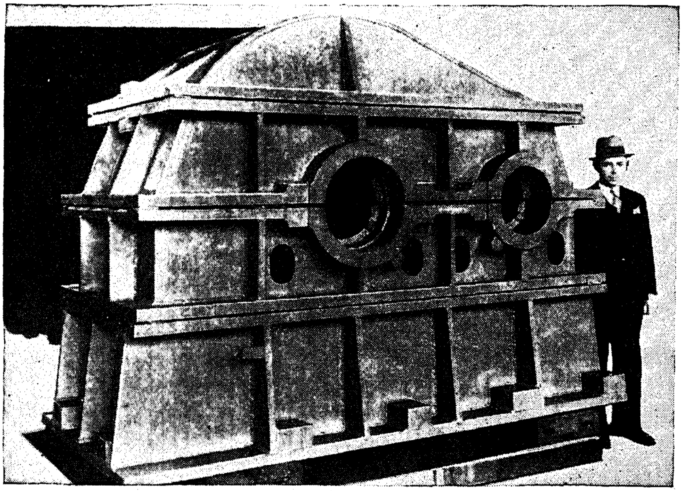


FIG. 60.—Gear housing for a rolling-mill drive, all parts of which are cut by machine gas-cutting. It is completely assembled by welding. (*Lukens Steel Co.*)

have avoided welded designs through fear of faulty and weak joints. With the advances made in welding technique, welding materials, and inspection methods, there is no reason why welded joints should not be just as reliable and free from faults as any riveted joint or any cast construction. The use of welding in the construction of pressure drums for boilers, refinery stills, and chemical processes has given satisfactory results over many years. The use of welded steel construction for the frames, housings, and connecting members of all types of machinery has reached the point where it must be definitely considered by

the designer when determining the material and construction to be used.

When designing for light weight, welding has very definite advantages. Standard rolled shapes, such as channels and I-beams, may often be used to advantage in welded structures. However, they should not be used without considering the possibility of using flat plates and bars bent in a bending brake to the shape required. In many cases, the use of such formed plates results in lighter, stronger, and better-looking construction with a smaller amount of welding.

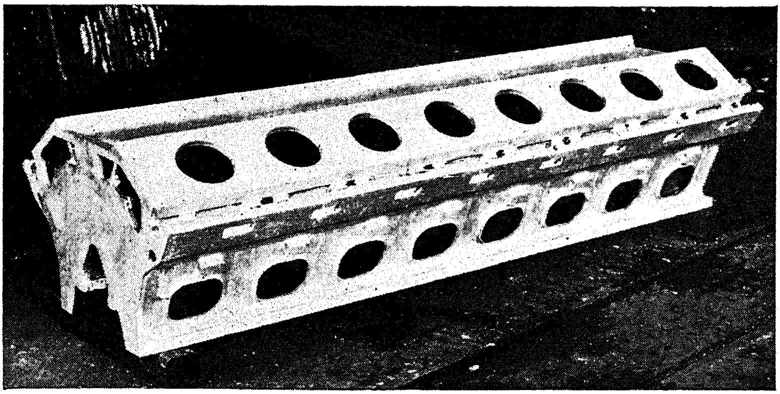


FIG. 61.—Welded Diesel engine crankcase, upper half. (*Lukens Steel Co.*)

No simple method has been developed to detect defective welds, and in many cases complete reliance is placed on the skill and technique of the welder.* Welds up to 3 in. in thickness can be examined by X rays, and this method is reliable.

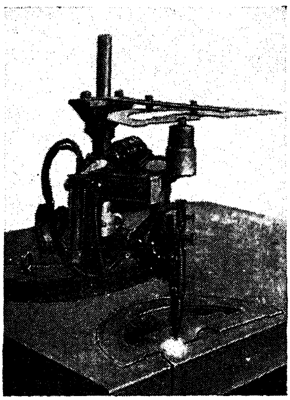
99. Types of Welding. Welding may be divided into five general classes, each of which has its particular field of use in manufacturing. These classes are: forge welding, Thermit welding, gas welding, electric-resistance welding, and electric-arc welding.

Forge welding consists of heating the parts in a forge or furnace until plastic and then hammering them together, a suitable flux being used to carry away the scale or oxide formed by contact

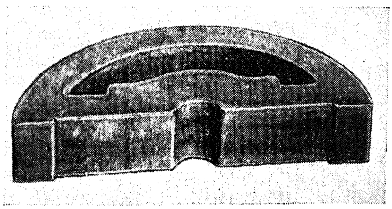
* A few methods of examining welds are: magnetic tests, X-ray photographs, X-ray stereoscopic pictures, X-ray diffraction, radium rays, physical tests of test coupons.

of the heated metal with the air. This process is practically obsolete, except in a modified form used in the manufacture of small pipes and tubes.

In Thermit welding, a suitable mold is built around the parts to be welded, and Thermit, a mixture of finely powdered aluminum and iron oxide, is confined in a crucible above the mold. When the Thermit is ignited, the iron is released by the heat of combustion and drops into the mold through an opening in the bottom of the crucible. The weld metal is essentially cast steel fused into the parts welded. This process is used principally



(a)



(b)

FIG. 62.—(a) Camograph. Typical example of quantity production. Motor-driven cam roller is electromagnetized and follows cam, while torch carried with it cuts a reproduction. (b) Gear blank, $5\frac{1}{2}$ in. thick, cut on camograph. Total length of cut $121\frac{3}{4}$ in. Time of cut 30 min. (*Air Reduction Sales Co.*)

in the repair of heavy machine parts and in the building up of defective castings.

Gas welding utilizes the heat produced by the combustion of either acetylene or hydrogen in a stream of pure oxygen. The flame is directed against the edges of the part to be welded, bringing the parent metal to the melting point; and extra metal required to fill the space between the parts is supplied by a welding rod of suitable material, melted in the gas flame. Acetylene welding is the most common form of gas welding and is widely used for repair work, for welding thin plates, and for welding gas, steam, and hydraulic pipe lines. A torch that supplies streams of pure oxygen around the heating flame makes an excellent device for cutting heavy slabs up to 12 in. thick. Flame cutting of irregular shapes by hand or by machine is becoming an important fabricating process.

In the electric-resistance process the parts are brought into contact, and a heavy current at low voltage is passed through the junction. Because of the high electrical resistance at the junction, the metal is rapidly brought up to the fusion temperature. Pressure, applied mechanically, forces the parts together and forms the weld. The resistance process is divided into classes such as spot welding, butt welding, flash welding, and seam welding. Butt and flash welding are economical where mass production justifies the special equipment required for each individual job. This process is widely used in the assembly of the bodies of automobiles, refrigerators, and other pressed-steel parts. The ordinary spot welder requires little special equipment and is used extensively in the manufacture of such parts as gear housings, switch housings, lamp reflectors, and other similar parts built in small lots. The field of resistance welding has been extended by the development of electronic controls such as the thyratron, so that many dissimilar metals can now be joined by welding, for instance, copper to aluminum, bronze to steel, and copper to steel.

In the electric-arc process, the heat is supplied by a continuous arc drawn between two electrodes. In the original process, now practically obsolete, the arc is drawn between carbon electrodes, the heat being reflected onto the parts to be welded. In the carbon-arc process, the work itself forms one electrode, a carbon rod being used for the second electrode. With the carbon electrode, it is difficult to make vertical and overhead welds, and excess carbon is likely to be present in the weld metal. The metallic-arc process is the most common electric welding process. The work forms one electrode, and the welding rod forms the second electrode. Overhead welding is possible, since molten metal from the tip of the welding rod is carried by the arc to the weld. The electric-arc process is readily adapted to welding machines with automatic regulation of arc length, speed, and other variables. Semi-automatic machines are used when the paths of the seams are irregular and not easily followed by fully automatic machines.

Molten steel has an affinity for oxygen and nitrogen, which make up the air; hence the weld metal is likely to contain gas pockets and nitrides, which weaken the weld and reduce the corrosion resistance. To prevent this, a shielded arc may be

used. The welding rod is heavily coated with a material which, in the heat of the arc, gives off large quantities of inactive gas, thereby protecting the weld metal from contact with the air. Welds made in this manner are about 20 per cent stronger than those made with bare welding rods.

The atomic-hydrogen arc-welding process is a recent development used to prevent oxidation of the metal. A reducing atmosphere is created by forcing a jet of hydrogen through the arc drawn between two tungsten electrodes. The heat of the arc separates the hydrogen into atoms, which later recombine giving back the heat of disassociation. The atomic-hydrogen atmosphere protects the weld metal.

FIG. 63.—Etched cross section showing a good weld in good parent metal. (*Lukens Steel Co.*)

100. Welding Properties of Materials. Most metals can be welded by some process, but some are more readily welded than others, and the properties of the weld depend upon many factors. At the temperatures reached, structural changes in the metal that change the physical properties and the corrosion resistance may take place. Some elements in the base metal, such as zinc, may vaporize during the welding and cause porous weld metal. Gaseous oxides may cause blowholes, soluble oxides in the molten metal reduce the strength and toughness of the weld, and insoluble oxides cause slag inclusions in the weld. Metals of high thermal expansion and low thermal conductivity are subject to high cooling stresses in the weld. The elements present as impurities or as alloys, the kind of metal used in the welding rod, the material used for shielding the rod, the fluxing material, and the welding procedure all affect the weld characteristics.

All the plain carbon steels except spring steel and tool steel (with carbon contents from 0.75 to 1.50 per cent) can be satisfactorily welded, but the lower-carbon steels are the most readily welded. Nickel, chromium, and vanadium improve the welding qualities slightly. Since the weld metal is essentially cast steel, it follows that cast-steel parts are easily welded by either the gas or electric processes.

The high strength and other desirable properties of alloy steels are chiefly due to their action on the carbon and to their response to heat treatment. The cast weld material is normally weaker than the heat-treated alloys, so that the weld is weaker than the base metal unless special precautions are taken. Special composition welding rods, usually of the shielded type, producing weld material of nearly the same analysis as the base metal, should be used, and the parts should be heat-treated after welding.

Cast iron is difficult to weld by any process, and even under the most favorable conditions the results are more or less unreliable. Satisfactory welds can be made only by care in preheating, preparation, and welding procedure; and important welds should be made only by experienced operators. Gas welding is, in general, superior to arc welding in strength, reliability, and machinability. Gas welding, however, requires careful preheating to prevent warpage and shrinkage stresses, whereas the more localized heating with the arc reduces the seriousness of these effects.

Practically all the common metals, including stainless steel, copper, bronze, nickel, Monel, and nickel silver can now be satisfactorily welded. Metals containing high percentages of lead, tin, zinc, aluminum, magnesium, and molybdenum are somewhat difficult to weld. This is due to the vaporizing of some of the ingredients, and to the fact that oxide forms and acts as an insulator interfering with the flow of current and heat.

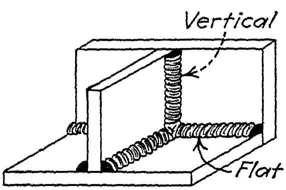
101. Strength of Welds. The properties of weld metal deposited by the gas or arc process, using bare welding rod, are shown in Table 22. With the shielded arc, the weld metal has better properties, as indicated in the table.

Butt welds may be assumed to have 80 per cent of the strength of the base metal if the welds are flush, and 100 per cent if they are reinforced 15 per cent, or if a reinforcing plate is used. Plug welds are as strong as the weld metal in shear.

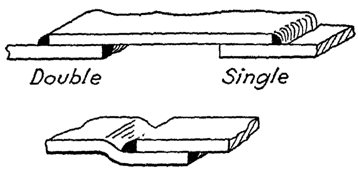
TABLE 22 —PROPERTIES OF WELD METAL

Engineering property	Shielded-arc weld metal	Light-coated and bare weld metal
Ultimate strength, tension	65,000–85,000	45,000–55,000
Yield stress, tension	50,000–55,000	28,000–32,000
Ultimate strength, shear		36,000–40,000
Endurance limit, reversed stress	28,000–30,000	12,000–16,000
Impact value, Izod, ft-lb	45–80	8–15
Per cent elongation in 2 in.	20–25	5–10

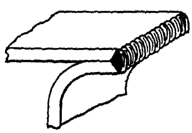
The following design stresses may be used when computing the strength of welds in material similar to structural steel. The joints should be designed so that the working stresses caused by the static and live loads combined will not exceed 11,500 psi in shear on the minimum section of the weld material, 13,000 psi in tension, and 15,000 psi in compression. The maximum stress due to bending must not exceed the working stresses given for tension or compression. The working stresses outlined are for low- and medium-carbon steels. Alloy steels, if welded with the proper technique and with shielded rods of the proper composition, will have much higher strengths.



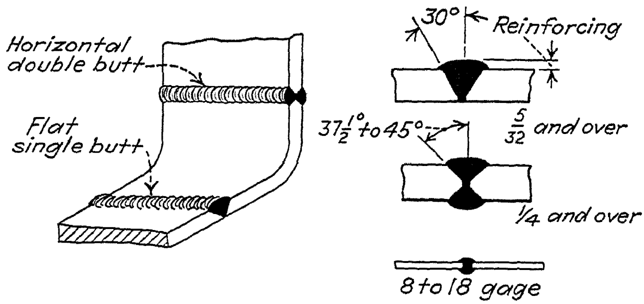
Fillet Welds
FIG. 64.



Lap Welds
FIG. 65.


Edge Weld
FIG. 66.

102. Strength of Fillet Welds. When the forces are applied as in Fig. 69, the weld metal near the corner A is subjected to direct tensile stress on the vertical face, and to direct shear on the lower face. In the usual form of weld, the two faces AB and AC in Fig. 69 are equal, and s_t is equal to s_s. The combined stresses due to



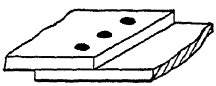
Butt Welds
FIG. 67.

these direct stresses are, from Eqs. (33) and (34),

$$\begin{aligned} s_{t\,\max} &= \tfrac{1}{2}(s_t + \sqrt{s_t^2 + 4s_s^2}) \\ &= \tfrac{1}{2}(s_t + \sqrt{5s_t^2}) \\ &= 1.618s_t \end{aligned}$$

and

$$\begin{aligned} s_{s\,\max} &= \tfrac{1}{2}\sqrt{s_t^2 + 4s_s^2} \\ &= 1.118s_s \end{aligned}$$



Plug or Rivet Welds
FIG. 68.

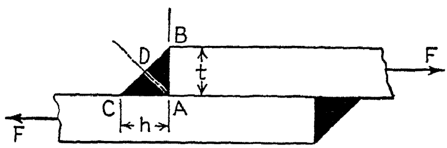


FIG. 69.

When the permissible tension and shear stresses are 13,000 and 11,500 psi, respectively, the permissible loads per linear inch of weld are

$$F_t = \frac{13{,}000}{1.618}\,t = 8{,}030t \text{ lb per in.} \tag{72}$$

and

$$F_s = \frac{11{,}500}{1.118}\,t = 10{,}300t \text{ lb per in.} \tag{73}$$

Hence the load capacity of a normal fillet weld in soft or medium-carbon steel may be taken to be 8,000t lb per lineal inch. This value has been used in computing the weld strengths in Table 23.

TABLE 23.—ALLOWABLE LOADS ON MILD-STEEL FILLET WELDS

Size of weld, in.	Allowable static load per linear inch of weld, lb			
	Bare welding rod		Shielded arc	
	Normal weld	Parallel weld	Normal weld	Parallel weld
$\frac{1}{8}$ by $\frac{1}{8}$	1,000	800	1,250	1,000
$\frac{3}{16}$ by $\frac{3}{16}$	1,500	1,200	1,875	1,500
$\frac{1}{4}$ by $\frac{1}{4}$	2,000	1,600	2,500	2,000
$\frac{5}{16}$ by $\frac{5}{16}$	2,500	2,000	3,125	2,500
$\frac{3}{8}$ by $\frac{3}{8}$	3,000	2,400	3,750	3,000
$\frac{1}{2}$ by $\frac{1}{2}$	4,000	3,200	5,000	4,000
$\frac{5}{8}$ by $\frac{5}{8}$	5,000	4,000	6,250	5,000
$\frac{3}{4}$ by $\frac{3}{4}$	6,000	4,800	7,500	6,000

It is generally assumed that the shear stress in a parallel fillet weld is uniformly distributed along the entire length of the weld. This is not true; the stresses are greatest at the ends of the weld, and the ratio of the maximum stress to the average stress increases with the weld length. For weld proportions used in practice, this variation may be neglected. In a parallel weld, the maximum shear stress is across the throat of the weld where the area is $0.707t$. Hence the shear stress on the side of the weld is 0.707 times $s_{s\,max}$, and the permissible load per

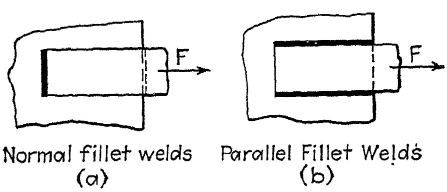


FIG. 70.

lineal inch is $0.707t$ times 11,500, or $8{,}130t$ lb. This is the same capacity as that found for a normal weld. However, when the external forces are not collinear, there will be bending in the weld and the permissible load should be reduced 20 per cent to $6{,}400t$. This value has been used in computing the values in Table 23.

103. Eccentric Loads. With axial loads on unsymmetrical sections such as angles, or channels welded on the flange edges,

the weld lengths should be proportioned so that the sum of the resisting moments of the welds about the gravity axis is zero. In Fig. 71, let L be the total weld length, L_a and L_b be the individ-

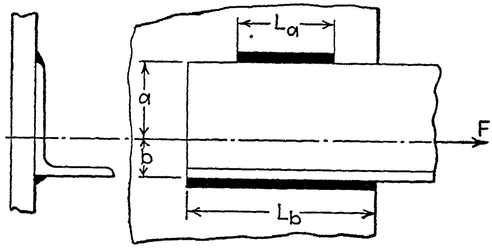


FIG. 71.

ual weld lengths, and a and b be the distances of the welds from the gravity axis. Then

$$aL_a s = bL_b s$$

and

$$L_b = L - L_a$$

from which

$$L_a = \frac{bL}{a + b}$$

and

$$L_b = \frac{aL}{a + b} \tag{74}$$

A general case of eccentric loading is shown in Fig. 72, where the fillet welds are subjected to the action of a load F acting at a

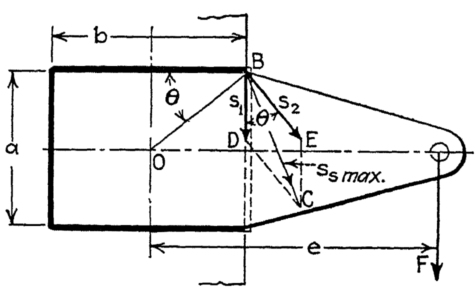


FIG. 72.

distance e from the center of gravity of the welds, O. The welds are subjected to a primary shear stress s_1, and to a secondary

shear stress s_2, which is proportional to the distance of the weld section from O and is maximum at the corners of the weld. Let s_2 be the maximum secondary stress; then the maximum total shear is the vector sum of s_1 and s_2. The vector sum BC is

$$s_{s\,\max} = \sqrt{s_1^2 + s_2^2 + 2s_1s_2 \cos\theta} \tag{75}$$

where

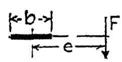
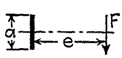
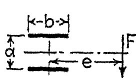
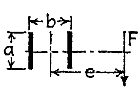
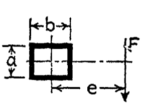
$$\cos\theta = \frac{b}{\sqrt{a^2 + b^2}}$$

and

$$s_1 = \frac{F}{tL}$$

and L is the total length of the welds. The value of the maximum secondary shear stress s_2 may be determined as follows.

TABLE 24.—VALUES OF J FOR FILLET WELDS

Type of weld	Moment of inertia J
	$\dfrac{tb^3}{12}$
	$\dfrac{ta^3}{12}$
	$\dfrac{tb(3a^2 + b^2)}{6}$
	$\dfrac{ta(a^2 + 3b^2)}{6}$
	$\dfrac{t(a + b)^3}{6}$

From Fig. 73, the moment of the shear on a weld area dA is

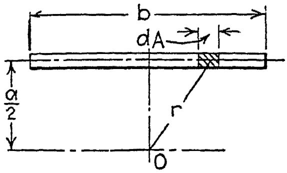


FIG 73.

$$dM = sr\,dA = \frac{2s_2r^2\,dA}{\sqrt{a^2+b^2}}$$

and

$$M = Fe = \frac{2s_2}{\sqrt{a^2+b^2}}\int r^2\,dA \quad (76)$$

The term $\int r^2\,dA$ is the polar moment of inertia of the weld, *i.e.*, J. Hence

$$s_2 = \frac{Fe\sqrt{a^2+b^2}}{2J} \quad (77)$$

Values of J for several combinations of eccentric-loaded fillet welds are given in Table 24.

104. Welded Pressure Vessels. The A.S.M.E. Boiler Code contains rigid rules for the welding, inspection, and testing of vessels to be used as containers of gases and liquids under pressure. This code should be thoroughly studied before attempting the design of any pressure vessel.

Vessels covered by the code are divided into three classes, each limited by the contents, pressure, and operating temperature. The principal limitations and the type of welding permitted in each class are shown in Table 25. The code applies only to unfired pressure vessels having a combination of diameter and working pressure such that $(P - 15)(D - 4)$ is greater than 60, and to those vessels having a combination of volume and working pressure such that $(P - 15)(V - 1.4)$ is greater than 22.5, where P is the pressure in pounds per square inch, D is the diameter in inches, and V is the volume in cubic feet.

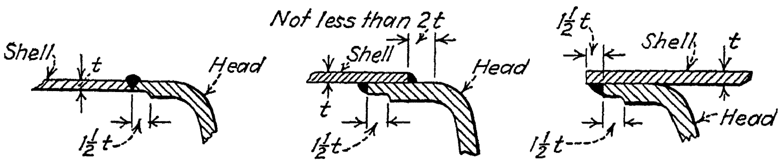


FIG. 74.—Methods of attaching heads by welds.

According to this code, dished heads placed concave to the pressure in boilers and Class 1 vessels must be butt welded as shown in Fig. 74*a*. The head in Class 3 vessels may be inserted

TABLE 25.—RULES FOR THE FUSION WELDING OF PRESSURE VESSELS

	Class of Fusion-welded Vessel		
	Class 1	Class 2	Class 3
Use of vessel	Any use	All vessels except containers of lethal gases or liquids	All vessels not over ⅝-in. plate thickness, except containers of lethal gases or liquids
Maximum operating temperature		300 F for vessels containing liquids (except steam only) 700 F for all others, provided the plate thickness does not exceed 1½ in	Not to exceed the boiling temperature at atmos pressure. Not to exceed 250 F.
Maximum pressure		400 psi except hydraulic pressure at atmospheric temperature	200 psi
Minimum joint efficiency	90%	80%	Variable
Allowable working stress, psi	90% of the values in Table 17	80% of the values in Table 17	Double-welded butt joints, 8,000 Single-welded butt joints for girth or head joints, 6,500 Double full-fillet welds for girth joints only, 7,000 Plug or intermittent welds for girth or head joints, 5,600 Single-welded butt joints for longitudinal joints and for material less than ¼ in. thickness, 5,600 For material from ¼ to ⅜ in thick, 7,000
Longitudinal joints	Double butt weld reinforced at the center of weld on each side of plate by at least 1/16 in. up to ⅝-in plate and up to ⅛ in. for heavier plate	Same as Class 1	Single butt weld for plates ¼ in. or less. Double butt weld for any thickness up to ⅝ in Double lap weld for ⅜ in. thickness or less
Circumferential joints	Double butt weld same as above	Double butt weld. Single butt for ⅝ in. thick or less	Butt or lap weld
Stress relieving	Must be stress relieved. Heat slowly and uniformly to 1100 to 1200 F or higher if possible without distortion. Hold at the temperature for 1 hr. per in. of thickness Cool slowly in a still atmosphere	Must be stress relieved when both the wall thickness exceeds 0.58 in., and the shell diameter is less than 20 in. For all cases where the ratio of diameter to the cube of the thickness is less than 118	Not required
Minimum thickness of shell plates, heads, and dome plates after flanging	⅛ in for shells up to 16 in. diameter 3/16 in. for shells of 16–24 in. diameter ¼ in. for shells of 24–42 in. diameter 5/16 in for shells of 42–60 in. diameter ⅜ in. for shells over 60 in. diameter		

with a driving fit and lap-welded inside and outside, except that heads less than 20 in. in diameter may be welded on the outside only. The distance from the weld to the point of tangency of the knuckle (corner radius) must be not less than twice the head thickness.

105. Some Practical Considerations. Welds should be placed symmetrically about the axis of the welded member unless the loading is unsymmetrical. For unsymmetrical members such as structural angles, the welded lengths are determined by the method previously outlined. When the strength of the weld is computed, $\frac{1}{2}$ in. should be subtracted from the length to allow for starting and stopping the weld, and welds having a length less than four times the width should not be considered. Intermittent welds should not be less than $1\frac{1}{2}$ in. long, spaced not less than 16 times the plate thickness or more than 4 in. in the clear.

Fillet welds should be laid out so as to make it possible to obtain good fusion at the bottom of the weld. It is desirable that the weld should not be subjected to direct bending stresses; hence corner welds should be avoided unless the plates are properly supported independently of the weld. No special preparation of the plate edges is required for fillet and edge welds. The tee welds may be made with plain fillet welds, or the plate may be beveled on one or both sides.

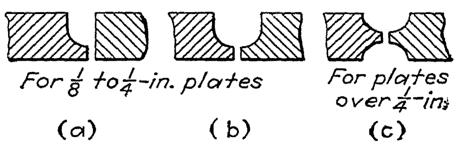


Fig. 75.

Butt welds in plates of less than $\frac{1}{8}$ in. thickness do not require beveling of the plate edge, plates from $\frac{1}{8}$ to $\frac{1}{4}$ in. should be beveled or cut to form a U-groove as shown in Fig. 75, and heavier plates should be cut to form a U-groove on both sides. When plates of unequal thickness are butt-welded, the edge of the thicker plate should be reduced so that it is of approximately the same thickness as the thinner plate.

Unreinforced holes should not be located in a welded joint, and when located near a weld, the minimum distance from the edge of the hole to the weld should be equal to the plate thickness for

plates from 1 to 2 in. thick, and never less than 1 in For plates over 2 in. thick, the minimum distance should be 2 in.

Illustrative Examples. Example 1. Two plates $\frac{1}{2}$ in. thick and $6\frac{1}{2}$ in wide are joined by a double lap joint having $\frac{1}{2}$-in. fillet welds. Determine the unit shear and tension stresses on the faces of the weld, and the maximum combined stress, when a tensile load of 24,000 lb is applied to the plates.

It may be assumed that each fillet weld transmits one-half the load and has an effective length of 6 in Then the average unit stresses on the weld faces are

$$s_t = s_s = \frac{12{,}000}{0.5 \times 6} = 4{,}000 \text{ psi}$$

The combined stresses are

$$s_{t\,max} = 1.618 s_t = 1.618 \times 4{,}000 = 6{,}472 \text{ psi}$$

and

$$s_{s\,max} = 1.118 s_s = 1.118 \times 4{,}000 = 4{,}472 \text{ psi}$$

It is often stated that the tension stress across the throat of the weld is the maximum, and that its value is twice the direct tension stress This is not true. If, in Fig. 69, an imaginary plane is passed through the throat, a tension stress s_n, normal to this plane, and a shear stress along this plane must be supplied to balance the direct tension load. Resolving the forces normal to the plane, noting that AD equals $t \cos 45$,

$$s_n t \cos 45 = s_t t \cos 45$$

and s_n is equal to s_t. Similarly, the shear stress on the throat is equal to the shear stress on the weld face. In this problem, all these are 4,000 psi, which is less than the combined stresses determined above

Example 2. Determine the thickness required for a welded drum of a 60-in. boiler that is to operate at a steam pressure of 900 psi.

Since this is a Class 1 vessel, the longitudinal joint must be butt-welded, the joint efficiency must be 90 per cent, and the permissible stress at the temperature of the steam is 11,000 psi. The thickness of plate required is determined by Eq. (362) page 425, and is

$$t = \frac{900 \times 60}{2 \times 11{,}000 \times 0.90} = 2.73, \text{ say } 2\tfrac{3}{4} \text{ in.}$$

Example 3. A 6- by 4- by $\frac{1}{2}$-in. angle is to be welded to a steel plate by fillet welds along the edges of the 6-in. leg. The angle is subjected to a tension load of 50,000 lb. Determine the weld lengths required if placed as shown in Fig. 71.

The line of action of the load may be assumed to be the gravity axis of the angle. From tables of the properties of structural shapes, the axis is 1.99 in. from the short leg. Hence the distance b in the figure is 1.99 in. and a is 4.01 in. Table 23 gives 3,200 lb as the permissible load per inch of

$\frac{1}{2}$-in. parallel weld. Hence the total length of weld required is

$$L = \frac{50{,}000}{3{,}200} = 15.62 \text{ in.}$$

The individual weld lengths are

$$L_a = \frac{15.62 \times 1.99}{6.00} = 5.18 \text{ in.}$$

and

$$L_b = \frac{15.62 \times 4.01}{6.00} = 10.44 \text{ in.}$$

Allowing $\frac{1}{2}$ in. for starting and stopping the weld, a should be made $5\frac{3}{4}$ in. and b should be 11 in.

Example 4. A 6- by 4- by $\frac{1}{2}$-in. angle is welded to its support by two fillet welds. A load of 4,200 lb is applied normal to the gravity axis of the angle at a distance of 15 in. from the center of gravity of the welds. Determine the maximum shearing stress in the welds, assuming each weld to be 3 in. in length and parallel to the axis of the angle.

The primary shear stress is

$$s_1 = \frac{4{,}200}{0.5 \times 3 \times 2} = 1{,}400 \text{ psi}$$

and the secondary shear is

$$s_2 = \frac{4{,}200 \times 15\sqrt{36 + 9}}{2\left(\frac{0.5 \times 3(3 \times 36 + 9)}{6}\right)} = 7{,}230 \text{ psi}$$

Combining these stresses, $s_{s\,\max}$ is

$$\begin{aligned} s_{s\,\max} &= \sqrt{1{,}400^2 + 7{,}230^2 + 2 \times 1{,}400 \times 7{,}230 \times 0.446} \\ &= 7{,}950 \text{ psi} \end{aligned}$$

CHAPTER VII

BOLTS AND SCREWS

Threaded bolts and screws are used to hold the removable heads of cylinders, machine members that must be readily disassembled, and parts of large machines that must be built in small units for ease in manufacturing, assembling, or shipping. Screws are also used for the transmission of power: for instance, the lead screws on machine tools, screws on presses, and similar devices. Screws are sometimes used as a means of adjustment or of obtaining accurate movement in measuring instruments such as micrometers.

106. Thread Forms. The form and proportions of common screw threads are shown in Figs. 76 to 82. The sharp tops and

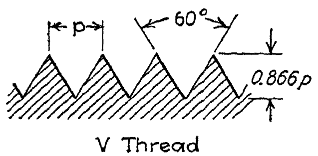


Fig. 76.

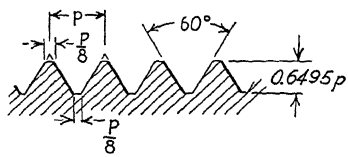


Fig. 77.

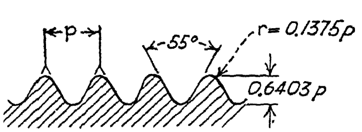


Fig. 78.

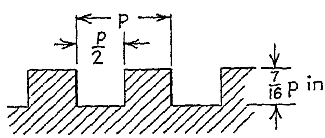


Fig. 79.

bottoms of the V thread tend to weaken the screw, especially when it is subjected to shock and repeated stresses. It is also difficult to maintain a sharp point on the cutting tool used in forming the thread and on the top of a thread cut in cast iron. In the Sellers thread the tops and bottoms are flattened off to

increase the strength and to make it easier to keep the threading tool sharp. The National Standard Threads (Table 26) are of the Sellers form with the number of thread per inch definitely standardized for each diameter of screw. The finer threads of the National Fine Thread series make it easier to pull the nuts up tight, and also make it harder to shake the nuts loose. The extra-fine thread is suitable for tough strong materials and is used in thin light sections where fine adjustments are needed or where vibration is present.

TABLE 26.—AMERICAN OR NATIONAL STANDARD COARSE AND FINE THREADS AND S.A.E. EXTRA-FINE THREADS

Nominal size and diameter	Coarse series (N.C)			Fine series (N.F)			Extra-fine series (E.F.)		
	Threads per in	Root diam, in.	Root area, sq in	Threads per in.	Root diam, in.	Root area, sq in.	Threads per in.	Root diam., in.	Root area, sq in.
Numbered sizes									
0 (0 060)		.	.....	80	0 0438	0 0015			
1 (0 073)	64	0.0527	0 0022	72	0 055	0 0024			
2 (0 086)	56	0 0628	0 0031	64	0 0657	0.0034			
3 (0 099)	48	0 0719	0 0041	56	0 0758	0 0045			
4 (0 112)	40	0.0795	0 005	48	0 0849	0.0057			
5 (0 125)	40	0 0925	0 0067	44	0 0955	0 0072			
6 (0 138)	32	0.0974	0 0075	40	0.1055	0 0087			
8 (0.164)	32	0.1234	0 012	36	0.1279	0 0128			
10 (0 190)	24	0.1359	0 0145	32	0 1494	0 0175			
12 (0.216)	24	0.1619	0.0206	28	0.1696	0 0226			
Inch sizes									
1/4	20	0.185	0 0269	28	0.2036	0 0326	32	0 2117	0 0352
5/16	18	0.2403	0 0454	24	0 2584	0.0524	32	0 2742	0 0591
3/8	16	0.2938	0.0678	24	0.3209	0.0809	32	0.3367	0.0890
7/16	14	0.3447	0.0933	20	0 3725	0.109	28	0 3937	0 1217
1/2	13	0 4001	0 1257	20	0 435	0.1486	28	0.4562	0 1635
9/16	12	0 4542	0 162	18	0.4903	0.1888	24	0 5114	0 2054
5/8	11	0 5069	0 2018	18	0 5528	0.240	24	0.5739	0.2586
3/4	10	0 6201	0 302	16	0.6688	0.3513	20	0 6887	0 3725
7/8	9	0 7307	0.4193	14	0.7822	0.4805	20	0.8137	0 5200
1	8	0.8376	0 551	14	0.9072	0.6464	20	0.9387	0.6921
1 1/8	7	0 9394	0 6931	12	1.0167	0.8118	18	1.0568	0.8772
1 1/4	7	1 0644	0 8898	12	1.1417	1 0238	18	1.1818	1.0969
1 3/8	6	1.1585	1.0541	12	1.2667	1.2602			
1 1/2	6	1.2835	1.2938	12	1.3917	1 5212	18	1.4318	1.6101
1 3/4	5	1 4902	1.7441	..	.....	. . ..	16	1.6733	2.1991
2	4 1/2	1.7113	2.3001	..	......	.. ..	16	1.9233	2 9053
2 1/4	4 1/2	1 9613	3.0212	..	... ..	....	16	2 1733	3.7096
2 1/2	4	2 1752	3.7161	..	......	....	16	2 4233	4.6123
2 3/4	4	2 4252	4.6194	..	.	.....	16	2.6733	5.6129
3	4	2 6752	5.6209	..	..	....	16	2.9233	6.7118

The pitch p is the axial distance between corresponding points on adjacent threads and is equal to the reciprocal of the number of threads per inch.

TABLE 27.—SELLERS STANDARD SQUARE THREADS

Bolt diam., in.	Threads per in.	Root diam., in.	Root area, sq in.
1/4	10	0 1625	0 0207
5/16	9	0.2153	0 0375
3/8	8	0.2658	0.0555
7/16	7	0 3125	0 0767
1/2	6 1/2	0.3656	0.1049
9/16	6	0.4167	0.1364
5/8	5 1/2	0.4666	0.1709
11/16	5	0.5125	0.2063
3/4	5	0.5750	0.2597
13/16	4 1/2	0.6181	0.3000
7/8	4 1/2	0.6806	0.3638
15/16	4	0.7188	0.4058
1	4	0.7813	0.4804
1 1/8	3 1/2	0.8750	0 6013
1 1/4	3 1/2	1.0000	0.7854
1 3/8	3	1.0834	0.9201
1 1/2	3	1.2084	1.1462
1 5/8	2 3/4	1.307	1 3414
1 3/4	2 1/2	1.400	1.5394
1 7/8	2 1/2	1.525	1.8265
2	2 1/4	1 612	2.0422
2 1/4	2 1/4	1.862	2.7245
2 1/2	2	2.063	3.3410
2 3/4	2	2.313	4.2000
3	1 3/4	2.500	4.9087
3 1/4	1 3/4	2.750	5.9396
3 1/2	1 5/8	2.962	6.8930
3 3/4	1 1/2	3.168	7.8853
4	1 1/2	3 418	8.8434

TABLE 28.—ACME SCREW THREADS

Threads per in.	Depth of thread, in.	Thickness at root of thread, in.
1	0 5100	0 6345
1 1/2	0 3850	0 4772
2	0.2600	0 3199
3	0.1767	0.2150
4	0.1350	0.1625
5	0.1100	0.1311
6	0.0933	0.1101
7	0.0814	0.0951
8	0 0725	0 0839
9	0.0655	0.0751
10	0.0600	0.0681

The lead is the axial distance a thread advances in one revolution. A single thread is one on which the lead equals the pitch; a double thread is one on which the lead equals twice the pitch; and a triple thread is one on which the lead equals three times the pitch.

Major diameter is the outside, or largest, diameter of the threads and is the nominal diameter. Minor diameter is the smallest diameter of the threads and is commonly called the root diameter. Basic pitch diameter is the mean of the major and minor diameters.

A 1-in. National Coarse Series right-hand thread with eight threads per inch and Class 2 fit is designated: 1 in.-8NC-2. If this thread is a left-hand thread it is designated: 1 in -8NC-2-LH.

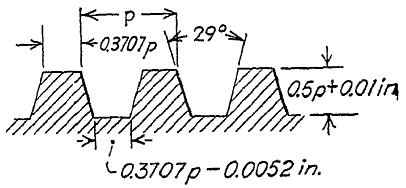


FIG. 80.

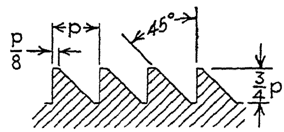


FIG. 81.

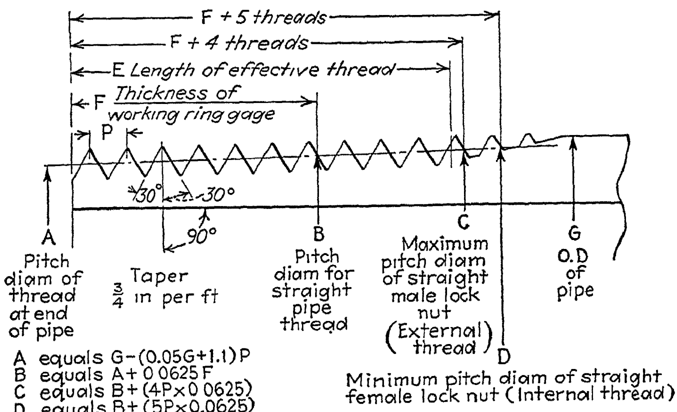


FIG. 82.—American, or Briggs, standard pipe threads.

The square, the Acme, and the buttress threads are used for power screws, being more efficient than the 60-deg. Sellers thread. The square thread has the highest efficiency, but is comparatively costly to make, and adjustment for wear is difficult. The Acme thread is not so costly, and adjustment for wear can be accomplished by using nuts split lengthwise; hence it is used for power drives where there must be little or no backlash, such as feed screws and lead screws of machine tools. Its efficiency is less than that of the square thread. When the power transmission

is in one direction only, the buttress thread is used, the flat driving side retaining the high efficiency of the square thread, and the sloping side permitting adjustment by means of a split nut.

107. Bolts. Bolt sizes are designated by the outside diameter of the thread and by the length under the head. Stock sizes vary in length by $\frac{1}{4}$ in. from 1 to 5 in., by $\frac{1}{2}$ in. from $5\frac{1}{2}$ to 12 in., and by inches for all longer bolts, other lengths being obtained by special order. The threaded length is about $1\frac{1}{2}$ times the diameter.

Through bolts are used where both the head and nut can be made accessible by the use of flange connections and are the most satisfactory form of screw fastenings, since they can be easily renewed when broken or when the threads strip. Machine bolts have rough bodies and either rough or finished heads and nuts. Since the body of the bolt is not finished, the hole should be drilled $\frac{1}{16}$ in. larger than the bolt. Coupling bolts are through bolts having finished bodies, and are used with reamed holes. When the bolt is subjected to shear loads, this is the proper construction.

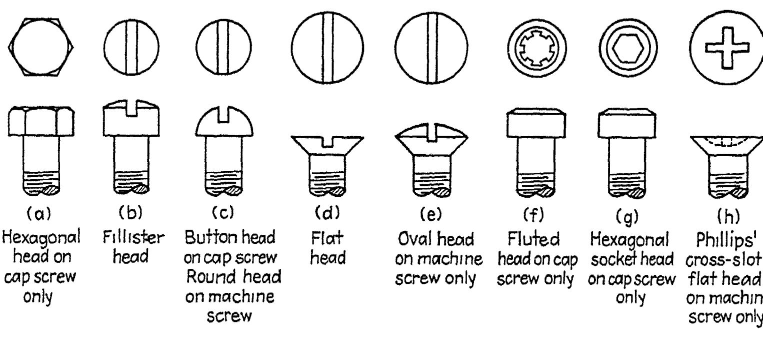


FIG. 83 —Forms of cap- and machine-screw heads. Cap-screw ends are chamfered to the thread root at 35 deg. Machine-screw ends are flat.

Cap screws have no nuts, the screw passing through a clearance hole in one member and threading into the mating member. They are desirable where lack of space or other considerations prevents the use of through bolts, but should not be used when they must be removed frequently as this may ruin the thread in the tapped hole. Cap screws are threaded for a length of two diameters plus $\frac{1}{4}$ in. and should enter the tapped hole at least one

diameter in steel, and at least $1\frac{1}{2}$ diameters in cast iron and aluminum.

Machine screws are small screws, in the National Coarse and Fine Thread series, designated by size numbers up to No. 12 (0.216 in.). The larger sizes, $\frac{1}{4}$, $\frac{5}{16}$, and $\frac{3}{8}$ in. diameter, are seldom used. Four A.S.A. standard slotted heads are shown in Fig. 83. The Phillips cross-slot head machine screws are obtainable in sizes up to $\frac{1}{2}$ in. and are used with flat, round, oval, and fillister heads.

Setscrews are used to prevent relative motion by means of pressure exerted on their points. The heads are square, the height and the distance across the flats being equal to the diameter of the threaded shank. Headless setscrews are made with either a screw-driver slot or hollowed out to receive hexagonal or spline-shaped wrenches. The various types of points used on setscrews

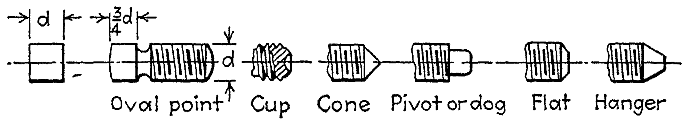


FIG. 84.—Forms of setscrew points.

are shown in Fig. 84. The cone and cup points raise burrs on the shaft, which are objectionable, and the shaft should be flattened off, or a conical seat drilled in the shaft to prevent this source of trouble. The proper size of setscrew may be determined by the empirical formula

$$d = \frac{D}{8} + \frac{5}{16} \text{ in.} \tag{78}$$

where d = setscrew diameter, in.
D = shaft diameter, in.

The maximum safe holding force of setscrews in pounds is given by the formula*

$$F = 2{,}500d^{2.31} \tag{79}$$

Studs are threaded on both ends and are used where through bolts are undesirable. The threads are either of the National Coarse or Fine series, depending on the material into which the stud is screwed. Studs are preferable to cap screws since they

* Based on experiments reported by B. H. D. Pinkney in *Machinery*, Oct. 15, 1914.

need not be removed when the joints are disassembled. To secure frictional resistance against turning when the nuts are removed, studs should enter the tapped holes at least $1\frac{1}{4}$ diameters, preferably $1\frac{1}{2}$ diameters in brittle materials. The holes

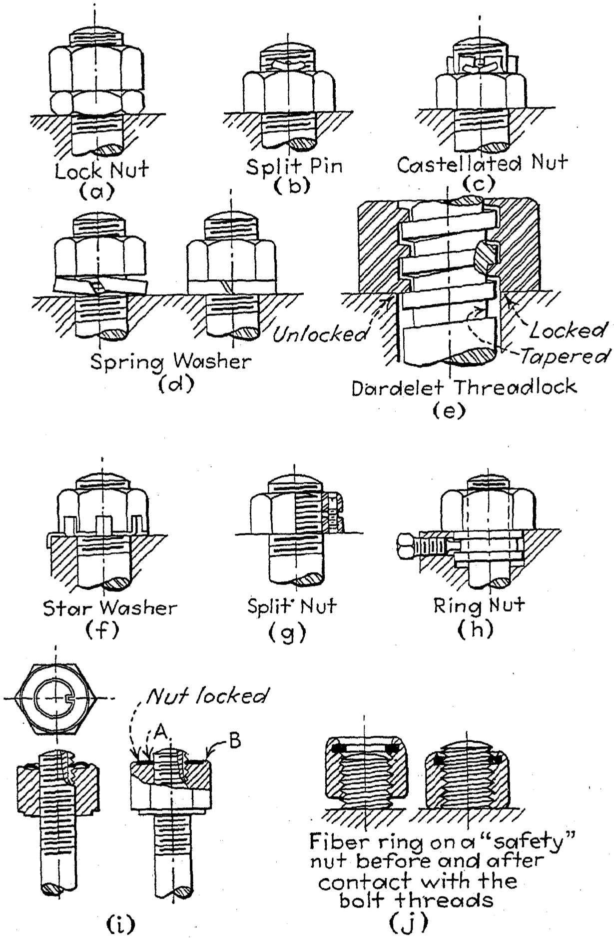


FIG. 85.—Locking devices.

should be drilled at least $\frac{1}{2}$ diameter deeper than the threaded length of the studs.

To prevent nuts from working loose, many types of special washers, pins, and nuts are used as locking devices. Some of the most common locking devices are shown in Fig. 85.

108. Effect of Initial Tension. As a result of the turning of the nut, a bolt is subjected to direct tension, compression on the threads, shear across the threads, and torsional shear in the body of the bolt. Since none of these stresses can be accurately determined, bolts are designed on the basis of the direct tension stress with a comparatively high factor of safety to allow for the undetermined stresses. Calculations and experimental evidence indicate that the tensile load in pounds applied by an experienced mechanic when tightening a bolt with a wrench of ordinary proportions, is about 16,000 times the bolt diameter. Subsequent loads may or may not increase this tension.

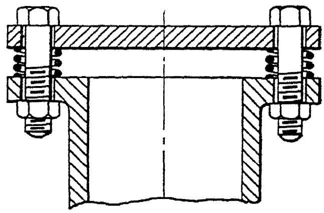


FIG. 86.

A bolted joint is shown diagrammatically in Fig. 86, the packing being represented by the coiled springs. Let

F = final load on bolt.
F_a = external or applied load.
F_i = initial load due to tightening.
C = final compressive load on packing or spring.
e = elongation of the bolt per unit load.
c = compression of packing per unit load.

When the bolt is first tightened, and before the external load is applied, the tension on the bolt and the compression on the packing will be equal, and the corresponding deformations will be

$$F_ie = \text{initial elongation of the bolt}$$
$$F_ic = \text{initial compression of the packing}$$

Application of the external load F_a increases the length of the bolt and the packing an amount equal to $(F - F_i)e$. Hence the final deformation of the packing is

$$Cc = F_ic - (F - F_i)e$$

and the compression load on the packing is

$$C = F_i - \frac{e}{c}(F - F_i)$$

The final tension load on the bolt is the sum of the packing load

and the applied external load; hence

$$F = C + F_a = F_i - \frac{e}{c}(F - F_i) + F_a$$

from which

$$F = \left(\frac{c}{c + e}\right) F_a + F_i = KF_a + F_i \tag{80}$$

This equation indicates that the final load carried by the bolt depends upon the initial tension, the external load, and the factor K, which is a measure of the relative stiffness of the packing and the bolt. With a metal-to-metal joint, or a packing such that the deformation of the packing is relatively small compared to that of the bolt, the value of c approaches zero, and the term KF_a vanishes. In this case the applied external load has no effect on the stress in the bolt unless $F > F_i$; and the final tension is equal to the initial tension produced by tightening the bolt. When a soft packing is used, so that the deformation of the packing is very large compared to that of the bolt, the value of e approaches zero, and the factor K becomes unity. In this case, the final tension on the bolt is the sum of the initial tension and the applied load. In any case, the tension must be sufficient to keep the joint tight so that, even with metal-to-metal joints, the initial load must be larger than the applied load. Table 29 may be used as a guide in selecting the proper value of K when the final load on the bolt is to be determined.

TABLE 29.—VALUES OF K FOR EQ (80)

Type of Joint	$K = \frac{c}{c + e}$
Soft packing with studs	1.00
Soft packing with through bolts	0 75
Asbestos	0.60
Soft-copper gasket with long through bolts	0.50
Hard-copper gasket with long through bolts	0 25
Metal-to-metal joints with through bolts	0.00

109. Allowable Stresses in Tension Bolts. As indicated in Art. 108, the initial load due to tightening the nut on a standard ½-in. National Coarse bolt is approximately 16,000 × ½, or 8,000 lb. The stress at the thread root produced by this load is

$$s_t = \frac{8,000}{0.1257} = 63,500 \text{ psi approximately}$$

This stress is equal to or greater than the ultimate strength of the materials generally used for bolts. Bolts smaller than $\frac{5}{8}$ in. are undesirable in any joint subjected to applied loads, and in many types of machines $\frac{3}{4}$-in. bolts are the smallest permitted. Smaller bolts must be used in some designs and, when used, should be made of alloy steels.

The torsional shear, combined with the direct tensile stress in the bolt, produces an equivalent tension about 25 per cent greater than the initial tensile stress. It is evident that the actual loads and stresses imposed on bolts are very indefinite, and that the actual design must be based on empirical formulas that reduce the apparent working stresses as the bolt diameter is reduced. One formula, proposed by Seaton and Routhwaite,* giving reasonable results when used for bolts made of steel containing from 0.08 to 0.25 per cent carbon and with diameters of $\frac{3}{4}$ in. and over, is

$$s_w = C(A_r)^{0.418} \tag{81}$$

from which the total load capacity of the bolt is

$$F_a = s_w A_r = C(A_r)^{1.418} \tag{82}$$

where F_a is the applied load (not including the initial tightening load), s_w is the permissible working stress, A_r is the root area.

The constant C may be taken as 5,000 for carbon-steel bolts of 60,000 psi ultimate tensile strength, and up to 15,000 for alloy-steel bolts, increasing in direct proportion to the ultimate strength of the steel. For bronze bolts, C may be 1,000.

Bolts 2 in. and larger are generally designed for a stress of 7,000 to 8,000 psi with carbon steels, and up to 20,000 psi with alloy steels, the initial tension being disregarded.

Illustrative Examples. Example 1. Determine the size of bolt required for the head of a 12-in. cylinder containing steam at 200 psi. Assume a hard gasket used in making up the joint.

The diameter of the bolt circle will be approximately 14 in, and 12 bolts may be assumed. Assuming that steam may enter under the head to the bolt circle, the load per bolt is

$$F_a = \frac{\pi 14^2}{4} \times \frac{200}{12} = 2{,}560 \text{ lb}$$

* SEATON and ROUTHWAITE, "Marine Engineer's Pocket Book."

Equation (82) gives

$$A_r = \left(\frac{2,560}{5,000}\right)^{\frac{1}{1\ 418}} = 0.625 \text{ sq in.}$$

Table 26 gives a $1\frac{1}{8}$-in. bolt with a root area of 0 693 sq in.

The initial stress in this bolt will be approximately 25,900 psi, and the applied stress will be 3,690 psi The probable final stress, using Eq. (80), is

$$s_t = \frac{KF_a}{A_r} + \frac{F_i}{A_r} = 0.25 \times 3{,}690 + 25{,}900 = 26{,}820 \text{ psi}$$

This stress is less than the yield stress of this material and is safe. A stress slightly higher than the yield stress might also be considered safe, since a slight tightening of the nut would take up any stretch of the bolt without causing failure. The distance from the cylinder wall to the bolt centers should be at least equal to the bolt diameter; hence the bolt-circle diameter should be increased to $14\frac{1}{4}$ or $14\frac{1}{2}$ in. The bolt spacing should be checked by means of Fig. 87.

Example 2. The cylinder of a stationary engine is $4\frac{3}{4}$ in. in diameter and is held to the crankcase by $\frac{1}{2}$-in. nickel steel bolts with National Coarse threads. The maximum gas pressure in the cylinder is 500 psi. Assume the ultimate strength of this steel to be 110,000 psi and the yield stress to be 85,000 psi. Determine the number of bolts required.

Since these bolts are less than $\frac{3}{4}$ in. in diameter, Eq (82) does not apply directly, and it is necessary to consider the initial tightening stress For a $\frac{1}{2}$-in. bolt, S_i is 63,500 psi. Subtracting this stress from the yield stress leaves 21,500 psi available for the applied gas load. Assuming a real factor of safety of 2 (based on the applied load) and a gasket factor of 0.75, the number of bolts is determined as follows. The load capacity per bolt is

$$F_a = A_r S_a = 0.126 \times \frac{21{,}500}{2} = 1{,}355 \text{ lb}$$

The total load on all bolts is

$$F_t = \frac{\pi \times 4.75^2}{4} \times 500 = 8{,}850 \text{ lb}$$

and the number of bolts required is

$$N = \frac{8{,}850}{1{,}355} = 6.53, \text{ say 6 bolts}$$

The probable stress in each bolt is

$$63{,}500 + \frac{0.75 \times 8{,}850}{0.126 \times 6} = 63{,}500 + 8{,}780 = 72{,}280 \text{ psi}$$

which is about 85 per cent of the yield stress and therefore satisfactory. In order to maintain an oil-tight joint it may be necessary to use eight or nine bolts.

110. Bolt Spacing. To permit the proper use of wrenches on the bolt heads and nuts, the spacing of the bolts should not be closer than the values indicated in Fig. 87. These values may be decreased slightly by the use of special wrenches. To maintain uniform contact pressure in a gasketed joint, the pitch should not be excessive, say five to six bolt diameters.

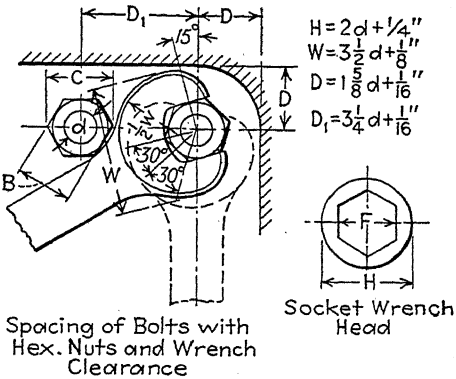


FIG. 87.

111. Bolts Subject to Impact Loading. The energy of a blow or impact load must be absorbed by an elongation of the bolt. The threaded portion of the bolt is weaker than the shank, not only because of the reduced area, but also because the groove formed by the thread is a region of highly localized stress, a condition that is undesirable when shock and repeated loads are encountered. The resistance to shock is measured by the amount of work that can be absorbed without rupture; hence, if the shank area is reduced by making it hollow or by machining it to a diameter slightly smaller than the root diameter of the thread, the shank will have greater elongation under any condition of impact loading and will absorb more of the impact energy. This will reduce the elongation and stress induced in the threaded portion and increase the ability of the bolt to resist impact. A bolt of this type is as strong as a regular bolt when subjected to

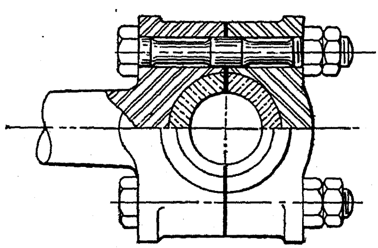


FIG. 88.

steady load, and much stronger when subjected to impact loads. Figure 88 shows this type of bolt in a connecting-rod end.

112. Bolts Subject to Shear. Bolts subjected to shear loads should be fitted tightly into reamed holes, and the plane of shear should never be across the threaded portion of the shank. In the best designs, the diameter of the shank is made slightly larger than the threaded portion in order to prevent injury to the threads by bearing pressure. When the bolt is subjected to both tension and shear loads, the root diameter can be approximated from the tension load, and the diameter of the shank may be approximated from the shear load. A diameter slightly larger than that required for either tension or shear can then be assumed, and the stresses due to combined tension and shear computed as outlined in Art. 55.

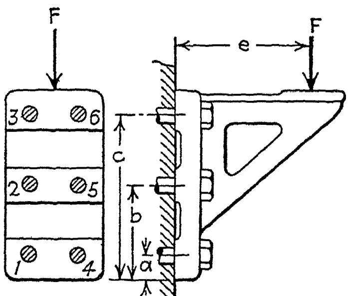


FIG. 89.

113. Eccentric Loading. In the bracket shown in Fig. 89, the load tends to rotate the bracket about its lower edge, and the bolts are not equally stressed. When the flange is heavy, it may be considered to be a rigid body, and the elongation of the bolts will be proportional to their distances from the lower edge. Hence the stresses will also be proportional to the distances from the lower edge. In practice, the bolts would all be made the same size as the most heavily loaded bolt. Then

$$T_1 = T_4 = \frac{a}{c} T_6$$

and

$$T_2 = T_5 = \frac{b}{c} T_6$$

Taking moments about the lower edge,

$$Fe = (T_1 + T_4)a + (T_2 + T_5)b + (T_3 + T_6)c \tag{83}$$

Substitution in these equations will determine the maximum tensile load on any bolt in the group. There is also a shear load that may be considered to be equally distributed between the

bolts if they are fitted in reamed holes, and distributed between two bolts if rough bolts are used in clearance holes.

Each bolt of the bracket shown in Fig. 90 is subjected to two shearing stresses; a primary shear equal to the applied load divided by the number of bolts; and a secondary shear caused by the twisting moment. The bracket will tend to rotate about the center of gravity of the bolts, at G, and the total secondary shear on each bolt will be proportional to its distance from G and normal to the line joining G and the bolt center. Taking moments about G,

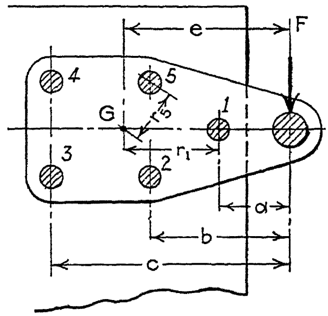


FIG. 90.

$$Fe = F_1r_1 + F_2r_2 + F_3r_3 \cdots \text{etc.}$$
$$= F_1r_1 + F_1\frac{r_2^2}{r_1} + F_1\frac{r_3^2}{r_1} \cdots \text{etc.} \tag{84}$$

from which the secondary shearing loads on each bolt may be determined. The total shearing stress on any bolt is the vector sum of the primary and secondary shear stresses on that bolt.

114. Power Screws. Screws used for the transmission of power develop considerable friction, and efficiency, wear, and heating become prime considerations in their design. It has already been noted that the square thread is used because of its higher efficiency, and that the Acme or buttress threads are used when adjustment of the nut to prevent backlash is necessary. Figure 91 shows a power screw arranged so that when a turning moment is applied to the nut, the screw will be advanced against an axial load F_a. The relation between the applied torque and the resisting load is expressed by the equation

$$T = \frac{F_a p}{2\pi e} \tag{85}$$

where the symbols have the meanings given below.

In the discussions that follow, let

A_n = total normal area of threads, sq in.
D_o = outside diameter of screw, in.
D_m = mean diameter of screw, in.

D_c = mean diameter of thrust collar, in.
e = efficiency.
f = coefficient of friction on threads.
f_c = coefficient of friction on thrust collar.
F_a = total axial resisting load, lb.
F_n = total force normal to thread surface, lb.
p = lead of threads, in.
p_n = unit normal force on threads, psi.
W = work done, in-lb; subscripts u, f, and c refer to useful work, thread friction, and collar friction.
T = turning moment or torque required to turn the nut (or screw), lb-in.
α = helix angle or lead angle of threads at mean diameter, deg.
ϕ = one-half the included thread angle measured on a plane through axis, deg.
θ = angle between the normal to thread surface and a line parallel to axis, deg.

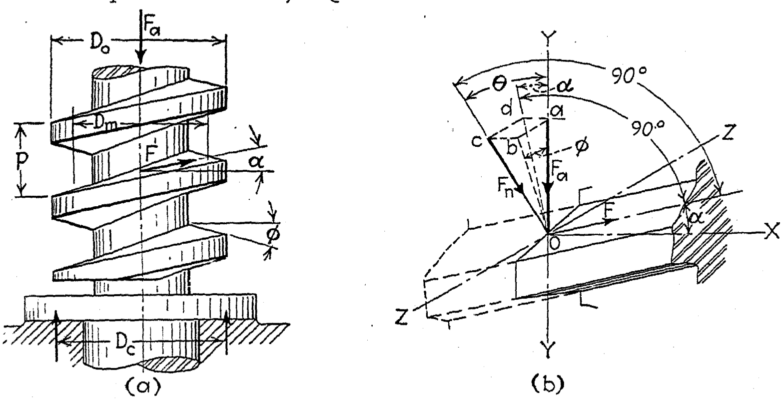


FIG. 91.

The forces acting on the screw threads are shown in Fig. 91. The total axial force exerted must equal the algebraic sum of the axial components of the forces normal to the thread surface and the friction forces. Hence

$$F_a = A_n(p_n \cos \theta - fp_n \sin \alpha) = F_n(\cos \theta - f \sin \alpha),$$

from which

$$F_n = \frac{F_a}{\cos \theta - f \sin \alpha}$$

When a torque T is applied to the nut by means of a lever, a gear, or any other turning medium, the screw will move forward p inches for one revolution of the nut. Hence, during one revolution

$$W_u = F_a p = F_n(\cos\theta - f\sin\alpha)p = \text{useful work}$$

$$W_f = \frac{fF_n p}{\sin\alpha} = \frac{fF_a p}{\sin\alpha(\cos\theta - f\sin\alpha)} = \text{work in overcoming thread friction}$$

$$W_c = f_c F_a \pi D_c = \text{work in overcoming collar friction}$$

$$W = 2\pi T = \text{work applied to turn the nut}$$

It is evident that

$$W = W_u + W_f + W_c$$

and by substitution

$$T = \frac{F_a p}{2\pi}\left[1 + \frac{f}{\sin\alpha(\cos\theta - f\sin\alpha)} + \frac{f_c\pi D_c}{p}\right] \tag{86}$$

The angle θ is dependent upon the lead and the included angle of the threads. From Fig. 91 it is found that

$$\cos\theta = \frac{1}{\sqrt{1 + \tan^2\phi + \tan^2\alpha}} \tag{87}$$

Equation (86) applies directly to horizontal screws and to "lifting screws" having a vertical axis, the load lifted being the axial force F_a. When the load is being lowered, the friction forces are reversed; hence for a "lowering screw"

$$T = \frac{F_a p}{2\pi}\left[\frac{f}{\sin\alpha(\cos\theta + f\sin\alpha)} + \frac{f_c\pi D_c}{p} - 1\right] \tag{88}$$

When the lead is large or the coefficient of friction small, the axial load may be sufficient to turn the nut, and the screw is said to "overhaul." In the limiting condition, no torque is required to lower the load, the axial load just balances the friction forces, and the right-hand side of Eq. 88 becomes zero. Hence the overhauling condition is reached when

$$1 - \frac{f}{\sin\alpha(\cos\theta + f\sin\alpha)} = \frac{f_c\pi D_c}{p} = \frac{f_c D_c}{D_m \tan\alpha}$$

or

$$\tan \alpha = \frac{f \cos \alpha + \dfrac{f_c D_c}{D_m} (\cos \theta + f \sin \alpha)}{\cos \theta} \tag{89}$$

This equation applies to any type of power screw. For the special case of the square-thread screw, ϕ is zero, $\tan \phi$ is zero, and $\cos \theta$ becomes $\cos \alpha$. Substituting these values in the equation and transposing, the overhauling condition for square threads is found to be

$$\tan \alpha \gtreqless \frac{fD_m + f_c D_c}{D_m - ff_c D_c} \tag{90}$$

In the usual construction, f_c is equal to f. When a ball or roller thrust bearing is used, f_c becomes practically zero, and the screw will overhaul when $\tan \alpha$ is equal to f.

115. Efficiency of Screw Threads. The efficiency of a screw thread is the ratio of the useful work to the work input, or

$$e = \frac{W_u}{W} = \frac{F_a p}{2\pi T} = \frac{W_u}{W_u + W_f + W_c} \tag{91}$$

$$= \frac{1}{1 + \dfrac{f}{\sin \alpha(\cos \theta - f \sin \alpha)} + \dfrac{f_c D_c}{D_m \tan \alpha}}$$

$$= \frac{\tan \alpha(\cos \theta - f \sin \alpha)}{\tan \alpha \cos \theta + f \cos \alpha + \dfrac{f_c D_c}{D_m} (\cos \theta - f \sin \alpha)}$$

For the special case of square threads, the efficiency is

$$e = \frac{\tan \alpha(1 - f \tan \alpha)}{\tan \alpha + f + \dfrac{f_c D_c}{D_m} (1 - f \tan \alpha)} \tag{92}$$

To show more clearly the relation between efficiency and helix angle, efficiency of square threads against helix angles has been plotted in Fig. 92. The curve reveals that for the ordinary values of f, the efficiency rises rapidly as the angle increases to 15 or 20 deg and more slowly until the maximum is reached at angles between 40 and 45 deg. However, as the helix angle is increased, the screw becomes more difficult to machine, and the mechanical advantage decreases. It should also be noted that any screw having an efficiency of over 50 per cent will overhaul, and will

not support an axial load without an applied torque. The actual angle selected must be a compromise based on the particular service requirements, and, in practice, angles as high as 30 deg are seldom used. Since the cos θ decreases with an increase in the included thread angle, it follows that triangular threads are less efficient than square threads.

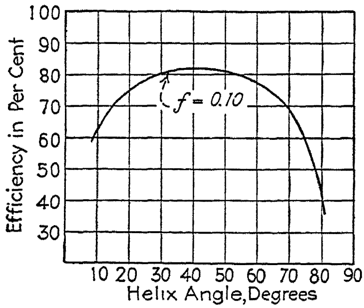


FIG. 92.—Efficiency of square threads not including thrust collars.

116. Coefficient of Friction. The coefficient of friction varies with the quality of lubrication, with the materials used, and with the unit pressure on the threads. Most power screws are made of steel with nuts of cast iron or bronze. When the unit pressure is less than 14,000 psi and the rubbing velocity less than 50 fpm, the coefficients shown in Table 30 will be obtained with average lubrication.

TABLE 30.—COEFFICIENT OF FRICTION FOR POWER SCREWS

Lubricant	Coefficient of Friction
Machine oil and graphite	0.07
Lard oil	0.11
Heavy machine oil	0.14

Tests made at the University of Illinois* indicate that plain thrust collars used with power screws have average values of the coefficient of friction as shown in Table 31.

TABLE 31.—COEFFICIENT OF FRICTION ON THRUST COLLARS

Material	Coefficient of running friction	Coefficient of starting friction
Soft steel on cast iron	0.121	0 170
Hardened steel on cast iron	0.092	0.147
Soft steel on bronze	0.084	0 ·101
Hardened steel on bronze	0 063	0.081

* HAM, CLARENCE W., and RYAN, DAVID G., *Univ. Ill Bull.*, Vol. 29, No. 81, June 7, 1932

In these tests, the coefficient of friction was found to be independent of the load and speed within the ranges used in common practice. Hardened steel on soft steel was found to be unsatisfactory, since galling, or seizing, occurred at fairly low pressures.

117. Differential Screws.* For some types of service a very slow advance of the screw is required, whereas in other services a very rapid movement is required. With a single screw, slow movement is obtained by using a small helix angle and hence a small pitch, which gives a weak thread. A rapid movement

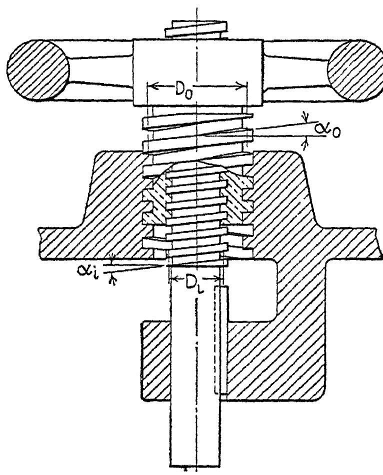


FIG. 93.

requires a large helix angle with the attendant mechanical difficulties in machining. To minimize these difficulties, an arrangement similar to Fig. 93 may be used, in which the revolving member is threaded on the outside as well as on the inside. When the threads are of the same "hand" and of different pitch the driven screw moves slowly, and the arrangement is called a differential screw. When the threads are of opposite hand, the driven screw moves rapidly and the arrangement is called a compound screw. As previously indicated, a square thread will not be self-locking when tan α is greater than f; hence when rapid movement with self-locking properties is desired, the compound screw with its smaller helix angles may

* KINGMAN, A. R., Determining Efficiency of Differential Screws, *Machine Design*, April, 1934, p. 25.

be used to advantage. When self-locking is not required, the choice between a single screw and a compound screw must be made on the basis of space requirements, cost, and efficiency.

The efficiency of a compound screw, not including the collar friction, is given by the equation

$$e = \frac{D_o \tan \alpha_o + D_i \tan \alpha_i}{D_o \dfrac{\tan \alpha_o + f_o}{1 - f_o \tan \alpha_o} + D_i \dfrac{\tan \alpha_i + f_i}{1 - f_i \tan \alpha_i}} \tag{93}$$

The efficiency of a differential screw is given by the equation

$$e = \frac{D_o \tan \alpha_o - D_i \tan \alpha_i}{D_o \dfrac{\tan \alpha_o + f_o}{1 - f_o \tan \alpha_o} - D_i \dfrac{\tan \alpha_i - f_i}{1 + f_i \tan \alpha_i}} \tag{94}$$

118. Stresses in Power Screws. A power screw is subjected to an axial load and to a turning moment, which induce in the screw direct tension or compression, torsional shear, shear across the root of the threads, and compression between the thread surfaces. If the axial load is compressive and the unsupported length is more than six or eight times the root diameter, the screw must be treated as a column. If possible, the thrust collar should be placed so that the screw is in tension.

The compressive stress or bearing pressure between the thread surfaces must be limited in order to reduce wear. In general, pressures of 2,000 psi should not be exceeded if the rubbing velocity is greater than 50 fpm, although pressures as high as 10,000 psi have been used with low velocities and adequate lubrication. With feed screws and the like, where accuracy and the maintaining of small backlash are important, pressures should be less than 200 psi.

Illustrative Example. A screw press is to exert a force of 12,000 lb with an applied torque of 5,000 lb-in. The unsupported length of the screw is 18 in. and a thrust bearing of hardened steel on cast iron is provided at the power end. The screw is to be made of S.A.E. 1045 steel having an ultimate strength of 76,000 psi and a yield stress of 38,000 psi. The design stresses are to be 12,500 psi in tension and compression, 7,500 psi in shear, and 2,000 psi in thread bearing. The nut is cast iron, and the permissible shear is 3,000 psi. Determine the dimensions of the screw and nut.

Neglecting any column action, the root diameter must be at least

$$D_r = \left(\frac{12,000}{\frac{\pi}{4} \times 12,500}\right)^{\frac{1}{2}} = 1.11 \text{ in.}$$

Allowing for the increase in stress due to column action and torsional shear, a trial mean diameter of $1\frac{1}{2}$ in. is assumed. The efficiency of the screw and thrust collar will be approximately 15 per cent, and the torque converted into useful axial work will be 0.15 times 5,000 or 750 lb-in. Equating the useful work per revolution to the work equivalent of this torque

$$12,000p = 2\pi 750$$

and

$$p = 0.395 \text{ in.}$$

Assuming square threads and a pitch of three threads per inch, the depth of the thread will be 0.167 in. and the outside diameter will be $1\frac{1}{2}$ plus 0.167 or 1.667 in. The next larger commercial size is $1\frac{3}{4}$ in. giving

$$\begin{aligned} D_o &= 1.75 \text{ in.} \\ D_r &= 1.75 - 2 \times 0.167 = 1.416 \text{ in.} \\ D_m &= 1.75 - 0.167 = 1.583 \text{ in.} \end{aligned}$$

The mean diameter of the thrust collar varies with the construction and may be assumed in this case to be 2 in. Using these dimensions

$$\tan \alpha = \frac{p}{\pi D_m} = \frac{0.333}{\pi \times 1.583} = 0.067$$

and the efficiency from Eq. (92) is

$$\begin{aligned} e &= \frac{0.067(1 - 0.14 \times 0.067)}{0.067 + 0.14 + \frac{0.14 \times 2}{1.583}(1 - 0.14 \times 0.067)} \\ &= 0.175, \text{ or } 17.5 \text{ per cent} \end{aligned}$$

which is greater than the assumed value of 15 per cent.

A portion of the applied torque is absorbed in friction in the thrust collar, and the torque transmitted to the screw is

$$T - \frac{f_c F_a D_c}{2} = 5,000 - 0.14 \times 12,000 \times 1 = 3,320 \text{ lb-in.}$$

This torque produces a torsional shear stress at the root of the threads.

$$s_s = \frac{Tc}{J} = \frac{3,320 \times 16}{\pi \times 1.416^3} = 5,940 \text{ psi}$$

Note that in some cases the torque may be applied in such a manner that the entire torque is transmitted through the screw.

The stress due to column action, assuming one end to be fixed and the other pivoted, is, from Eq. (18),

$$s_c = \frac{F_a}{A_r}\left[\frac{1}{1 - \frac{s_y}{4n\pi^2 E}\left(\frac{L}{K}\right)^2}\right]$$

$$= \frac{12{,}000}{1.58}\left[\frac{1}{1 - \frac{38{,}000}{4 \times 1 \times \pi^2 \times 30 \times 10^6}\left(\frac{18}{0.354}\right)^2}\right] = 8{,}300 \text{ psi}$$

The combined stresses at the root of the threads, are

$$s_{s\,max} = \tfrac{1}{2}\sqrt{8{,}300^2 + 4 \times 5{,}940^2} = 7{,}270 \text{ psi}$$

and

$$s_{t\,max} = \tfrac{1}{2}[8{,}300 + \sqrt{8{,}300^2 + 4 \times 5{,}940^2}] = 11{,}420 \text{ psi}$$

both of which are less than the permissible stresses.

Since the bearing pressure on the threads is limited, the length of the nut must be determined. The bearing capacity of one turn of the thread is

$$As_b = 2{,}000\,\frac{\pi}{4}\,(D_0^2 - D_i^2) = 2{,}000\,\frac{\pi}{4}\,(1.75^2 - 1.416^2) = 1{,}680 \text{ lb}$$

and the minimum length of nut is

$$\frac{F_a p}{As_b} = \frac{12{,}000 \times 0.333}{1{,}680} = 2.38, \text{ say } 2\tfrac{3}{8} \text{ in.}$$

The shear stress across the threads of the nut is

$$s_s = \frac{F_a}{\pi D_o}\,\frac{2}{L} = \frac{12{,}000 \times 2}{\pi \times 1.75 \times 2.375} = 1{,}840 \text{ psi}$$

which is less than the stress permitted for the cast iron.

The student should rework this problem using Sellers standard square threads.

CHAPTER VIII

KEYS, COTTERS, AND KNUCKLE JOINTS

Pulleys, gears, levers, and similar devices used to transmit power to or from shafts must be rigidly attached to the shaft by shrinkage, setscrews, keys, or cotters. Shrink fits are suitable only for permanent assemblies, setscrews for light service, and cotters for axial loads. When the parts must be disassembled,

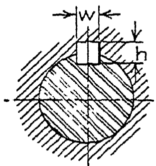


FIG. 94.

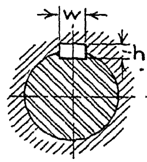


FIG. 95.

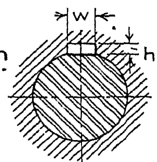


FIG. 96.

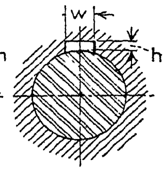


FIG. 97.

and when torsional loads are transmitted, keys are generally used. The key is a piece of metal fitted into mating grooves in the shaft and mating member and transmitting power by shear across the key. The cutting of the keyway in the shaft reduces its strength and rigidity by an amount that depends upon the shape and size of the keyway. For a discussion of this effect see Art. 131.

119. Types of Keys. The square key, with the key sunk half in the shaft and half in the hub, is the type most commonly used. Flat keys are used where the weakening of the shaft is serious. Although there is no universal standard,* the key usually has sides equal to one-fourth the shaft diameter.

* For standard sizes, see:

AESC bulletin B-17-e (1927), American Society of Mechanical Engineers.

KENT, "Mechanical Engineer's Handbook," John Wiley & Sons, Inc.

MARKS, "Mechanical Engineers' Handbook," 4th ed., McGraw-Hill Book Company, Inc., 1941.

Types of keys are shown in Figs. 94 to 103. Tangent, Kennedy, and Barth keys are used on large heavy-duty shafts. Round keys have the advantage that the keyway may be drilled and reamed after the mating parts are assembled. Small round keys are used for fastening cranks, handwheels, and other parts that

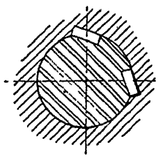


FIG. 98.

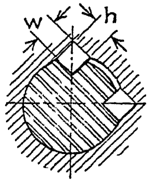


FIG. 99.

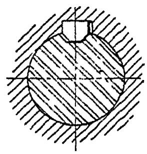


FIG. 100

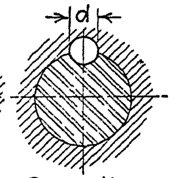


FIG. 101.

FIG. 98.—This key has the disadvantage of being difficult to fit. It is usually made in two parts, each tapered, to ensure a tight fit. Heavy forces can be transmitted in only one direction, and when the drive is reversible, two keys are used as shown.

FIG. 99.—This type of key is suitable for heavy duty. Shafts under 6 in. in diameter use one key; larger shafts use two keys placed 90 or 120 deg apart. To permit easy assembly, large hubs are bored to fit the shaft, and then rebored $\frac{1}{64}$ in. off center providing clearance on one side of the bore.

FIG. 100.—The double bevel ensures that the key will fit tightly against the top of the keyway when the drive is in either direction, and lessens the tendency to twist. The key does not require a tight fit and the small clearance permits easy assembly and removal.

FIG. 101.—This key has the advantage of not requiring close fits to prevent twisting, of being easily removed, and of being easily adjusted. It has the disadvantage of weakening the shaft because of its deep keyway.

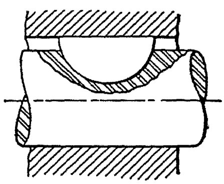


FIG. 102.

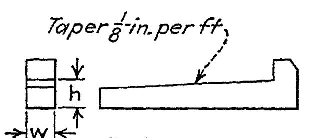
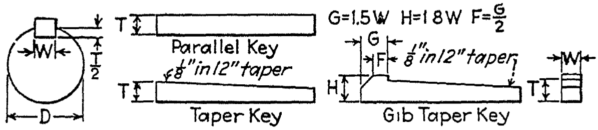


FIG. 103.

do not transmit heavy loads. A few manufacturers use round keys for heavy-duty shafts over 6 in. in diameter, since the absence of sharp corners reduces the high local stresses common to rectangular keyways, thus increasing the strength of the shaft. The keys may be either straight or tapered. The taper is usually $\frac{1}{8}$ in. per ft, and the diameter one-fourth the shaft diameter for shafts under 6 in., and one-fifth the shaft diameter for shafts 6 in. and over.

Saddle keys are used only for light work or in cases where relative motion between the shaft and its mating hub is required for adjustment and a keyway cannot be used in both. They are also used to hold parts during assembly until the permanent keyway can be located and machined. Since the power is transmitted by friction, the top or outer side of the key is tapered, insuring a large radial pressure.

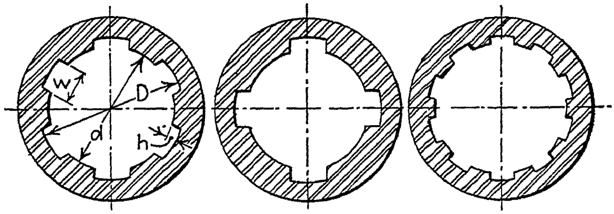
TABLE 32.—STANDARD KEYS
(A.S.M.E.)



Diam of shaft D	Width of key W	Thickness of key T		Depth of keyway $T/2$		Height of gib H	Length of gib G	Length of flat F	Tolerance on stock keys (—)	Size of setscrew to be used with parallel key
		Square	Flat	Square	Flat	For square keys				
1/2 – 9/16	1/8	1/8	3/32	1/16	3/64	1/4	1/4	1/8	0.0020	1/4
5/8 – 7/8	3/16	3/16	1/8	3/32	1/16	3/8	5/16	3/16	0.0020	1/4
15/16 – 1 1/4	1/4	1/4	3/16	1/8	3/32	7/16	3/8	3/16	0.0020	3/8
1 5/16 – 1 3/4	3/8	3/8	1/4	3/16	1/8	11/16	11/16	3/8	0.0020	3/8
1 13/16 – 2 1/4	1/2	1/2	3/8	1/4	3/16	7/8	3/4	3/8	0.0025	1/2
2 5/16 – 2 3/4	5/8	5/8	7/16	5/16	7/32	1 1/8	15/16	1/2	0.0025	5/8
2 7/8 – 3 1/4	3/4	3/4	1/2	3/8	1/4	1 3/8	1 1/8	9/16	0.0025	5/8
3 3/8 – 3 3/4	7/8	7/8	5/8	7/16	5/16	1 1/2	1 1/4	5/8	0.0030	3/4
3 7/8 – 4 1/2	1	1	3/4	1/2	3/8	1 7/8	1 1/2	3/4	0.0030	3/4
4 3/4 – 5 1/2	1 1/4	1 1/4	7/8	5/8	7/16	2 1/4	1 7/8	15/16	0.0030	7/8
5 3/4 – 7 3/8	1 1/2	1 1/2	1	3/4	1/2	2 5/8	2 1/4	1 1/8	0.0030	1
7 1/2 – 9 7/8	1 3/4	1 3/4	...	7/8		3 1/8	2 5/8	1 5/16	0.0030	1 1/4
10 – 12 1/2	2	2	..	1		3 5/8	3	1 1/2	0.0030	1 1/4

Feather keys are used when there must be relative axial motion between the shaft and the mating member. The key is made a tight fit and fastened in either the shaft or the hub and made a sliding fit in the mating member. The pressure in bearing on feather keys should not exceed 1,000 psi; and if the members are to slide when under load, the pressure should be reduced below this value.

Splines are permanent keys made integral with the shaft and fitting in keyways broached in the hub. The dimensions of splined shafts, as given in the standards of the S.A.E., are shown in Fig. 104.



	6 spline	4 spline	10 spline
Permanent fit	$d = 0.90D$ $w = 0.25D$ $h = 0.05D$	$d = 0.85D$ $w = 0.241D$ $h = 0.075D$	$d = 0.91D$ $w = 0.156D$ $h = 0.045D$
To slide when not under load	$d = 0.85D$ $w = 0.25D$ $h = 0.075D$	$d = 0.75D$ $w = 0.241D$ $h = 0.125D$	$d = 0.86D$ $w = 0.156D$ $h = 0.07D$
To slide when under load	$d = 0.80D$ $w = 0.25D$ $h = 0.10D$		$d = 0.81D$ $w = 0.156D$ $h = 0.095D$

Shaft dimensions 0.001 in. under nominal for small shafts and 0.002 in. for large shafts

FIG. 104.—S.A.E. standard splines.

120. Stresses in Keys. When the keyway is cut in both the shaft and the hub, force is transmitted by compression on the surfaces *ab* and *de*, Fig. 105. These compression forces act as a couple tending to roll the key, and, if the key is fitted on all four sides, induce a resisting couple acting on the surfaces *cd* and *af* as indicated by the forces marked F'. The crushing or bearing force is found approximately by considering the force F to act at the circumference of the shaft.

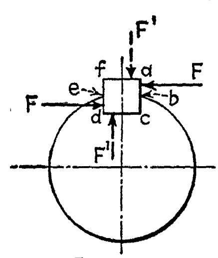


FIG. 105.

Let T = torsional moment transmitted, lb-in.

D = shaft diameter, in.

L = length of the key, in.

w = width of the key, in.

h = depth of the key, in.

Then

$$F = \frac{2T}{D} \tag{95}$$

The crushing stress on the surfaces *ab* and *de* is

$$s_c = \frac{2F}{Lh} = \frac{4T}{DLh} \tag{96}$$

and the shearing stress on the area *eb* is

$$s_s = \frac{F}{Lw} = \frac{2T}{DLw} \tag{97}$$

The key should be equally strong in crushing and in shear, and this condition is satisfied if Eqs. (96) and (97) are solved for the torsional moments and equated. Hence

$$\frac{DLhs_c}{4} = \frac{DLws_s}{2}$$

from which

$$\frac{h}{w} = \frac{2s_s}{s_c} \tag{98}$$

When the key is fitted on all four sides, the permissible crushing stress for the usual key materials is at least twice the permissible stress in shear. Assuming s_c equal to $2s_s$, the equation indicates that a square key is the proper shape. In this case, it is necessary to check the key for shear strength only. When the key is not fitted on all four sides, the permissible crushing stress is about 1.7 times the permissible shear stress, and the key must be checked for crushing. When the key is made of the same material as the shaft, the length of key required to transmit the full power capacity of the shaft is determined by equating the shear strength of the key to the torsional shear strength of the shaft. Hence

$$\frac{2T}{DLw} = \frac{Tc}{J} \times \frac{1}{0.75} = \frac{16T}{\pi D^3} \times \frac{1}{0.75}$$

and if $w = \dfrac{D}{4}$

$$L = 1.18D, \qquad \text{or approximately} \qquad L = 1.2D$$

In this equation, the value 0.75 is the effect of the keyway on the shaft strength (see Art. 131).

121. Taper Pins. Standard taper pins may be used as fasteners for light work by placing them tangent to the shaft or on

a diameter as shown in Figs. 106 and 107. Hollow shafts are frequently connected by means of a sleeve and taper pins, as in

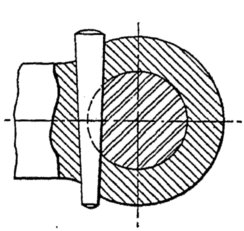


Fig. 106.

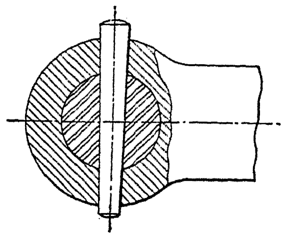


Fig. 107.

Fig. 108. The pin holes should be drilled and reamed with a taper reamer after the parts have been assembled. The diameter of the large end of the pin should be one-fourth the shaft diameter, and the taper is $\frac{1}{4}$ in. per ft.

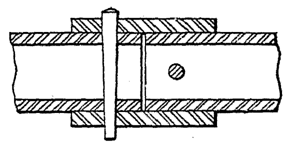


Fig. 108.

122. Taper Bushings. For light work, the hub may be taper-bored, and fitted with a taper bushing. The bushing is usually split axially into two or three parts so that when it is pressed into the hub it can exert a large radial pressure on the shaft. Since the power is transmitted by friction, these bushings are not suitable when accurate alignment must be maintained.

123. Cotter Joints. In a cotter joint, the key or cotter transmits power by shear on an area perpendicular to the length of the key instead of by shear on an area parallel to the length. The

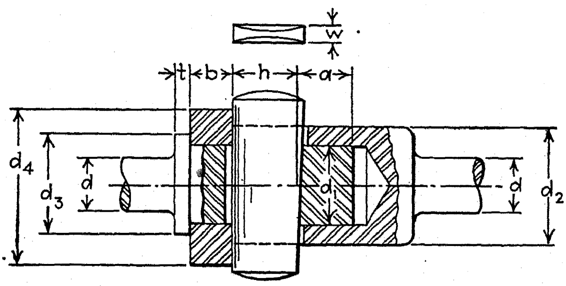


Fig. 109.

cotter is usually a flat bar tapered on one side to ensure a tight fit. Two types of cotter joints are shown in Figs. 109 and 110.

The following example will illustrate the method of determining the stresses and dimensions of a typical cotter joint.

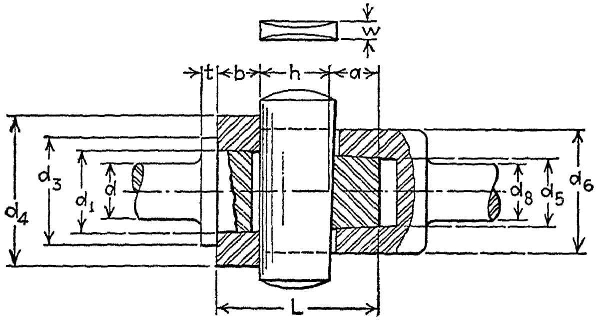


FIG. 110.

Example. The cotter joint shown in Fig 109 is commonly used for long pump rods and similar machine members Design this joint to support a load varying from 6,000 lb in tension to 6,000 lb in compression, using steel with the following allowable stresses: s_t equal to s_c equal to 7,500, s_b equal to 9,000, and s_s equal to 5,000 psi

The rod proper is in direct tension, and

$$F = \frac{\pi d^2}{4} s_t$$

and

$$s_t = \frac{4F}{\pi d^2} \tag{99}$$

from which

$$d = \sqrt{\frac{4 \times 6,000}{7,500\pi}} = 1.01, \text{ say 1 in.}$$

If the rod is very long between supported sections, it must be considered to be a column, and the diameter determined accordingly

The maximum tension stress in the rod end is at the section through the cotter hole, and

$$F = \left(\frac{\pi d_1^2}{4} - wd_1\right) s_t \tag{100}$$

The total load must also be sustained by bearing between the cotter and the rod end, and

$$F = wd_1 s_b, \text{ very nearly} \tag{101}$$

For equal strengths in the rod end and cotter, equate Eqs. (100) and (101). Then

$$\left(\frac{\pi d_1^2}{4} - wd_1\right) s_t = wd_1 s_b$$

and

$$w = \frac{\pi d_1 s_t}{4(s_t + s_b)} \tag{102}$$

from which

$$w = \frac{\pi d_1 7{,}500}{4(7{,}500 + 9{,}000)} = 0.357 d_1$$

However, in practice w is usually made equal to $d_1/4$. If this value is substituted in Eq. (101),

$$d_1 = \sqrt{\frac{4F}{s_b}} = \sqrt{\frac{4 \times 6{,}000}{9{,}000}} = 1.63, \text{ say } 1\tfrac{5}{8} \text{ in.}$$

and

$$w = \frac{d_1}{4} = \frac{13}{32} \text{ in.}$$

Substitute these values in Eqs. (100) and (101), and the stresses in tension in the rod end and in shear in the cotter are found to be safe

The cotter carries the load in shear on two areas; therefore

$$F = 2hws_s$$

and

$$h = \frac{F}{2ws_s} \tag{103}$$

from which

$$h = \frac{6{,}000}{2 \times 0.40625 \times 5{,}000} = 1\ 475, \text{ say } 1\tfrac{1}{2} \text{ in.}$$

The diameter and thickness of the collar on the rod end are determined by the compressive load, since the only load on the collar when the joint is in tension is that due to driving in the cotter. The magnitude of this load is unknown, but it is at least as large as the compressive load on the rod. The compression on the collar is

$$F = \frac{\pi}{4}(d_3^2 - d_1^2)s_c$$

and

$$d_3 = \sqrt{\frac{4F}{\pi s_c} + d_1^2} \tag{104}$$

from which

$$d_3 = \sqrt{\frac{4 \times 6{,}000}{\pi \times 7{,}500} + 1.625^2} = 1.91, \text{ say } 2 \text{ in.}$$

The shear on the collar is

$$F = \pi d_1 t s_s$$

and

$$t = \frac{F}{\pi d_1 s_s} \tag{105}$$

from which

$$t = \frac{6{,}000}{\pi \times 1.625 \times 5{,}000} = 0.236, \text{ say } \tfrac{1}{4} \text{ in.}$$

The force between the cotter and socket is

$$F = (d_4 - d_1)ws_b$$

and

$$d_4 = \frac{F}{ws_b} + d_1 \qquad (106)$$

from which

$$d_4 = \frac{6{,}000}{0.40625 \times 9{,}000} + 1.625 = 3.27, \text{ say } 3\tfrac{1}{4} \text{ in.}$$

In tension, the greatest stress in the socket is at the section through the cotter hole, and

$$F = \left[\frac{\pi}{4}(d_2^2 - d_1^2) - w(d_2 - d_1)\right] s_t \qquad (107)$$

from which

$$6{,}000 = \left[\frac{\pi}{4}(d_2^2 - 1.625^2) - \frac{13}{32}(d_2 - 1.625)\right] 7{,}500$$

and

$$d_2 = 1.96, \text{ say } 2 \text{ in.}$$

Since this value is less than d_4, it may be desirable to save machine work by making d_2 equal to d_4

If the rod end and socket are made of the same material, as they are in this example, the distances a and b will be equal. The cotter may shear out the rod end, and

$$F = 2ad_1s_s, \text{ nearly}$$

and

$$a = \frac{F}{2d_1s_s} \qquad (108)$$

from which

$$a = \frac{6{,}000}{2 \times 1.625 \times 5{,}000} = 0.369, \text{ say } \frac{3}{8} \text{ in.}$$

In practice, a and b are usually made from $\frac{3}{4}d$ to d, or in this case say $\frac{7}{8}$ inch.

This completes the design of the joint except for the taper of the cotter key, which is usually made from $\frac{1}{4}$ to $\frac{1}{2}$ in. per ft.

124. Taper Joint and Nut. The piston rod is often joined to the piston by a tapered rod end provided with a nut as shown in Fig. 111. In a joint of this type, the nut takes the tension load, and the taper and collar take the compression load. Computations for the dimensions are made in a manner similar to that used in the preceding article. The small diameter of the taper d_2

must be larger than the thread diameter; and the large diameter of the taper will then depend on the thickness of the piston and on the portion of the compression to be carried on the collar.

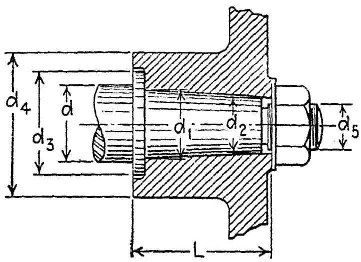


FIG. 111.

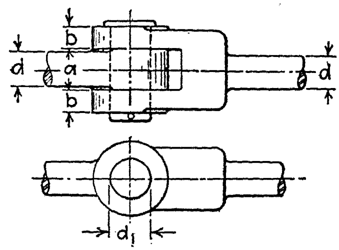


FIG. 112.

125. Pin or Knuckle Joints. These joints are used to connect two rods or bars when a small amount of flexibility or angular movement is necessary. Common uses are with valve and eccentric rods, diagonal stays, tension links in bridge structures, and lever and rod connections of many kinds. A typical rod end and forked knuckle joint are shown in Fig. 112.

Consider shear on the pin and tension in the main rod:

$$F = 2\frac{\pi d_1^2}{4} s_s \tag{109}$$

and

$$F = \frac{\pi d^2}{4} s_t \tag{110}$$

from which

$$d_1 = d\sqrt{\frac{s_t}{2s_s}} \tag{111}$$

and, assuming s_s equal to $0.65s_t$,

$$d_1 = 0.877d \tag{112}$$

If the pin is loose in the hole in the forks or side plates, consider the load to be uniformly distributed along the middle portion and uniformly varying over the forks. Then, in the forks, one-half the load will act at a distance of $b/3$ from the inner edge. The bending moment will be a maximum at the center of the pin and is

$$M = \frac{1}{2}F\left(\frac{a}{4} + \frac{b}{3}\right) \tag{ 13}$$

and the maximum bending stress is

$$s_t = \frac{Mc}{I} = \frac{1}{2}\,\frac{F\left(\frac{a}{4} + \frac{b}{3}\right)}{\pi d_1^3/32} \tag{114}$$

from which

$$d_1 = \sqrt[3]{\frac{16F\left(\frac{a}{4} + \frac{b}{3}\right)}{\pi s_t}} \tag{115}$$

In practice, d_1 is usually made equal to d, a equal to $1\frac{1}{8}d$, and b equal to $\frac{3}{4}d$. A pin of these dimensions is sufficiently strong to resist both the shear and bending stresses.

126. Practical Considerations. Shaft diameters are usually determined by deflection limits and are in general stronger than necessary; hence keys designed to transmit the full power capacity of the shaft may be excessive in size. Practical considerations require that the hub length should be at least $1.5D$ to obtain a good grip and to prevent rocking on the shaft. In general, this is the minimum length of key that should be used.

To facilitate removal, keys are often made with gib heads and tapered. In some cases, a tapped hole is provided in the end to accommodate a drawbolt. When it is necessary to drive the key out, the point may be hardened to resist the battering action. Gib-head keys should be provided with a cover or guard to prevent the possibility of injury when hands or clothing come into contact with the rotating shaft.

When a tapered key is used, a high pressure that tends to rupture the hub is exerted. This also forces the shaft against the opposite side of the bore, and, if a single key is used and the bore is slightly oversize, the hub will be forced into a position eccentric to the shaft. Also, since the shaft and hub are held only at two contact lines, there is a tendency for the hub to rock on the shaft, a condition which may eventually loosen the key. To overcome this tendency, the hub is often bored eccentric and fitted with two keys at 90 deg, thus obtaining a three-line contact. Rocking and eccentricity may also be prevented by using a light press fit between the shaft and the hub.

Cotters are usually driven in to make a tight joint and the initial stresses are indeterminate, and, as in the case of bolts, the

applied loads may or may not increase these stresses (see Art. 108). In many cases, therefore, it is necessary to alter the proportions as obtained in Art. 123 to meet the conditions of service and the materials used. Empirical formulas, based on wide experience in a particular service, may be of more practical use than the theoretical analysis. These empirical formulas as found in texts covering each particular field of design should be considered by the student.

CHAPTER IX

SHAFTS

Shafts transmitting power by torsion may be divided into two general classes: transmission shafts and machine shafts. *Transmission shafts* are those used to transmit power between the source and the machines absorbing the power, and include countershafts, line shafts, head shafts, and all factory shafting. *Machine shafts* are those forming an integral part of the machine itself.

127. Stresses in Shafts. Shafts may be subjected to torsional, bending, or axial loads or to a combination of these loads. If the load is torsional, the principal stress induced is shear; if bending, the principal stresses are tension and compression. When a shaft is subjected to a combination of loads, the maximum resulting stresses are determined by Eqs. (33) and (34). Shafts are generally made of ductile materials, and the maximum shear stress, Eq. (33), is generally assumed to control the size of the shaft. Since the ratio of the twisting moment to the bending moment and the ratio of the ultimate tensile stress to the ultimate shear stress of the shaft material will determine the method of failure, many designers prefer to determine the diameter of the shaft by both the maximum-shear theory and the maximum-normal-stress theory and use the larger diameter.

When the loading is torsional only, the maximum stress and the angular deformation are

$$s_s = \frac{Tc}{J} \qquad \text{and} \qquad \theta = \frac{TL}{JG} \tag{116}$$

where s_s = torsional shear stress, psi.
T = torsional moment, lb-in.
c = distance from neutral axis to outermost fiber, in.
J = polar moment of inertia of cross-sectional area about axis of rotation, in.^{4}
L = length of shaft, in.

θ = angular deformation in length L, radians.
G = modulus of rigidity in shear, psi.

For round shafts, Eq. (116) becomes

$$s_s = \frac{16T}{\pi d^3} \qquad \text{for solid shafts} \tag{117}$$

and

$$s_s = \frac{16Td_o}{\pi(d_o^4 - d_i^4)} = \frac{16T}{\pi d_o^3} \times \frac{1}{1 - K^4} \qquad \text{for hollow shafts} \tag{118}$$

where d_o = outside diameter, in.
d_i = inside diameter, in.
K = ratio of inside to outside diameter.

When the shaft is subjected to bending only, the maximum stress is given by the beam formula, and is

$$s_t = \frac{Mc}{I} = \frac{32M}{\pi d^3} \qquad \text{for a solid shaft} \tag{119}$$

and

$$s_t = \frac{32M}{\pi d_o^3} \times \frac{1}{1 - K^4} \qquad \text{for a hollow shaft} \tag{120}$$

where s_t = tensile or compressive stress, psi.
M = bending moment, lb-in.
I = rectangular moment of inertia of cross-sectional area about neutral axis, in.^{4}

When the shaft is subjected to both torsional and bending loads, the stresses s_t and s_s from Eqs. (118) and (120), when substituted in Eqs. (33) and (34) give

$$s_{s\,\max} = \frac{16}{\pi d_o^3} \sqrt{M^2 + T^2} \times \frac{1}{1 - K^4} \tag{121}$$

and

$$s_{t\,\max} = \frac{16}{\pi d_o^3} (M + \sqrt{M^2 + T^2}) \frac{1}{1 - K^4} \tag{122}$$

These equations are applicable to solid shafts if K is made equal to zero. The term $\sqrt{M^2 + T^2}$ is often referred to as the equivalent twisting moment, and the term $\frac{1}{2}(M + \sqrt{M^2 + T^2})$ as the equivalent bending moment.

In certain installations the shaft may be subjected to an axial load F_a in addition to the torsional and bending loads. If there

is no column action, the average axial stress $4F_a/\pi(d_o^2 - d_i^2)$ must be added to the stress s_t from Eqs. (119) and (120). In this case Eqs. (121) and (122) become

$$s_{s\,\max} = \frac{16}{\pi d_o^3}\sqrt{\left(M + \frac{F_a d_o(1+K^2)}{8}\right)^2 + T^2}\left(\frac{1}{1-K^4}\right) \tag{123}$$

and

$$s_{t\,\max} = \frac{16}{\pi d_o^3}\left[M + \frac{F_a d_o(1+K^2)}{8} + \sqrt{\left(M + \frac{F_a d_o(1+K^2)}{8}\right)^2 + T^2}\right]\frac{1}{1-K^4} \tag{124}$$

If the axial load produces column action, the column action may be taken care of by multiplying the term $F_a d_o(1 + K^2)/8$ in Eqs. (123) and (124) by a constant α [as in Eq. (125)], which is equal to the ratio of the maximum to the average intensity of stress resulting from column action of the axial load only.

In all the above equations the stresses s_s, s_t, $s_{t\,\max}$, and $s_{s\,\max}$ are the design stresses obtained by the use of the proper factor of safety.

128. Code for Transmission Shafting Design. The American Society of Mechanical Engineers is the sponsor of a Code for the Design of Transmission Shafting approved by the American Engineering Standards Committee. This Code is based upon the assumption that the shaft is made of a ductile material whose ultimate tensile strength is twice the ultimate shear strength. For this case, the shaft diameter is controlled by the maximum-shear theory regardless of the ratio of the twisting moment to the bending moment. The A.S.M.E. Code equation* for a hollow shaft subjected to torsion, bending, and an axial load is Eq. (123) with shock and fatigue factors introduced.

$$d_o^3 = \frac{16}{\pi s_s}\sqrt{\left[K_m M + \frac{\alpha F_a d_o(1+K^2)}{8}\right]^2 + (K_t T)^2} \times \left(\frac{1}{1-K^4}\right) \tag{125}$$

where d_o = shaft diameter, in.

F_a = axial tension or compression, lb.

* See the Code for the Design of Transmission Shafting, sponsored by the A.S.M.E., approved by the American Engineering Standards Committee, November, 1929.

K = ratio of inside to outside diameter of hollow shafts.
K_m = combined shock and fatigue factor to be applied to the computed bending moment.
K_t = combined shock and fatigue factor to be applied to the computed torsional moment.
M = maximum bending moment, lb-in.
T = maximum torsional moment, lb-in.
s_s = maximum stress permissible in shear, psi.
α = ratio of the maximum intensity of stress resulting from the axial load, to the average axial stress.

The value of α is obtained by considering the axial load, or thrust, as a load on a column of diameter d having a length equal to the distance between bearings. A straight-line formula commonly used for columns having a slenderness ratio less than 115 gives

$$\alpha = \frac{1}{1 - 0.0044(L/k)} \tag{126}$$

where L = length between supporting bearings, in.
k = radius of gyration of the shaft, in.

When the slenderness ratio is greater than 115, Euler's equation gives

$$\alpha = \frac{s_y}{n\pi^2 E}\left(\frac{L}{k}\right)^2 \tag{127}$$

where s_y = yield stress in compression, psi.
n = constant for the type of column end support.
E = modulus of elasticity, psi.

For free end supports, n equals unity, and for fixed bearings n may be taken as 2.25.

When the shaft is subjected to bending loads only, α, F, and T vanish. When the shearing stress, s_s is replaced by its equivalent tension stress $s_t/2$, Eq. (125) becomes

$$d_o = \sqrt[3]{\frac{32}{\pi s_t} K_m M} \times \frac{1}{\sqrt[3]{1 - K^4}} \tag{128}$$

When the shaft is subjected to torsional loads only, α, F, and M vanish, and Eq. (125) becomes

$$d_o = \sqrt[3]{\frac{16}{\pi s_s} K_t T} \times \frac{1}{\sqrt[3]{1 - K^4}} \tag{129}$$

The maximum allowable stresses to be used with Eqs. (125), (128), and (129) are given in Table 33. The values for working stress are those suitable for static loading conditions, and are such that on this basis a solid shaft will have a factor of safety of $2\frac{1}{3}$ based on the elastic limit, and from 4 to 4.5 based on the ultimate strength in tension. The properties of the steels commonly used for shafting are given in Table 34.

TABLE 33.—MAXIMUM PERMISSIBLE WORKING STRESSES FOR SHAFTS

Grade of shafting	Simple bending	Simple torsion	Combined stress
"Commercial steel" shafting without allowance for keyways...	16,000	8,000	8,000
"Commercial steel" shafting with allowance for keyways....	12,000	6,000	6,000
Steel purchased under definite specifications..	60% of the elastic limit but not over 36% of the ultimate in tension	30% of the elastic limit but not over 18% of the ultimate in tension	30% of the elastic limit but not over 18% of the ultimate in tension

TABLE 34.—PROPERTIES OF SHAFTING MATERIALS

Material	% carbon	Ultimate strength, psi			Elastic limit, psi			Elongation, %
		Tension	Compression	Shear	Tension	Compression	Shear	
Commercial:								
Cold-rolled	0.10–0.25	70,000	70,000	35,000	35,000	35,000	18,000	35
Turned. ...	0.10–0.25	60,000	60,000	30,000	30,000	30,000	15,000	35
Hot-rolled or forged ..	0.15–0.25	65,000	65,000	32,500	36,000	36,000	16,250	26
	0.25–0.35	70,000	70,000	35,000	40,000	40,000	17,500	24
	0.35–0.45	75,000	75,000	37,500	45,000	45,000	18,750	22
	0.45–0.55	80,000	80,000	40,000	50,000	50,000	20,000	20
$3\frac{1}{2}$% nickel.	0.15–0.25	85,000	85,000	42,500	55,000	55,000	21,250	26
Chrome vanadium ..	0.25–0.35	90,000	90,000	45,000	60,000	60,000	22,500	25

129. Shock and Fatigue Factors. Since a rotating shaft is subjected to completely reversed stress, a fatigue factor, *i.e.*, K_m, of at least 1.5 must always be used in Eq. (125). When the bending and torsional loads are subject to variations in intensity or to shock, the stresses will be greater than those indicated by static conditions, and values of K_m and K_t taken from Table 35 will allow for these additional stresses. Note that K_m and K_t are values of the shock factor b as used in Eq. (49) page 72 in determining the factor of safety.

TABLE. 35.—COMBINED SHOCK AND FATIGUE FACTORS TO BE USED WITH EQS (125), (128), AND (129)

Type of loading	Rotating shafts		Stationary shafts	
	K_m	K_t	K_m	K_t
Gradually applied and steady loads	1.5	1 0	1.0	1 0
Suddenly applied loads with minor shock only.	1.5–2 0	1.0–1.5	1.5–2 0	1 5–2.0
Suddenly applied loads with heavy shock	2 0–3.0	1.5–3.0		

Example. A hollow steel shaft is to transmit 19.8 hp at 250 rpm. The loading is such that the maximum bending moment is 10,000 lb-in., the maximum torsional moment 5,000 lb.-in., and the axial compressive load 4,000 lb. The shaft is supported on rigid bearings 6 ft apart and is subjected to minor torsional loads suddenly applied. The maximum allowable shear is 6,000 psi. The ratio of the inside diameter to the outside diameter is 0 75.

Since the shaft diameter is not known, the values of k and α can not be determined. It is necessary to find or assume a trial value of d, because d is included on the right-hand side of the equation and also affects the value of α. The value of α varies from 1 to 2.02 for slenderness ratios varying from 0 to 115, and in this case a value of 2.00 corresponding to a slenderness ratio of 115 can be assumed. Then the trial value of k is $L/115$ equal to $\frac{72}{115}$, or 0.625, and the trial value of d is $4k$ or 2.50 in.

Then from Eq. (125):

$$d_o = \sqrt[3]{\frac{16}{\pi \times 6{,}000}\sqrt{\left(1\ 5 \times 10{,}000 + \frac{2.0 \times 4{,}000 \times 2.5(1+0.75^2)}{8}\right)^2 + (1.5 \times 5000)^2}} \times \frac{1}{\sqrt[3]{1 - 0.75^4}}$$

from which d_o equals 2.93 in. By the use of this value of d_o, k is equal to

0.73, L/k equal to 98.6, and α equal to 1.76. By the substitution of these values in the equation and solving for d_o, the shaft diameter is found to be 2.94 in. This checks the last trial value of 2.93 and is the required diameter. For a transmission shaft, the next larger standard size is $2\frac{15}{16}$ in. For a machine shaft a diameter of $2\frac{15}{16}$ in. with an inside diameter of $2\frac{3}{16}$ in. is satisfactory, provided the deflections are within the specified limits.

130. Shafts of Brittle Materials. The design of shafts as outlined applies to ductile materials to which the maximum shear theory is applicable. Occasionally, brittle materials are used, and the maximum-normal-stress theory applies. Then Eq. (34) gives

$$s_{t\,\max} = \frac{s_t}{2} + \frac{1}{2}\sqrt{s_t^2 + 4s_s^2}$$

from which

$$s_{t\,\max} = \frac{16}{\pi d_o^3}\left[K_m M + \sqrt{(K_m M)^2 + (K_t T)^2}\right] \times \frac{1}{1 - K^4}$$

and

$$d_o^3 = \frac{16}{\pi s_t}\left[(K_m M) + \sqrt{(K_m M)^2 + (K_t T)^2}\right] \times \frac{1}{1 - K^4} \qquad (130)$$

131. Effect of Keyways. The keyway cut into the shaft materially affects the strength or load-carrying capacity of the shaft since highly localized stresses occur at and near the corners of the keyway, and the effect of these is more pronounced when shock and fatigue conditions prevail. Mathematical analyses of the stresses around the keyway are complex and are seldom made in design. Experimental work by H. F. Moore* indicates that the static weakening effect of the keyway is given by the formula

$$e = 1.0 - 0.2\frac{w}{d} - 1.1\frac{h}{d} \qquad (131)$$

where e = shaft strength factor or ratio of strength of shaft with keyway to the same shaft without keyway.
w = width of keyway.
h = depth of keyway.
d = shaft diameter.

The Code for Transmission Shafting recommends the use of an efficiency of 75 per cent for keyed shafts, a value somewhat lower than that indicated by the equation.

* MOORE, PROF. H. F., *Univ. Ill. Eng. Exp. Sta. Bull.* 42.

The same experiments indicate that the angular twist of a shaft with a keyway is given by the formula

$$k = 1.0 + 0.4\frac{w}{d} + 0.7\frac{h}{d} \tag{132}$$

where k is the ratio of the angular twist of the shaft with the keyway to the same shaft without the keyway. Since the twist is increased only in that part of the shaft containing the keyway, and since the hub of the mating member tends to stiffen the shaft, the increase in angular twist may be disregarded except for long keyways used with sliding or feather keys.

132. Transmission Shafts. Transmission shafts serve primarily to transmit power by torsion and are therefore subjected principally to shearing stresses. Belt pulleys, gears, chain sprockets, and similar members carried by the shaft introduce bending loads, which cannot in general be determined and it is customary to assume simple torsion, allowance for the unknown bending stresses being made by using lower design stresses for those shafts in which experience indicates that the bending stresses are severe. Formulas including such allowances are given in Table 36. These formulas should not be used when the center distance is more than 8 ft for countershafts and line shafts, or about 10 diameters for head shafts. Pulleys, gears, and couplings should be placed as near the supporting bearings as possible to reduce the bending stress.

Torsional deformation in transmission shafts should be limited to 1 deg in 20 diameters. Lateral deflection caused by bending should not exceed 0.01 in. per ft of length.

TABLE 36.—TRANSMISSION-SHAFT DIAMETERS

Service	Load factor	Cold-rolled or drawn shafting	Hot-rolled and turned shafting
Transmission shafts, torsion only	$\begin{cases} K_t = 1.0 \\ K_m = 1.0 \end{cases}$	$\sqrt[3]{\frac{40\text{ hp}}{N}}$	$\sqrt[3]{\frac{50\text{ hp}}{N}}$
Line shafts, limited bending.....	$\begin{cases} K_t = 1.0 \\ K_m = 1.5 \end{cases}$	$\sqrt[3]{\frac{70\text{ hp}}{N}}$	$\sqrt[3]{\frac{90\text{ hp}}{N}}$
Head shafts..................	$\begin{cases} K_t = 1.0 \\ K_m = 2.5 \end{cases}$	$\sqrt[3]{\frac{110\text{ hp}}{N}}$	$\sqrt[3]{\frac{135\text{ hp}}{N}}$

N = rpm, and hp = horsepower transmitted.

133. Commercial Shafting. Transmission shafts are commonly made of cold-rolled stock. For diameters over 3 in. many engineers prefer hot-rolled stock, and for diameters over 6 in. forged stock is generally used. Cold-rolled shafting is stronger than hot-rolled stock of the same analysis, but the resistance to deformation is the same. Cold-rolled shafting is made in diameters increasing by $\frac{1}{16}$ in. from $\frac{1}{2}$ to $2\frac{1}{2}$ in., by $\frac{1}{8}$ in. up to 4 in., and by $\frac{1}{4}$ in. up to 6 in. However, transmission shafts are standardized in $\frac{1}{4}$-in. steps from $\frac{15}{16}$ in. to $2\frac{7}{16}$ in., and by $\frac{1}{2}$-in. steps up to $5\frac{15}{16}$ in. Standard stock lengths are 16, 20, and 24 ft.

134. Machine Shafts. The deflection of a machine shaft is usually as important as its strength, and in many cases more important. In most cases a shaft that is rigid enough is also strong enough, and the shaft should be designed for stiffness and checked for strength. The strength of the material does not affect the stiffness, which depends only on the dimensions and the modulus of elasticity of the material, so that shafts of similar dimensions and loading have the same deflection, regardless of the kind of steel used. Hence, if the dimensions of a shaft are limited by the permissible deformations, a plain carbon-steel shaft may be just as satisfactory as a shaft of higher cost alloy steel.

The permissible deformation of a machine shaft depends upon the service for which it is intended. The deflection of shafts carrying gears should be limited to $0.005/f$ in. at the gear, where f is the width of the gear face in inches. For very smooth-running gears, the deflection should be much smaller than this. The deflection of any machine shaft supported on plain bearings should not exceed $0.0015L$, where L is the distance from the load point to the center of the bearing, in inches.

The angular twist of machine shafts should be limited to 6 min per ft for ordinary service, to $4\frac{1}{2}$ min per ft for variable loads, and to 3 min per ft for suddenly reversed loads and long feed shafts. The angular rigidity of machine-tool spindles should prevent a movement of more than $\frac{1}{64}$ in. at the circumference of the face plate. Milling cutter spindles should have an angular twist of less than 1 deg at the edge of the cutter.

135. Determination of Shaft Size for Strength. The shaft diameter required for strength can be found from Eq. (125), when the bending moment, the torsional moment, and the

conditions of loading are known. The torsional moment is determined from the equation

$$T = \frac{\text{hp} \times 33{,}000 \times 12}{2\pi N} = \frac{63{,}000 \times \text{hp}}{N} \tag{133}$$

where T = torsional moment, lb-in.
hp = horsepower transmitted.
N = shaft speed, rpm.

The torsional moment may differ at various points along the shaft if the power is taken off through gears, belts, or chains.

The bending moment is most easily found by the replacing of each individual load and bearing reaction by its vertical and horizontal components. All vertical components may then be considered as loads on a beam, and the vertical bending moments can be determined. Similarly, all horizontal components may be considered as loads on a beam, and the horizontal bending moments can be determined. The vector sum of the vertical and horizontal moments at any section of the beam is the total bending moment at that section.

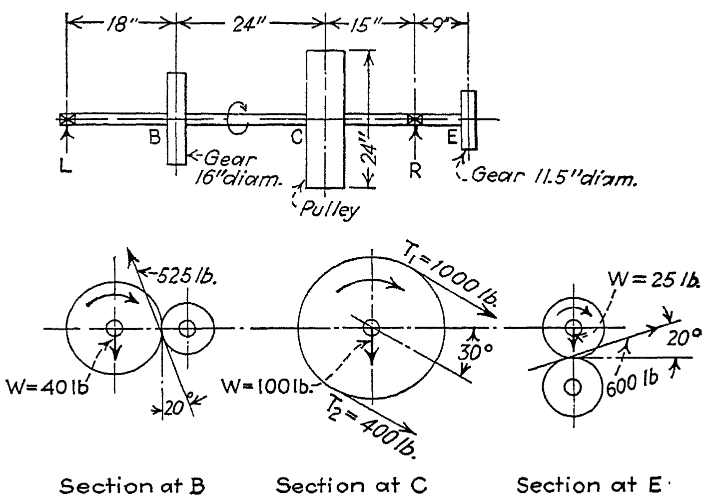


FIG. 113.

Example. Assume a shaft $5\frac{1}{2}$ ft long supported on two bearings and driven by a belt pulley and driving two gears as shown in Fig. 113. The permissible shearing stress allowing for keyways is 6,000 psi. The torsional moment is suddenly applied with moderate shock.

Resolve each load into vertical and horizontal components.

Load	Lb	Vertical component	Horizontal component
Force B	525	+493	+ 179
Weight B	40	− 40	
Force C	1,400	−700	−1,212
Weight C	100	−100	
Force E	600	+205	− 564
Weight E	25	− 25	

The negative sign indicates forces acting down or back.

Considering the vertical forces only, a beam loaded as shown in Fig. 114

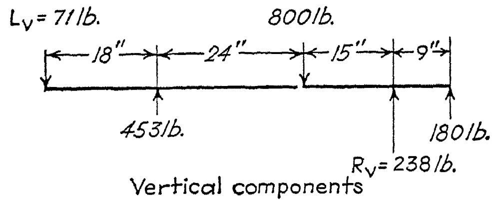


FIG. 114.

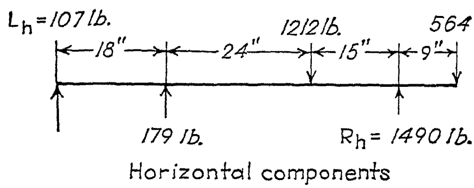


FIG. 115.

results. The vertical reactions at L and R are

$$L_v = 71 \text{ lb}$$

and

$$R_v = 238 \text{ lb.}$$

The vertical bending moments at L, B, C, R, and E are

$$M_{Lv} = 0$$
$$M_{Bv} = -71 \times 18 = -1{,}278 \text{ lb-in.}$$
$$M_{Cv} = -71 \times 42 + 453 \times 24 = 7{,}890 \text{ lb-in.}$$
$$M_{Rv} = 180 \times 9 = 1{,}620 \text{ lb-in.}$$
$$M_{Ev} = 0$$

The horizontal forces acting on the beam are shown in Fig. 115 and the vertical and horizontal moments are given in Table 37. The total moments, found by combining the vertical and horizontal moments, are also tabulated.

The maximum bending moment is 11,814 lb-in. located at the pulley.

TABLE 37

Section	Bending moments		
	Vertical	Horizontal	Total
L	0	0	0
B	1,278	1,926	2,311
C	7,890	8,790	11,814
R	1,620	5,076	5,336
E	0	0	0

The maximum torsional moment is also at the pulley and is

$$T = 493 \times 8 = 3{,}944 \text{ lb-in.}$$

Substitution of the maximum bending and torsional moments in Eq. (125) gives

$$d = \sqrt[3]{\frac{16}{\pi 6{,}000}\sqrt{(1.5 \times 11{,}814)^2 + (1.5 \times 3{,}944)^2}} = 2.51 \text{ in., say } 2\tfrac{1}{2} \text{ in.}$$

If the maximum bending and torsional moments are not at the same section of the shaft, it is necessary to determine the diameter required by the combination of moments at each point of load application, and to select the largest diameter.

Since this calculation gives the diameter required for strength only, the shaft should now be checked for deflection and twist, if these have specified maximums.

136. Graphical Determination of Bending Moments. When there are many loads to be considered, the use of graphical methods for determining the bending moments will expedite the solution. The method is best explained by an illustrative solution, and for this purpose, the data in the preceding article are used. The shaft and the loads are shown in Fig. 113. The resultant loads are shown in Fig. 117 as forces acting through the center of the shaft.

In Fig. 117 the total belt load is shown to scale, by the vector F_C. To the belt load add the weight of the pulley and belt, obtaining the resultant load R_C. Similarly, to each of the gear loads add the gear weights, obtaining the resultant loads R_B and R_E. The resultant loads scale 1,453, 488, and 591 lb, respectively.

The total bending moment at any section of the beam is the vector sum of the bending moments produced at that section by the individual loads. Consequently, if the vectors representing the individual moments can be obtained graphically, they can be added graphically to determine the total moment at any desired section of the beam.

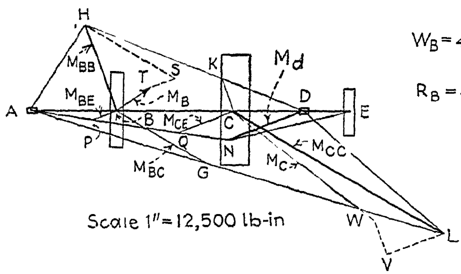


FIG 116.

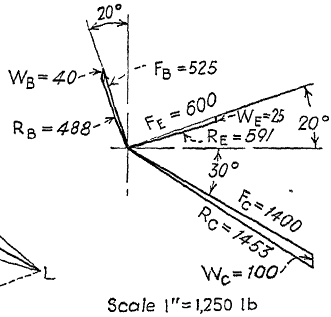


FIG 117.

Consider all the loads removed except the load at the gear B. The bending moment under the gear will then be

$$M_{BB} = \frac{488 \times 39}{57} \times 18 = 6{,}010 \text{ lb-in.}$$

In Fig. 116 draw BH parallel to R_B and equal to M_{BB}, *i.e.*, 6,010 lb-in. Then $AHDE$ is the bending-moment diagram for the beam, and CK, parallel to BH, represents the bending moment at C produced by the load R_B acting alone.

Now consider the beam with the loads at C acting, the resultant of these loads being R_C. In a manner similar to that just used, CL is made equal to M_{CC} and parallel to R_C, and $ALDE$ is the bending-moment diagram of the beam for the load R_C. Then BG parallel to CL represents the bending moment at B produced by the load R_C.

Similarly, ANE is the bending-moment diagram for the load R_E, and BP and CQ represent the moments at B and C, respectively, produced by this load.

At the section B, there are three bending moments, M_{BB}, M_{BC}, and M_{BE}. Add these vectors as indicated by the vector polygon $BHST$. Then BT, their vector sum, represents to scale the bending moment at B, or M_B. Similarly, M_C and M_D,

the bending moments at C and D, are found. By scaling the vectors, M_B is found to be 2,300 lb-in., M_C 12,000 lb-in., and M_D 5,330 lb-in. The maximum bending moment having been found to be 12,000 lb-in., the shaft diameter can be determined by substitution in Eq. (125), as in the preceding example.

137. Design of Shafts for Deflection. As previously stated, most machine shafts must be designed for a specified maximum deformation, and as the rigidity, rather than the strength, usually determines the shaft size, the deformation computations should be made first, and then the shaft should be checked for strength. When there are a number of loads acting, the principle of superposing deformations may be used to advantage; *i.e.*, the total deformation at any point is found by adding the deformations produced at this point by each load acting separately.

Example. Assume the same shaft and loading as in the previous example and in Fig. 113. Determine the required shaft diameter if the deflection at any load is limited to 0.025 in.

Consider the shaft with the single vertical load at B. From the beam formulas, the vertical deflection at the gear B is found to be

$$y_{vBB} = \frac{Fa^2b^2}{3EIL} = \frac{453 \times 18^2 \times 39^2 \times 64}{3 \times 30{,}000{,}000 \times \pi d^4 \times 57} = \frac{0.890}{d^4},$$

and the deflection at the pulley C is

$$\begin{aligned} y_{vCB} &= \frac{Fbx}{6EIL}(L^2 - b^2 - x^2) \\ &= \frac{453 \times 18 \times 15 \times 64}{6 \times 30{,}000{,}000 \times \pi d^4 \times 57}(57^2 - 18^2 - 15^2) \\ &= \frac{0.631}{d^4} \end{aligned}$$

and the deflection at the gear E is

$$\begin{aligned} y_{vEB} &= (\text{slope at } L) \times 9 = \frac{Fb}{6EIL}(L^2 - b^2 - 3x^2) \times 9 \\ &= \frac{453 \times 18 \times 64}{6 \times 30{,}000{,}000 \times \pi d^4 \times 57}(57^2 - 18^2 - 0) \times 9 \\ &= \frac{0.448}{d^4} \end{aligned}$$

Note that the deflections y_{vBB} and y_{vCB} are up, whereas y_{vEB} is down.

In a similar manner, the deflections caused by the vertical components of the loads at C and E, and the deflections caused by the horizontal components are found. The deflections are tabulated in Table 38. Evidently

TABLE 38

Load	Deflection					
	Vertical			Horizontal		
	B	C	E	B	C	E
B	$+\frac{0\ 890}{D^4}$	$+\frac{0\ 631}{D^4}$	$-\frac{0\ 448}{D^4}$	$+\frac{0\ 352}{D^4}$	$+\frac{0\ 250}{D^4}$	$-\frac{0\ 018}{D^4}$
C	$-\frac{0.223}{D^4}$	$-\frac{1.265}{D^4}$	$+\frac{0.297}{D^4}$	$-\frac{1.510}{D^4}$	$-\frac{8\ 600}{D^4}$	$+\frac{2\ 020}{D^4}$
E........	$-\frac{0.170}{D^4}$	$-\frac{0.200}{D^4}$	$+\frac{0.218}{D^4}$	$+\frac{0.532}{D^4}$	$+\frac{0.627}{D^4}$	$-\frac{0\ 684}{D^4}$
Total	$+\frac{0\ 497}{D^4}$	$-\frac{0.834}{D^4}$	$+\frac{0\ 067}{D^4}$	$-\frac{0.626}{D^4}$	$-\frac{7.723}{D^4}$	$+\frac{1.318}{D^4}$

the maximum total deflection at a load point is at the pulley C, and by vector addition of the vertical and horizontal deflections

$$y_C = \frac{1}{d^4}\sqrt{0.834^2 + 7.723^2} = \frac{7.76}{d^4}$$

from which

$$d^4 = \frac{7\ 76}{y_C} = \frac{7.76}{0.025} = 310.2$$

and

$$d = 4.20 \text{ in.}$$

In this case, the diameter required for strength is less than that required for deflection, and so the shaft diameter should be made $4\frac{1}{4}$ in.

138. Graphical Determination of Deflection. The mathematical determination of deflection becomes very tedious when there are many loads, and especially so when the shaft is made up of sections of different diameters. Graphical determination of the deflection can be conveniently applied with sufficient accuracy for most purposes.

Texts on mechanics show that the second derivative of the deflection equation of any shaft is expressed by the relation

$$\frac{d^2y}{dx^2} = \frac{M}{EI} \tag{134}$$

where y = deflection at a distance x from one end of shaft, in.
M = bending moment at section x, lb-in.
E = modulus of elasticity of material, psi.
I = rectangular moment of inertia of shaft area at same section, in.^{4}

From the known loads and reactions acting on the shaft, the bending-moment diagram is plotted. When the moments are divided by the values of EI for the corresponding sections of the shaft and plotted, a diagram representing the variations of M/EI over the entire length of the shaft is obtained. Double integration of this diagram gives the deflection curve of the shaft. The process of graphical integration is best shown by an illustrative example.

Example. Assume a shaft loaded as shown in Fig. 118. Determine the deflection curve.

The bending moments are found to be 192,000 lb-in. at the 10,000-lb load, and 224,000 lb-in. at the 16,000-lb load. These values determine the moment diagram, curve I, Fig 118. From the left reaction to the section B, the shaft has a diameter of 4 in., and the value of I is 12.57 in ^{4} From curve I, the moment at section B is 144,000 lb-in. Then, at this section

$$\frac{M}{EI} = \frac{144{,}000}{30{,}000{,}000 \times 12.57} = 0\ 000382 \text{ in.}^{-1}$$

The diameter changes at this section from 4 to 6 in. and M/EI changes Hence

$$\frac{M}{EI} = \frac{144{,}000}{30{,}000{,}000 \times 63.62} = 0.000075 \text{ in.}^{-1}$$

Similarly, the values of M/EI at the sections C, D, E, F, and G are found. Plotting these to scale, curve II is obtained. To integrate this curve select any section such as K_2 and draw K_2k_2. By any means desired, measure the area $A_2k_2K_2$, and plot this area to scale in curve III, locating the point k_3. Measure the area $A_2b_2B_2$ and plot as B_3b_3, locating point b_3. Measure the area $A_2b_2mc_2C_2$ and plot, locating the point c_3. Continue in this manner until the entire curve III has been plotted. This curve represents the first integral of curve II, and therefore represents dy/dx, the slope of the deflection curve. The ordinates of curve III are measured in abstract units since areas in curve II are ordinates times abscissae, or inches^{-1} times inches. To eliminate the constant of integration, find the mean ordinate of curve III and plot this value as the line XX. This is the base line from which to measure values of dy/dx.

Integrate curve III by measuring the areas between the curve and the base line XX. Thus K_4k_4 represents to scale the area A_3k_3hX, and D_4d_4 represents the area $A_3b_3c_3d_3nX$. Remember that areas below the base line

XX are negative and areas above are positive. The complete integration of curve III gives curve IV, which is the deflection curve for the shaft. Ordinates measured from the base line A_4G_4 represent deflections in inches, to the proper scale

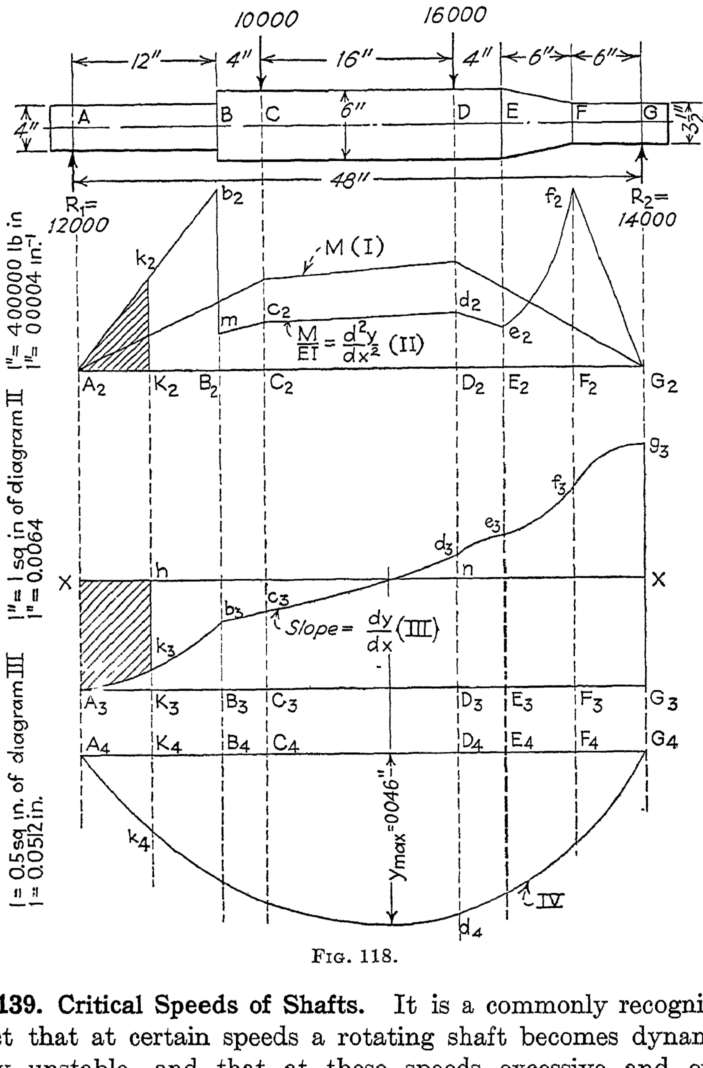


FIG. 118.

139. Critical Speeds of Shafts. It is a commonly recognized fact that at certain speeds a rotating shaft becomes dynamically unstable, and that at these speeds excessive and even dangerous deflections may occur. The speeds at which a shaft becomes dynamically unstable are called the critical speeds and

correspond to the speeds at which the number of natural vibrations, or natural frequency, equals the number of revolutions per minute.

Consider the shaft in Fig. 119 with the disk of mass M located between the supports. Also consider that the center of mass of the disk is at a distance e from the axis of rotation of the shaft. When the shaft is rotating with an angular velocity ω, the disk rotates at the same angular velocity; but since it is not rotating about an axis through its center of mass. there is set up a centrif-

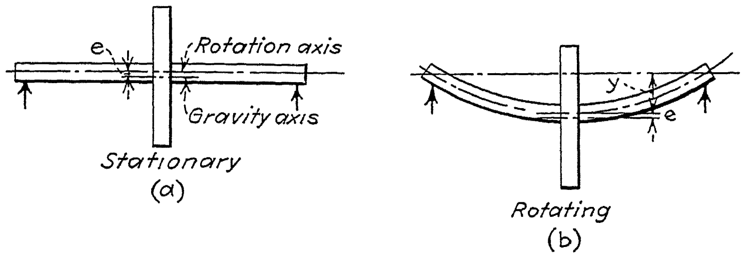


FIG. 119.

ugal force Mv^2/r, which tends to deflect the shaft an amount y. The radius of the path of the center of mass of the disk is then $y + e$, and the centrifugal force becomes

$$F_c = \frac{12Mv^2}{y + e} = \frac{M(y + e)\omega^2}{12} \tag{135}$$

where y and e are measured in inches and ω is in radians per second. This force causes the deflection y to increase until the flexural resisting moment of the shaft balances the bending moment produced by the centrifugal force. The relation between the deflection in any shaft and the force F producing that deflection is expressed by the equation

$$y = C\frac{FL^3}{EI}$$

or

$$F = \frac{EI}{CL^3}y$$

where C is a constant depending on the type of beam and the method of loading.

When the condition of equilibrium is reached,

$$F_c = F$$

or

$$\frac{M(y + e)\omega^2}{12} = \frac{EI}{CL^3} y$$

from which

$$y = \frac{e\omega^2}{\dfrac{12EI}{CML^3} - \omega^2} \tag{136}$$

Examination of this equation shows that the deflection becomes infinite when the angular velocity is such that

$$\omega_{cr}^2 = \frac{12EI}{CML^3}$$

and

$$\omega_{cr} = \sqrt{\frac{12EI}{CML^3}} = \sqrt{\frac{12K}{M}} = \sqrt{\frac{12Kg}{W}} \tag{137}$$

In this equation, K is the ratio F/y for the beam under consideration, so that if the equation for the deflection of the beam is known, the critical speed ω_{cr} can be computed. Thus if the rotating mass M is at the center of the shaft of negligible weight, and the shaft is simply supported in end bearings,

$$y = \frac{FL^3}{48EI}$$

and

$$\frac{F}{y} = \frac{48EI}{L^3} = K \tag{138}$$

Hence

$$\omega_{cr} = \sqrt{\frac{48EI}{L^3}\frac{12g}{W}} = \sqrt{\frac{576EIg}{WL^3}} \tag{139}$$

for a *central disk.*

When the mass M is at a distance a from the left support and at a distance b from the right support, the deflection under the load is

$$y = \frac{a^2b^2F}{3EIL}$$

and

$$\frac{F}{y} = \frac{3EIL}{a^2b^2} = K$$

Hence

$$\omega_{cr} = \sqrt{\frac{36EILg}{Wa^2b^2}} \tag{140}$$

for a *non-central disk.*

140. Critical Speed of a Shaft with Several Disks. When there are several disks or masses M_1, M_2, M_3, . . . M_n, on the shaft, the critical speed of the shaft may be found approximately by the equation

$$\omega_{cr} = \frac{\omega_1\omega_2\omega_3 \ . \ . \ . \ \omega_n}{\sqrt{(\omega_1\omega_3\omega_4\cdots\omega_n)^2 + (\omega_1\omega_2\omega_4\cdots\omega_n)^2 + \cdots + (\omega_2\omega_3\omega_4\cdots\omega_n)^2}} \tag{141}$$

where ω_1, ω_2, etc., are the critical speeds of the shaft alone, and of each mass considered by itself.

141. Critical Speed of a Uniform Shaft. The critical speed of a shaft supporting a uniform load, or of a shaft that supports no loads except its own weight, is found by considering the shaft to be made up of a number of short lengths of known mass. The critical speeds of the weightless shaft carrying these individual masses can then be computed and combined by means of Eq. (141) to determine the critical speed of the entire shaft. By this procedure, the critical speed of a shaft of uniform section, simply supported in end bearings and not supporting any concentrated masses, is found to be

$$\omega_{cr} = \sqrt{\frac{\pi^4 EIg}{WL^3}} = \sqrt{\frac{15.30Wg}{y}} \tag{142}$$

where W = total uniform load or shaft weight, lb.
L = distance between supports, in.
y = deflection produced in a simple uniformly loaded beam by the distributed weight W, in.

The speed determined by this equation is the lowest of a series of critical speeds. Other critical speeds occur at 4, 9, 16, 25, etc., times the lowest critical speed.

A shaft of uniform section, uniformly loaded, and rigidly supported in end bearings has for its lowest critical speed

$$\omega_{cr} = \sqrt{\frac{5.0625\pi^4 EIg}{WL^3}} = \sqrt{\frac{15.528Wg}{y}} \tag{143}$$

and others at $(\frac{5}{3})^2$, $(\frac{7}{3})^2$, $(\frac{9}{3})^2$, $(\frac{11}{3})^2$, etc., times this speed.

142. Critical Speeds in Terms of the Weight and Deflection. In cases where the total deflection of the shaft at each supported disk is known, or can be easily determined by computation or graphical analysis, it is more convenient to state the critical speeds in terms of the disk weights and the deflections. In Eq. (137), change radians per second to revolutions per minute, and substitute the deflection y for its equivalent CFL^3/EI. Then, for a shaft with a single disk,

$$N_{cr} = \frac{60\omega_{cr}}{2\pi} = \frac{30}{\pi}\sqrt{\frac{12F}{My}} = \frac{30}{\pi}\sqrt{\frac{12g}{y}} \tag{144}$$

Similarly, Eq. (141) for a shaft supporting n disks becomes

$$N_{cr} = \frac{30}{\pi}\sqrt{\frac{12g(W_1y_1 + W_2y_2 + \cdots + W_ny_n)}{W_1y_1^2 + W_2y_2^2 + \cdots + W_ny_n^2}} \tag{145}$$

where W_1, W_2, etc., are the weights of the disks, and y_1, y_2, etc., are the deflections at the respective disks.

143. Operating Speeds. Damping effects in the shaft will reduce the total deflection when the shaft is rotating at a critical speed, but large deflections will occur that may cause serious trouble and possibly structural damage to the machine. This is particularly true if the operating speed is alternately above and below the critical speed. For this reason, the operating speed should be well removed from the neighborhood of any of the series of speeds at which these extreme deformations and vibrations may occur. In some machines regular impulses may be transmitted to the shaft and if the timing of these impulses approximates the natural frequency, or critical speed of the shaft, trouble may be expected although the shaft speed may be safely removed from the critical speed. This often happens in shafts connected to multiple-cylinder internal-combustion engines.

In order to avoid vibration troubles, some machines are mounted with very flexible shafts that, when rotated at high speeds, allow the disks to rotate on their own centers of gravity. De Laval used this principle in his early high-speed turbines. Centrifugal separators that operate at speeds of from 20,000 to 50,000 rpm also employ these flexible shafts.

In general, if the operating speed of any shaft is removed at least 20 per cent from any critical speed, there will be no vibration troubles.

CHAPTER X

COUPLINGS AND CLUTCHES

Commercial shafts are limited in length by manufacturing and shipping requirements, so that it is necessary to join sections of long transmission shafts with couplings. Couplings are also required to connect the shaft of a driving machine to a separately built driven unit. Permanent couplings are referred to simply as couplings, while those which may be readily engaged to transmit power, or disengaged when desired, are called clutches.

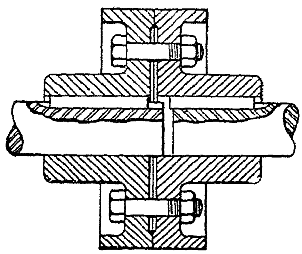


FIG. 120.—Flange coupling.

144. Rigid Couplings. The flange coupling shown in Fig. 120 is the most common coupling. It has the advantage of simplicity and low cost, but the connected shafts must be accurately aligned to prevent severe bending stresses and excessive wear in the bearings. The length of the hub is determined by the length of key

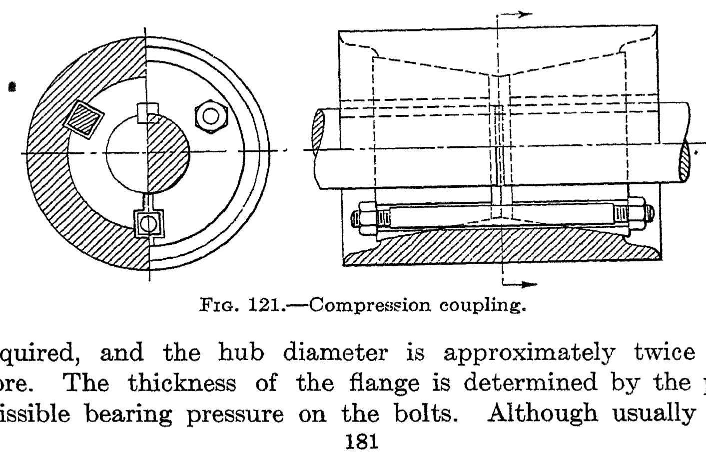


FIG. 121.—Compression coupling.

required, and the hub diameter is approximately twice the bore. The thickness of the flange is determined by the permissible bearing pressure on the bolts. Although usually not

critical, the shearing stress on the cylindrical area where the flange joins the hub should be checked.

When large flanges are objectionable, compression couplings similar to the coupling in Fig. 121 may be used. Power is transmitted by keys between the shafts and the cones and by friction between the cones and the outer sleeve. The inner cones may be split so that they will grip the shaft when drawn together.

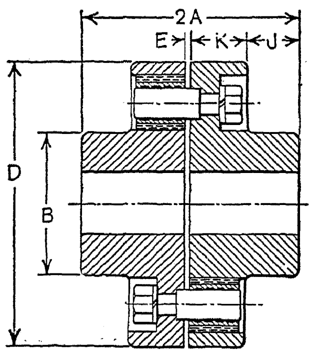


FIG. 122.—Rubber bushed coupling.

145. Flexible Couplings. Slight misalignment is usually encountered when connecting the shafts of separately built units, and flexible couplings are required. These couplings use rubber bushings in the bolt flanges, or leather, fabric, or flexible steel disks bolted at alternate points to the flanges, or metallic connections that provide the necessary flexibility. Several forms are shown in the accompanying illustrations.

Forms of the popular universal joint are shown in Figs. 125 and 126. The construction of this joint permits the positive transmission of power between shafts intersecting at a compara-

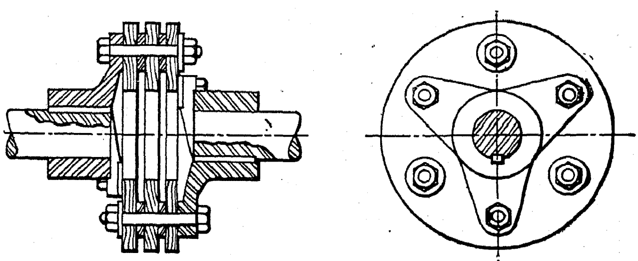


FIG. 123.—Flexible disk coupling.

tively large angle. Texts on mechanism* and kinematics of machinery show that there is an axial movement of the driven shaft that must be provided for by the use of a spline or feather-key connection, and that when the driving shaft rotates at uniform angular velocity, the driven shaft turns with a variable

* VALLANCE and FARRIS, "Principles of Mechanism," p. 315, The Macmillan Company.

angular velocity. When the driven shaft must have uniform angular velocity, two universal joints, placed as shown in Fig. 125*a*, must be used.

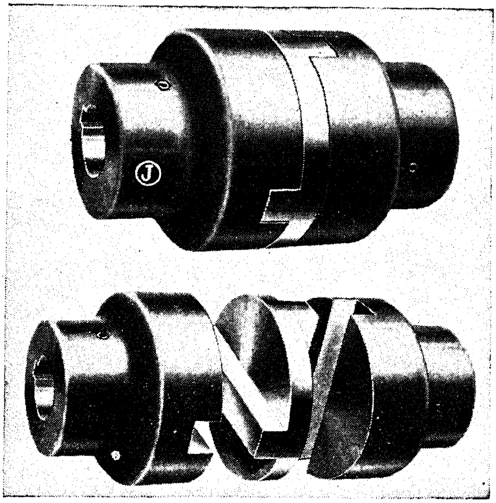


FIG. 124.—Oldham coupling. (*W. A. Jones Foundry & Machine Co., Chicago, Ill.*)

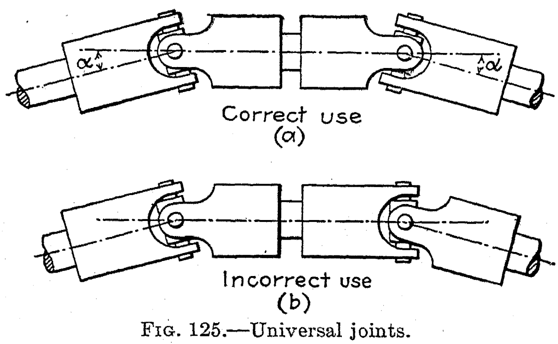


FIG. 125.—Universal joints.

146. Clutches. When the driving machine or machine member is to rotate continuously, and the driven member must be arranged to rotate or remain stationary as desired, or when the driving machine (such as an internal-combustion engine) is such that it must be brought up to speed before being connected to the load, it is necessary to use a clutch. Clutches may be positive in action, or they may depend upon friction for their torque-transmitting capacity.

The simplest positive clutch is the jaw clutch shown in Fig. 127. When power is transmitted in one direction only, the jaws may be cut at an angle on the trailing side to make a saw-tooth clutch which is somewhat stronger and also easier to engage. The proportions are more or less empirical, the jaw size being determined by the permissible compression on the material used.

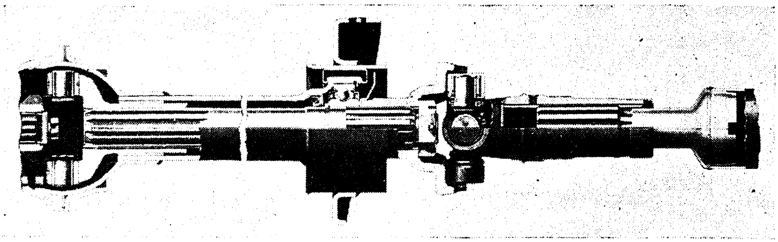


FIG. 126.—Propeller shaft and universal joints. (Chevrolet ¾-ton truck, 1942.)

The objection to this type of clutch is that the jaws must be moved into engagement when both parts are moving at the same velocity or the resultant shock will impose high stresses on the connected members. In automobile transmissions, a clutch of this type having jaws in the form of gear teeth is used to connect the shaft from the engine clutch to the main transmission shaft in the direct or high-speed position. In recent transmissions, with the so-called silent second, where helical gears are used and not shifted in and out of engagement, a similar jaw clutch is used. In order to bring both parts of the clutch to the same speed before engagement, a small synchronizing clutch with friction surfaces is used. A typical installation is shown in Fig. 128.

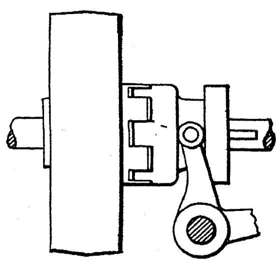


FIG. 127.—Jaw clutch.

147. Plate Friction Clutches. In Fig. 129 are shown two flanges, one keyed rigidly to the driving shaft, and the other fitted to the driven shaft by a feather key or spline so that it may be moved along the shaft. By a suitable mechanism, the flange *B* may be pressed against flange *A* so that torque may be transmitted by friction between the flanges. The amount of torque transmitted is dependent upon the axial pressure, the mean radius of the friction surfaces, and the coefficient of

friction. Hence

$$T = F_f r_m = f F_a r_m \tag{146}$$

where T = torque, lb-in.
F_f = friction force, lb.
F_a = axial force, lb.
r_m = mean radius, in.
f = coefficient of friction, (see Table 39).

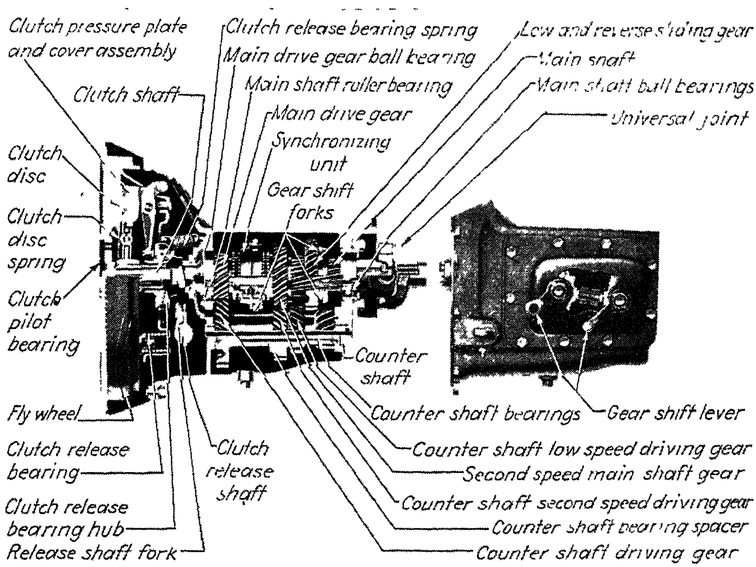


FIG. 128.—Automobile transmission and clutch. (Ford, 1942.)

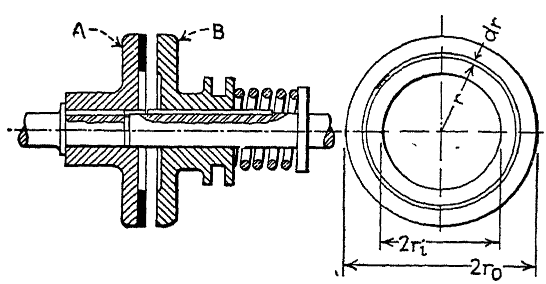


FIG. 129.

148. Uniform Pressure Distribution. If the pressure between the friction surfaces is assumed to be uniformly distributed

over the entire surface, then in Fig. 129 the pressure on an annular ring of radius r and width dr, is

$$dF_a = p2\pi r\,dr$$

where p = unit pressure, psi.

The frictional force on this ring is

$$dF_f = f\,dF_a = fp2\pi r\,dr$$

and the torque or turning moment is

$$dT = r\,dF_f = fp2\pi r^2\,dr$$

By integration, the total torque is found to be

$$T = \int_{r_i}^{r_o} dT = 2\pi f p\,\frac{(r_o^3 - r_i^3)}{3}$$

and the total axial force is

$$F_a = \int_{r_i}^{r_o} dF_a = 2\pi p\,\frac{(r_o^2 - r_i^2)}{2}$$

By substitution of these values of T and F_a in Eq. (146), the value of the mean radius is found to be

$$r_m = \frac{2}{3}\left(\frac{r_o^3 - r_i^3}{r_o^2 - r_i^2}\right) \tag{147}$$

where r_o = outer radius of friction disks, in.

r_i = inner radius of friction disks, in.

149. Uniform Wear. The condition of uniform pressure distribution assumed in the preceding paragraph is probably correct for new contact surfaces, but on account of relative motion between the surfaces during the engaging period, there is a certain amount of wear, especially when lubrication is absent, as is usually the case in clutches depending on friction for power transmission. In order that the surfaces will remain in contact, the wear in the axial direction must be the same for all values of r. But wear is proportional to the work done by friction, which is in turn proportional to the product of the normal pressure and the velocity of rubbing. Hence

$$W = \text{wear} = kpv = Kpr$$

from which

$$p = \frac{W}{Kr} = \frac{C}{r} \tag{148}$$

where C is a constant, since W and K are constants.

Referring again to Fig. 129,

$$T = \int_{r_i}^{r_o} 2f\pi pr^2\,dr$$

and, after substituting for p its value C/r,

$$T = 2\pi fC \int_{r_i}^{r_o} r\,dr = 2\pi fC\left(\frac{r_o^2 - r_i^2}{2}\right) \tag{149}$$

The total normal or axial pressure is

$$F_a = 2\pi \int_{r_i}^{r_o} pr\,dr = 2\pi C \int_{r_i}^{r_o} dr = 2\pi C(r_o - r_i) \tag{150}$$

Substitution of these values of T and F_a in Eq. (146) gives

$$r_m = \frac{r_o^2 - r_i^2}{2(r_o - r_i)} = \frac{r_o + r_i}{2} \tag{151}$$

Several types of plate clutches are shown in Figs. 128, 138, 139, and 140.

These indicate various methods of construction, of applying the axial pressure, and of releasing the clutch by moving the friction surfaces out of contact.

150. Cone Clutches. The equations developed for disk clutches apply to cone clutches. The pressure must be measured perpendicular to the cone surface and, of course, is the unit pressure times the area of the cone surface. An examination of Fig. 130 shows that the axial pressure necessary to produce the normal pressure is

$$F_a = F_n \sin\alpha \tag{152}$$

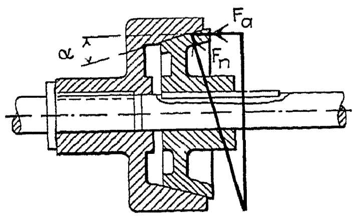


FIG. 130.

The clutch may be designed for free disengagement, in which case the tan α must be greater than the coefficient of friction, and a spring must be used to keep the cone surfaces in contact. If the clutch is not to disengage of its own accord, the tan α must

be less than the coefficient of friction and a force must be exerted to disengage the clutch. The angle α varies from $7\frac{1}{2}$ to 30 deg, depending upon the type of disengagement desired and upon the clutch facing material. The most common angle is about $12\frac{1}{2}$ deg. The proper angles may be obtained from the coefficients of friction given in Table 38.

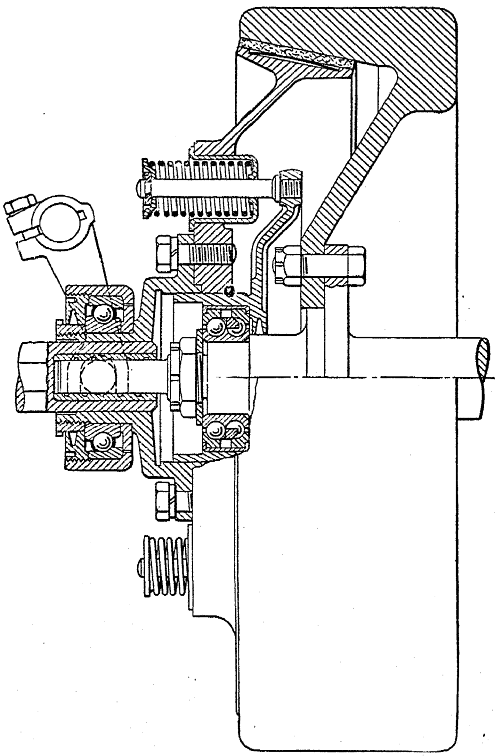


FIG. 131.—Cone clutch as formerly used in automobiles.

The axial pressure necessary may be exerted by suitable springs or by other means. If exerted by springs, the springs must be designed to sustain the load impressed when the clutch is released, since the spring is then still further compressed, and the load increases in direct proportion to the compression of the spring. The increase in spring load is usually 15 to 20 per cent of the operating load.

151. Block Clutches. In this type of clutch, power is transmitted by friction between a block, or series of blocks, pressed against the surface of a drum. A single block or shoe of such a

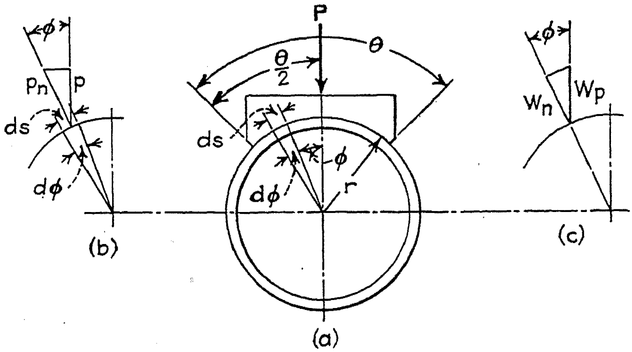


FIG. 132.

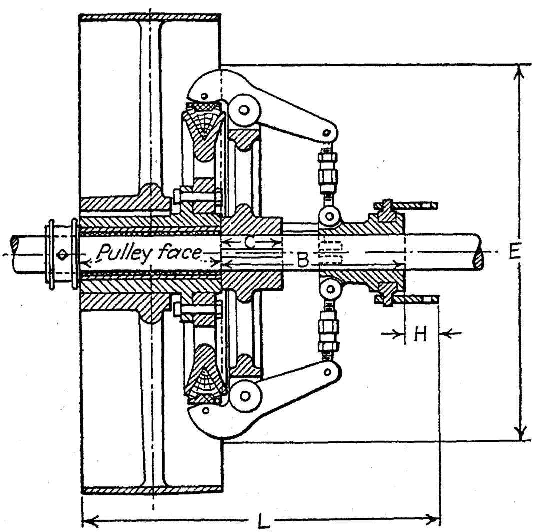


FIG. 133.—V-groove friction clutch pulley.

clutch is shown in Fig. 132. If the arc subtended by the block is not more than 60 deg, it may be assumed that the pressure is uniformly distributed over the contact area. Then the torque that can be transmitted by the single block of a clutch is

$$T = wfp_nr^2\theta \tag{153}$$

where T = torque, lb-in.
w = width of block, in.
f = coefficient of friction.
p_n = unit normal pressure, psi.
r = radius of drum, in.
θ = angle of contact, radians.
L = length of the chord subtended by the block, in.

The total force required to force the block against the drum is

$$P = 2p_nwr \sin \frac{\theta}{2} = p_nwL = \frac{TL}{fr^2\theta} = \frac{T}{fr}, \quad \text{nearly} \tag{154}$$

When the arc of contact is large, the unit pressure normal to the surface of contact is less at the ends than at the center. If the wear in the direction of the applied force is assumed to be uniform, then from Fig. 132c the wear normal to the drum surface is

$$W_n = W_p \cos \phi$$

The normal wear is proportional to the normal pressure times the rubbing velocity, which in this case is constant. Hence

$$p_n = \frac{KW_n}{V} = \frac{KW_p}{V} \cos \phi = C \cos \phi$$

where K and C are constants, and W_n and W_p are the normal wear and the wear parallel to the applied load, respectively

The normal pressure on the arc ds is

$$p_nw\, ds = Cwr \cos \phi\, d\phi$$

and the total pressure on the block is

$$P = \int (p_nw\, ds) \cos \phi = Crw \int_{-\frac{\theta}{2}}^{+\frac{\theta}{2}} \cos^2 \phi\, d\phi = \frac{Cwr}{2} (\theta + \sin \theta) \tag{155}$$

and the total torque transmitted is

$$T = \int fr(p_nw\, ds) = Cfr^2w \int_{-\frac{\theta}{2}}^{+\frac{\theta}{2}} \cos \phi\, d\phi = 2Cfr^2w \sin \frac{\theta}{2} \tag{156}$$

Solving Eqs. (155) and (156) for C and equating,

$$T = \frac{4fPr \sin \dfrac{\theta}{2}}{\theta + \sin \theta} \tag{157}$$

152. Expanding-ring Clutches. A clutch of this type consists of a drum or shell, attached to the driving shaft, and a split ring or band placed inside this shell and connected to the driven shaft. The ring may be expanded by a cam or wedge so that it presses against the inside of the driving shell. The pressure is commonly assumed to be uniformly distributed over the entire contact surface, although this is probably only approximately true.

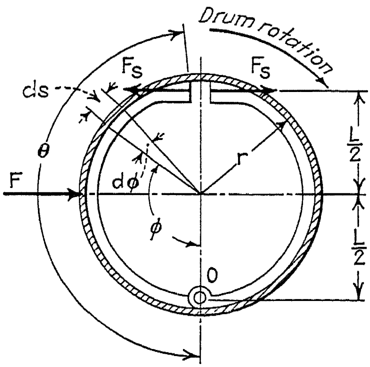


FIG 134.

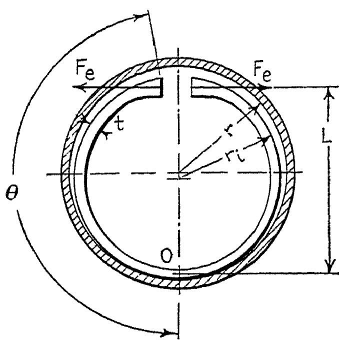


FIG. 135.

In Fig. 134, the force applied to the ends of the split ring to expand the ring is F_s. If the pressure is uniformly distributed over the contact surface,

$$T = 2 \int fpwr\, ds = 2 \int_0^{\theta} fpwr^2\, d\phi = 2fpwr^2\theta \tag{158}$$

where θ is one-half o the total arc of contact in radians.

If the ring is made in two sections as shown in Fig. 134, the force F_s required to separate the parts is found by taking moments about the pivot point O. The moment of the normal forces acting on the area ds, about the pivot point O, is

$$M_o = \int dM_o = \int_0^{\theta} pwr\, d\phi \left(\frac{L}{2} \sin \phi\right)$$

$$= \frac{pwrL}{2}(-\cos \theta + 1) \tag{159}$$

When θ is nearly equal to π radians (180 deg), $\cos\theta$ is approximately equal to -1, and

$$M_o = pwrL \text{ for each half of the band} \tag{160}$$

The moment of F_s about O must be equal to M_o. Hence

$$F_sL = M_o = pwrL$$
$$F_s = pwr \tag{161}$$

If the ring is made in one piece, then there is an additional force required to expand the inner ring before contact is made with the inner surface of the shell. Reference to Fig. 135 shows* that

$$\frac{M}{EI} = \frac{1}{r_1} - \frac{1}{r}$$

where r_1 = original radius of ring, in.
M = bending moment about O, lb-in.

In this case, M is equal to F_eL, and I is $wt^3/12$, the moment of inertia of the cross-sectional area of the ring. Hence

$$F_e = \frac{Ewt^3}{12L}\left(\frac{1}{r_1} - \frac{1}{r}\right) = \frac{Ewt^3}{6L}\left(\frac{1}{d_1} - \frac{1}{d}\right) \tag{162}$$

where d_1 = original diameter of ring, in.
d = inner diameter of drum, in.

Usually d_1 is from $\frac{1}{64}$ to $\frac{1}{16}$ in. smaller than d.

The total force required to expand the ring and to produce the necessary pressure between the contact surfaces is

$$F = F_s + F_e = pwr + \frac{Ewt^3}{6L}\left(\frac{1}{d_1} - \frac{1}{d}\right) \tag{163}$$

153. Band Clutches. Band clutches are especially suitable for mine hoists and other services where heavy loads are accompanied by severe shock. The clutch usually consists of a flexible steel band lined with wood or composition blocks, or with asbestos fabric, one end of which is fixed to either the driving or driven member, and the other end pulled around the circumference of a drum on the mating member. Figure 136 illustrates a clutch of this type.

* TIMOSHENKO and LESSELS, "Applied Elasticity," p. 230, Westinghouse Technical Night School Press

The ratio of tensions on the two ends of the band is expressed by the equation developed for belt tensions in Art. 223, page 304.

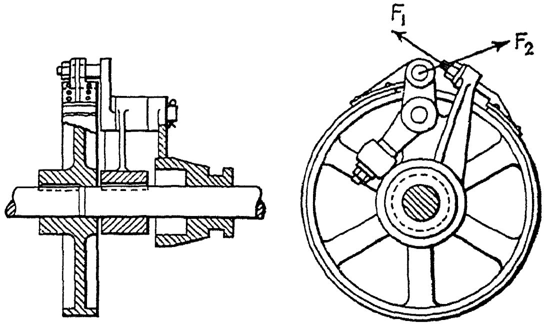


FIG. 136.—Band clutch.

Hence

$$\frac{F_1}{F_2} = e^{f\theta} \tag{164}$$

and the torque transmitted is

$$T = (F_1 - F_2)r \tag{165}$$

where T = torque, lb-in.
F_1 = maximum tension in band, lb.
F_2 = tension at opposite end of band, lb.
r = radius of the friction drum, in.
e = 2.718.
f = coefficient of friction.
θ = arc of contact, radians.

The width of the clutch band is determined by the permissible unit pressure, normal to the drum. The maximum normal pressure is at the high-tension end of the band and is

$$p_{\max} = \frac{F_1}{wr} \tag{166}$$

where w = width of band, in.

154. Unidirectional Clutches. It is often necessary to use a clutch that will transmit power when the driver is rotating in one direction but will automatically release when the direction of rotation is reversed, or when the driven shaft is rotating faster

than the driving member. An overriding clutch of this kind is shown in Fig. 137. The pockets of the inner member are slightly tapered, and when rotation is in the clockwise direction the rollers are forced to the large end of the pockets so that the inner member can rotate without driving the outer sleeve. When the direction of rotation is counterclockwise, friction causes the rollers to wedge between the inner member and the outer sleeve, so that power can be transmitted. The free-wheeling unit on recent models of automobiles is an overriding clutch of this kind.

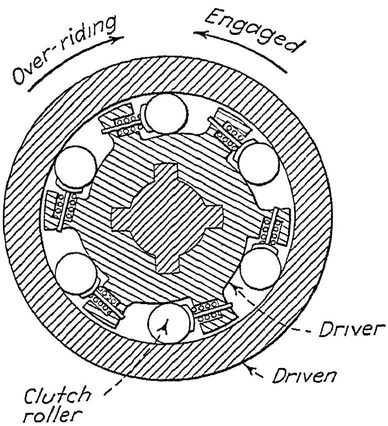


FIG. 137.—Overriding clutch.

155. Automatic Clutch Couplings. Some clutches are constructed so that the friction surfaces are held in engagement by the action of centrifugal force. These are used where the driving member has a low starting torque and must be brought up to speed before picking up the load. The centrifugal members are attached to the driving unit, and as the speed increases the friction shoes will gradually engage and bring the driven member up to speed. Used with internal-combustion engines, this clutch will disengage when the engine is idling, and will engage and pick up the load when the engine is speeded up.

156. Automobile Clutches. Clutches used in automobiles have passed through several important stages in their development. The early models were nearly all of the cone type, using leather or woven asbestos facings. Clutch facings at that time were not suitable for heavy unit pressures (10 to 15 psi being used) and large clutch surfaces were necessary. As the motors became more powerful, multiple-disk clutches were used to obtain the required area without increasing the diameter or the required axial pressure to abnormal values. Metal-to-metal friction surfaces with high unit pressures became popular, and these clutches were operated in an oil bath to prevent scoring of the metal surfaces when slipping under high pressure. The use of oil reduced the coefficient of friction far below that obtainable

with dry friction surfaces. As better facing materials were developed, multiple-disk dry-plate clutches came into general use.

All three types mentioned are now practically obsolete in automotive work, but have been successfully adapted to machine-tool and similar installations. A remarkable improvement in clutch facings permits the use of comparatively small diameter clutches, of one or two disks, to transmit the full power of present-day motors. The lever-release single- or double-plate clutch is

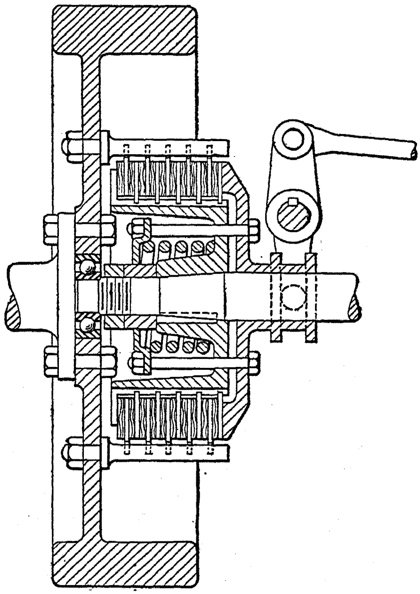


FIG. 138.—Multiple-disk clutch for automotive service.

in almost universal use in the automobiles of this country at the present time. This type of clutch is illustrated in Figs. 128 and 139.

157. Design Requirements and Constants. A clutch of good design must have ability to withstand and dissipate heat, adequate reserve-torque capacity, and long life. For the high speeds encountered in automotive practice, the driven members must have a low moment of inertia, all parts must be accurately balanced, and for the operator's convenience, the clutch must have positive release, smooth engagement, low operating force, and ease of repair.

To permit easy engagement and to prevent excessive wear during the engagement period, the facing on the driven disk should be flexible, and the largest possible area should be in contact during engagement. In automotive clutches, this may be accomplished by slitting the spring steel disks radially, and

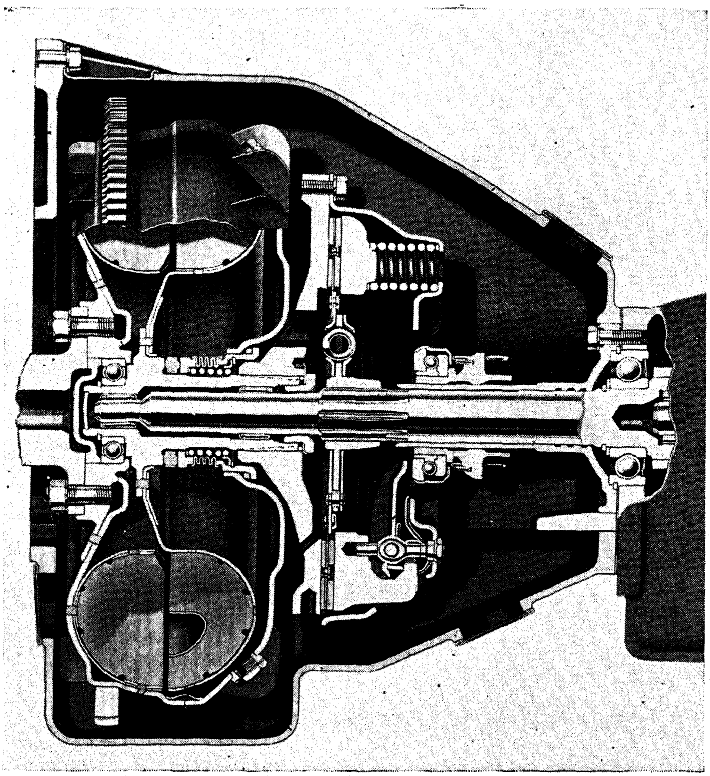


FIG. 139.—Single-disk plate clutch combined with a hydraulic coupling. (Chrysler, 1941.)

warping the adjacent segments in opposite directions. The facings are then fastened to the convex sides of the alternate segments so that engagement will be gradual and contact will extend over the entire facing surface. The driving disks of automotive clutches are usually castings thick enough to provide large heat conducting areas. When high-carbon spring-steel driving disks are used, they have a tendency to warp into conical

shape. The new high tensile-strength alloy cast irons give the best service.

The unit pressure on the contact surfaces varies with the facing material and with the type of service for which the clutch is designed. Suggested unit pressures are given in Table 39. When selecting the unit pressure, it should be remembered that reducing the normal pressure reduces the wear and therefore

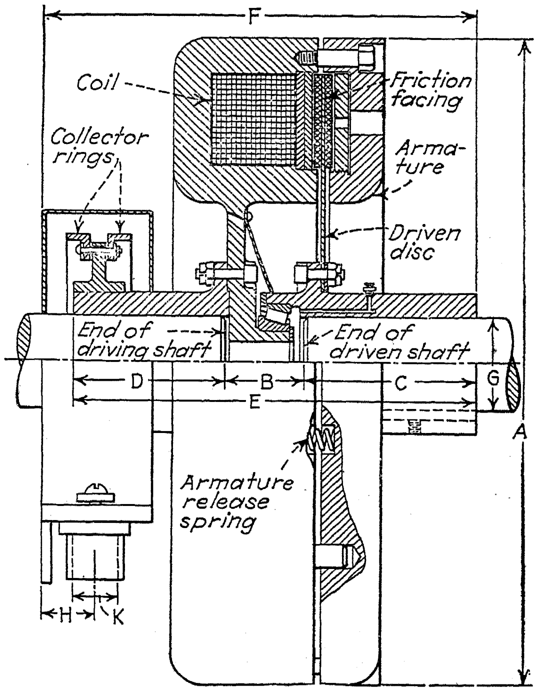


FIG. 140.—Magnetic clutch.

increases the life of the clutch between adjustments or facing renewals, and hence, clutches that must be operated frequently, or that are subject to considerable slippage while the driven shaft is being brought up to speed, should be designed with lower pressures than clutches that are infrequently used. Facings of high density give the longest life, and woven facings wear longer and have less scoring effect on the pressure plates and drums than molded facings, but are more expensive. Wood blocks give excellent service in industrial clutches.

The coefficient of friction depends upon the facing material and may vary with the normal pressure and with the temperature. Complete data on these variations are lacking, and it is customary to assume the coefficient to be constant for all pressures. New facings generally have higher coefficients of friction than do old facings, and to allow for this change the coefficient used in design is that of old facings. Design values of the coefficient of friction may be taken from Table 39. When

TABLE 39 —DESIGN VALUES FOR CLUTCH FACINGS

Facing material	Coefficient of friction	Unit pressure, psi
Cast iron on cast iron:		
Dry	0 20	40–60
Oily	0.07	
Cast iron on steel:		
Dry	0.30	40–60
Oily	0.10	
Leather on cast iron:		
Dry	0.50	10–12
Oily	0.15	10–12
Cork on cast iron:		
Dry	0.35	1 25–2
Oily	0.30	
Asbestos fabric:		
Dry	0.35–0 45	30–60
Oily	0.25	
Molded asbestos	0 25–0 35	30–60
Wood on cast iron	0.30	25–50

the contact surfaces are sliding over each other, as in starting, the coefficient is decreased. Also, when starting, the inertia of the driven parts must be overcome. Hence, to provide sufficient starting capacity, clutches should be designed for overload capacities of 75 to 100 per cent. If the load is variable or subject to shock, additional service factors should be used. Suggested service factors are given in Table 40.

158. Hydraulic Couplings. Many hydraulic transmissions have been developed in recent years; but only a few have been successful. There are two general types: (1) the displacement type transmitting power by liquid under pressure from a variable-

displacement pump to a constant-displacement hydraulic motor on the driven shaft; (2) the turbo-type transmitting power by the kinetic energy of a liquid discharged from a driving impeller against the vanes of a turbine runner on the driven shaft.

TABLE 40.—SERVICE FACTORS FOR CLUTCHES

Type of Service	Factor Not Including Starting Factor
Driving machine:	
Electric motor, steady load	1.0
Fluctuating load	1.5
Gas engine, single cylinder	1.5
Multiple cylinder	1.0
Diesel engine, high speed	1.5
Large, slow speed	2.0
Driven machine:	
Generator, steady load	1.0
Fluctuating load	1.5
Blower	1.0
Compressor, depending on the number of cylinders	2.0–2.5
Pumps, centrifugal	1.0
Single acting	2.0
Double acting	1.5
Line shaft	1.5
Wood working machinery	1.75
Hoists, elevators, cranes, shovels	2.0
Hammer mills, ball mills, crushers	2.0
Brick machinery	3.0
Rock crushers	3.0

The displacement type is positive in action and is especially adapted to installations where the power transmitted is not large but where close speed regulation over a large range of torque must be maintained, and where the speed of the driven shaft must remain constant in spite of the load changes. By varying the amount of liquid circulated (usually by varying the stroke of the pump plunger) the speed of the driven shaft may be varied from zero to a maximum. This type of drive is used successfully for machine-tool feeds, ship controls, aeronautic propeller controls, ordnance controls, and many similar devices.

The hydraulic coupling is a turbo-type transmission consisting of a rotating driving impeller, corresponding to a centrifugal pump, and a rotating driven runner corresponding to the runner

of a hydraulic turbine. The right-hand member shown in Fig. 141 is the driving impeller. Both members have radial guide vanes in the passages A and C, and when the impeller is rotated, liquid in the passage A is given a rotational velocity. Centrifugal force causes the liquid to flow outward between the radial vanes, and at B the axial component of the velocity carries the liquid across the gap into the runner where it moves inward through the passage C to D, where it again enters the impeller.

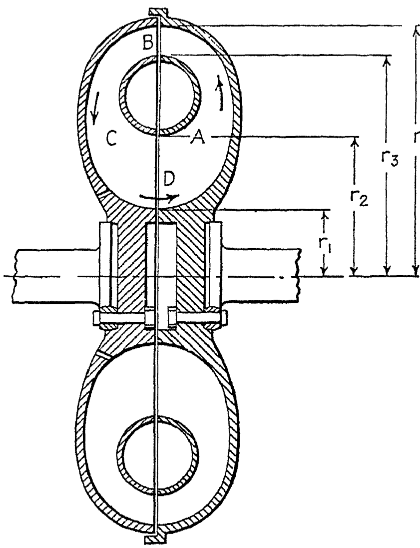


FIG. 141 —Hydraulic coupling.

At B the fluid has a rotational velocity v_1 and a kinetic energy of $\frac{1}{2}Mv_1^2$. At D the rotational velocity has decreased to v_2 and the kinetic energy to $\frac{1}{2}Mv_2^2$. Energy equal to $\frac{1}{2}M(v_1^2 - v_2^2)$ has been imparted to the runner producing driving torque.

The torque imparted to the runner is always equal to the torque developed by the impeller, and since liquid circulation is dependent upon centrifugal force, the runner must always rotate at a lower speed than the impeller. It follows that the efficiency of power transmission is always equal to 100 less the slip in per cent. The slip or loss of speed between the impeller and the runner is due to friction and turbulence losses and amounts to about 1 or 2 per cent at full torque; but when space limits the diameter, slip as high as 4 per cent may be used. The limit of torque trans-

mission is reached when the slip increases to the point where liquid circulation becomes turbulent. By designing for a maximum torque, at 100 per cent slip, slightly less than the maximum torque capacity of the driving-power source, it will be impossible to stall the driving engine or motor. Load changes cause changes in the speed of the driven runner; hence these couplings are not suitable where precise speed control is required.

The torque capacity is given by the equation

$$T = KSN^2W(r_o^2 - r_i^2) \tag{167}$$

where T = torque transmitted, lb-in.
K = a coefficient = $36/10^7$, approximately.
S = slip, per cent.
N = impeller speed, rpm.
W = weight of fluid circulating in operating portion of coupling, lb.
r_o = mean radius of outer passage, in. $= \frac{2}{3}\left(\frac{r_2^3 - r_1^3}{r_2^2 - r_1^2}\right)$.
r_i = mean radius of inner passage, in. $= \frac{2}{3}\left(\frac{r_4^3 - r_3^3}{r_4^2 - r_3^2}\right)$.
r_1, r_2, r_3, and r_4 are shown in Fig. 141.

It is readily seen that the power transmitted depends upon the coupling diameter, speed of rotation, fluid density, amount of fluid circulated, and the slip. The power loss* is largely due to friction of the fluid. The best fluid has been found to be a light mineral oil with a viscosity of about 150 sec Saybolt at 100 F.

The amount of power transmitted may be regulated by varying the mass of fluid circulating in the coupling. Two methods are in general use. In one, calibrated nozzles permit the fluid to leak from the periphery of the coupling and collect in a sump from which an independent pump returns it to the driving impeller. A valve in the pump discharge regulates the liquid feed and, hence, the mass of fluid in the circulating passages. The second method omits the pump but uses a reservoir surrounding the coupling and rotating with the impeller, as in Fig. 142. Centrifugal force causes the fluid to hug the outside of the reservoir chamber where a scoop, adjustable in position, picks up the fluid

* See BRUCKNER, R. E., A Simple Method for Calculating Hydraulic Coupling Performance, *Prod. Eng*, December, 1942.

and returns it to the impeller. By changing the position of the scoop, the amount of fluid in the circulating passages, and hence the power transmitted, can be changed at will. To disconnect the coupling, the fluid must be discharged from the circulating passages. This is done by opening valves in the outer portion of the coupling members.

To reduce the drag torque or creeping tendency of the driven member, a baffle has recently been applied at the point D on the

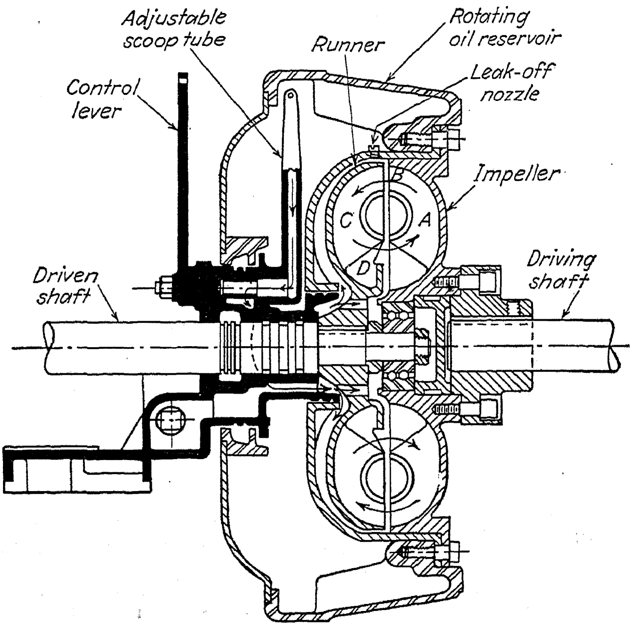


FIG. 142.—Hydraulic coupling.

driven runner, in Fig. 142. This baffle is a radial rib blocking off approximately the inner third of the circulating passage. When starting, the impeller rotates at the regular operating speed imparting a high velocity to the fluid which on entering the runner passages clings to the outer surface of the slow moving runner. This stream is broken up by the baffle ring, reducing the drag. As the runner speed increases, the fluid circulates above the baffle, and the drag and slip reach their normal value.

159. Hydraulic Torque Converter. In the two-element couplings the torque imparted to the runner is equal to that

developed in the impeller. By adding a stationary vane in the circulating circuit, the coupling becomes a torque converter and the torque imparted to the driven runner does not equal the

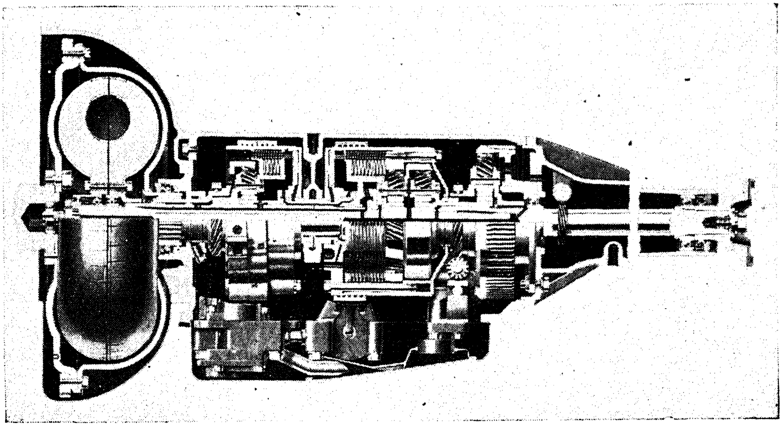


FIG. 143.—Hydramatic transmission. (Cadillac, 1941.)

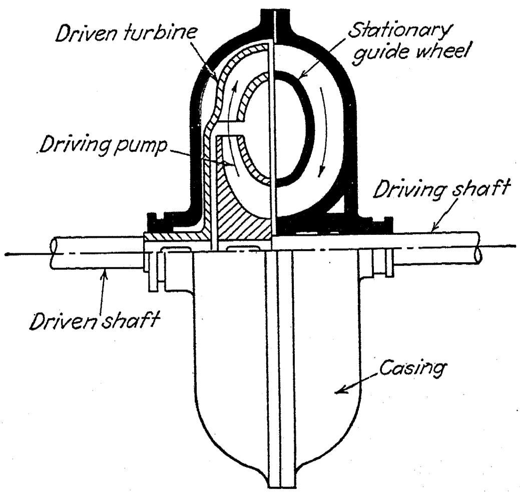


FIG. 144.—Hydraulic torque converter.

torque developed by the impeller. One or more stationary blades with an equal number of blades on the runner may be used. A simple single-stage torque-converter is shown in Fig. 144.

The power transmitted is given by the equation

$$T = KN^2D^5$$

where T = torque, lb-in.
K = design coefficient.
N = driven shaft speed, rpm.
D = outer diameter of vanes, in.

The coefficient K varies with the design and must be determined for any given design and vane arrangement. It can then be used to determine the torque capacity of any other geometrically similar converter.

The power* absorbed by the impeller and transmitted to the circulating fluid is proportional to the cube of the speed and the impeller must be designed to absorb the driving engine or motor power at operating speed. The engine will then theoretically operate at approximately constant speed regardless of the speed of the driven runner. Actually, the speed of an internal-combustion engine will drop 15 to 20 per cent from maximum speed of the runner to stalling speed. The runner operates from zero rpm, or stalling speed, to approximately two-thirds of the engine speed, at which speed the output torque is approximately equal to the engine torque.

These converters, when well designed, have a fairly flat efficiency curve ranging from 70 per cent at full runner speed and at 25 per cent runner speed, to a maximum efficiency of 85 per cent at about 80 per cent of full speed. This flat efficiency curve permits operation at various torque-speed ratios with fairly high efficiencies.

* For a discussion of power and design see P. M. Heldt, "Torque Converters," P. M Heldt, Nyack, N.Y.

CHAPTER XI

BRAKES

Brakes, like clutches, depend upon the friction between two surfaces for their action, the difference being that clutches are used to keep the driving and driven members moving together, whereas brakes are used to stop a moving member or to control its speed. The capacity of any brake depends upon the unit pressure between the braking surfaces, the coefficient of friction, and the heat-radiating capacity of the brake. The heat-radiating capacity is important, since, during the operating of the brake, the contact surfaces are sliding over each other, and the work of friction generates a large amount of heat that must be dissipated to avoid overheating the brake and burning the facing material.

160. Block Brakes. The equations developed in Art. 151 for block clutches apply equally well to block brakes, and the torque transmitted when the blocks are pressed against a flat or conical surface is

$$T = fP_n r_m \tag{168}$$

where T = torque applied at the braking surface, lb-in.

P_n = total normal force, lb.

f = coefficient of friction.

r_m = mean radius of braking surface, in.

When the blocks are pressed radially against the outer or inner surface of a cylindrical drum,

$$T = \frac{4fPr \sin \dfrac{\theta}{2}}{\theta + \sin \theta} \tag{169}$$

where θ is the angle of contact in radians.

When θ is less than 60 deg, the torque may be computed with sufficient accuracy by the use of the uniform-normal-pressure equation; hence

$$T = wfp_n r^2 \theta = fPr \tag{170}$$

The position of the fulcrum of the brake arm is of considerable importance in block brakes. In Fig. 145, if the force F_o applied at the end of the brake arm is made zero, there will be a small force P applied to the brake surface because of the weight of the arm. There will then be a frictional force F acting at a distance c above the fulcrum A and producing a moment about A of magnitude Fc. The direction of this moment is such that it increases the force P, which in turn increases the friction and the frictional moment, so that this brake is self-energizing. If the direction of rotation of the drum is reversed (in this case made counterclockwise), or if the fulcrum A is located above the line of action of F, the moment will act to release the braking

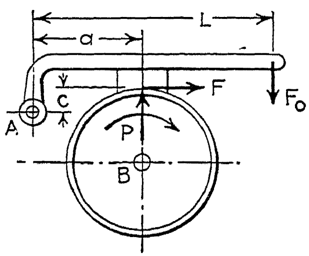


FIG 145

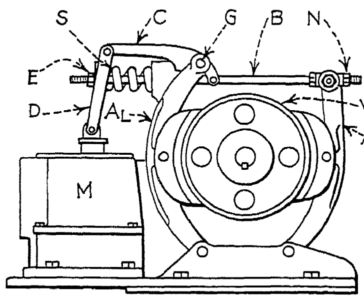


FIG. 146.—Solenoid-operated brake.

action. The moment due to the frictional force may be eliminated by placing the fulcrum on the line of action of the friction force F.

A good example of a block brake as used for elevator and hoisting service is shown in Fig. 146. This brake is positive in action, the brake shoes being pressed against the brake wheel W by means of the spring S, which is compressed between the end of the arm A_L and the spring retainer E on the end of the rod B. The brake is released by a magnet enclosed in the case M, which, when energized by electric current, pulls down on the rod D. As a result, the arm C rotates on its fulcrum G forcing apart the tops of the arms A_R and A_L, thus compressing the spring and releasing the brake. Releasing of both brake shoes is insured by an adjustable stop which limits the outward movement of the arm A_R. Adjustment for wear is made by means of the adjusting nuts at N.

161. Analysis of a Block Brake. A block brake similar to the one described in the preceding paragraph is shown diagrammatically in Fig. 147. The dimensions of a typical brake are given in the figure. The coefficient of friction may be taken as 0.35 and the spring may be assumed to exert a pressure of 3,000 lb when the brake is set.

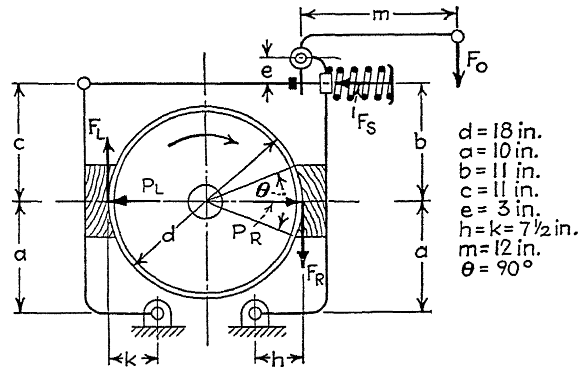


FIG. 147.

The normal pressure against the right-hand brake shoe is found by taking moments about the lower pin joints; thus

$$\Sigma M = F_R h + P_R a - F_s(a + b) = 0$$

from which

$$P_R = \frac{(a + b)F_s - F_R h}{a} \tag{171}$$

The relation between the friction force and the total shoe pressure, from Eq. (169), is

$$F_R = \frac{4fP_R \sin \frac{\theta}{2}}{\theta + \sin \theta}$$

and, by substituting this value in Eq. (171),

$$P_R = \frac{(a + b)F_s}{a + \dfrac{4hf \sin \frac{\theta}{2}}{\theta + \sin \theta}} = \frac{(10 + 11)3{,}000}{10 + \dfrac{4 \times 7.5 \times 0.35 \times 0.707}{1.57 + 1}} = 4{,}890 \text{ lb}$$

In the same way,

$$P_L = \frac{(10 + 11)3{,}000}{10 - \dfrac{4 \times 7.5 \times 0.35 \times 0.707}{1.57 + 1}} = 8{,}860 \text{ lb}$$

Note that in this case the shoe pressures are not equal. If the brake is to act only when the drum is revolving clockwise, the pressures, and hence the friction forces, may be equalized by making the distance b somewhat larger than the distance c, or by making h and k equal to zero.

The total braking capacity of this brake is expressed by

$$T = (F_R + F_L)r = \frac{4f \sin \dfrac{\theta}{2}}{\theta + \sin \theta} (P_R + P_L)r$$

$$= \frac{4 \times 0.35 \times 0.707}{1.57 + 1} (4{,}890 + 8{,}860)9 = 47{,}600 \text{ lb-in.}$$

The force that must be applied to the operating arm by the magnet to release the brake is

$$F_o = \frac{eF_s'}{m} = \frac{3 \times 3{,}000 \times 1.10}{12} = 825 \text{ lb}$$

The factor 1.10 is used, since the spring is further compressed when in the release position. The release pressure on the spring is usually 10 to 15 per cent greater than the required braking pressure.

After the forces acting are determined, the individual members of the brake may be considered separately to determine the required size of each. The arms carrying the brake shoes are treated as simple beams, the operating arm as two cantilever beams, and the pins at the various joints are oscillating high-pressure bearings.

162. Location of the Brake-shoe Pivot. If the friction forces tend to rotate the shoe on its supporting pin, the facing will not wear evenly, the unit pressure being greater at the toe of the shoe. This may cause overheating and brake chatter. If the wear is to be evenly distributed, the point of support must be located so that there is no frictional turning moment on the shoe. Referring to Fig. 148, which represents a brake shoe

operating on the outside of a brake drum, the pin is located on the axis of symmetry of the shoe. When the drum is rotating clockwise, the tangential friction forces tend to rotate the shoe in the counterclockwise direction, increasing the pressure on the top toe of the shoe.

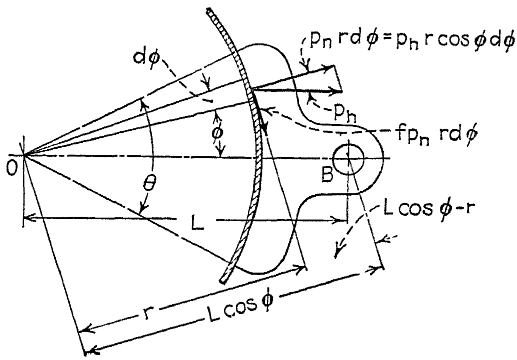


FIG. 148.

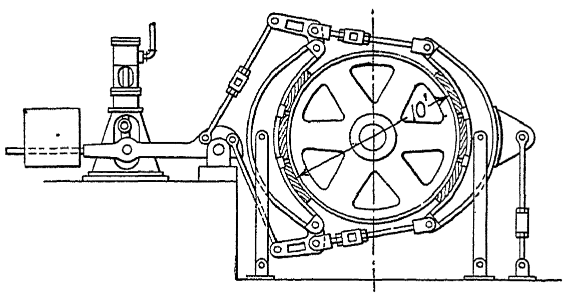


FIG. 149.—Mine-hoist brake, block type.

In the figure, consider a short length of friction surface, ds, subtended by an angle $d\phi$. The normal pressure on this small surface is $p_n r\, d\phi$ and the tangential frictional force is $fp_n r\, d\phi$. The moment of this friction force about the pin is

$$dM = fp_n r(L \cos \phi - r)d\phi$$

If the wear is to be uniform, p_n is equal to $p_h \cos \phi$, and the total moment about the pin is

$$M_B = \int_{-\frac{\theta}{2}}^{+\frac{\theta}{2}} fp_h r \cos \phi(L \cos \phi - r)d\phi = 0 \qquad (172)$$

and by integration

$$L = \frac{4r \sin \frac{\theta}{2}}{\theta + \sin \theta} \tag{173}$$

which is the distance from the center of the drum to the pivot point of the shoe, when the pivot is on the axis of symmetry of the shoe. If the shoe is not symmetrical, the length L can be determined by the introduction of the proper limits in Eq. (172) before integration.

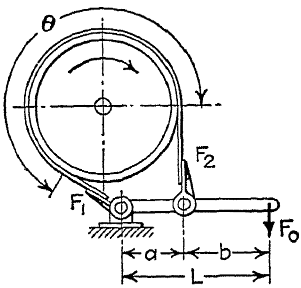


FIG. 150.

163. Band Brakes. The band brake consists of a rope, belt, or flexible steel band (lined with friction material) acting against the surface of a cylindrical drum. A simple band brake is shown in Fig. 150. The band acts in a manner similar to a belt wrapped around a pulley, and the ratio of the tension forces at the band ends is expressed by the belt equation

$$\frac{F_1}{F_2} = e^{f\theta} \tag{174}$$

where F_1 = force on high-tension side, lb.
F_2 = force on low-tension side, lb.
e = 2.718.
f = coefficient of friction.
θ = angle of contact, radians.

The torque developed by the braking action is

$$T = (F_1 - F_2)r \tag{175}$$

The force required at the end of the operating lever is

$$F_o = \frac{a}{a + b} F_2 \tag{176}$$

Note that if the direction of rotation is reversed, F_2 becomes larger than F_1, and a larger force F_o is required to operate the brake. With the operating arm in the position shown, and with the brake drum rotating clockwise, the friction may cause the

brake to lock itself, and the arm should therefore be held up when the brake is out of action. For this purpose a supporting hook or a counterweight on the lever to the left of the fixed support may be used.

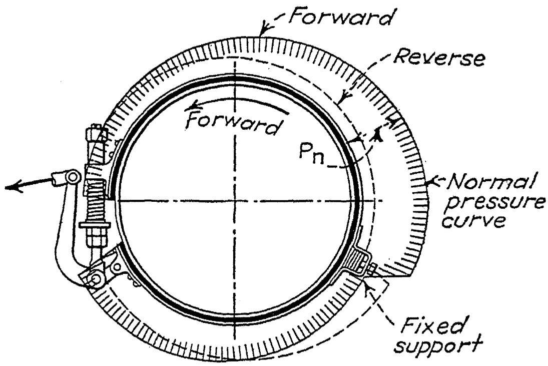


FIG. 151.—Pressure variation along the surface of a band brake.

The band width depends upon the maximum unit pressure that occurs at the high-tension end and is determined from

$$p_{max} = \frac{F_1}{wr} \tag{177}$$

The pressure distribution around the drum of a band brake is shown in Fig. 151, and the average unit normal pressure (which is required in heating computations) is

$$p_{avg} = \frac{F_1}{wrf\theta}\left(\frac{e^{f\theta} - 1}{e^{f\theta}}\right) \tag{178}$$

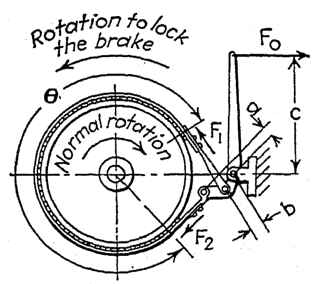


FIG. 152.

164. Self-locking Brakes. In cranes and hoists, it is desirable to use brakes that will automatically prevent the load from lowering when the hoisting power is removed. This end can be accomplished by a brake that will allow the drum to rotate freely in one direction but will lock itself when the drum begins to rotate in the opposite direction. A band brake of this type is shown in Fig. 152. In this brake, when the distance a is made greater than the distance b, and moments are taken about the

fixed support, with counterclockwise rotation.

$$\Sigma M = F_1 b - F_2 a + F_o c = 0$$

and

$$F_o = \frac{F_2 a - F_1 b}{c} = \frac{F_2 a - F_2 e^{f\theta} b}{c} = \frac{F_2}{c}(a - be^{f\theta})$$

If the brake is to be self-locking, the required operating force F_o must be zero. Hence, the last parenthesis must be zero, and the brake is self-locking when

$$(a - be^{f\theta}) \lesseqgtr 0$$

or

$$\frac{a}{b} \leqq e^{f\theta} \tag{179}$$

When the rotation is clockwise, any friction between the drum and the band will cause the band to loosen on the drum, but

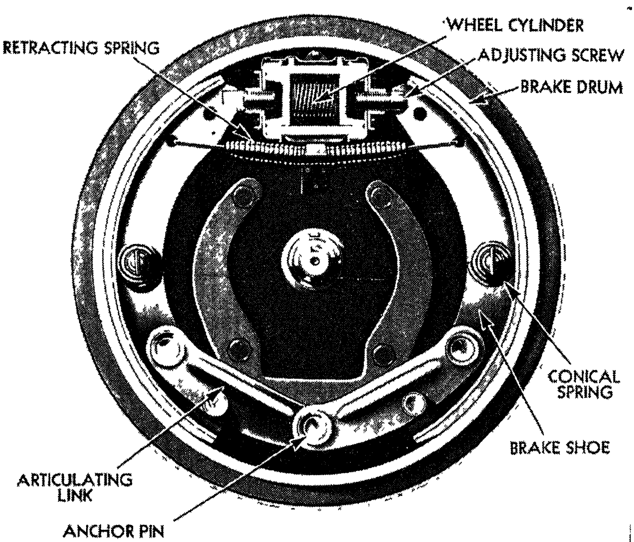


FIG. 153.—Shoe-type automobile brake with articulating link. (Chevrolet, 1942.)

when the rotation is counterclockwise, friction and the unequal moment arms will cause the band to tighten on the drum locking the brake. The brake may be released by the application of a force acting to the left at the end of the release lever.

165. Self-energizing Automobile Brakes. Band brakes were used on nearly all of the earlier automobiles, but these were exposed to dirt and water and the heat radiation was poor. These conditions, together with the tendency toward smaller wheels and larger tires, have forced the use of internal shoe brakes. Nearly all automotive brakes are now internal and self-energizing, that is, friction makes the shoe tend to follow the rotating brake drum wedging itself between the drum and

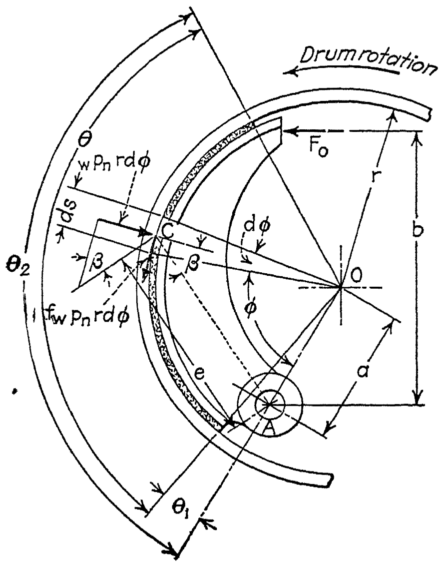


FIG. 154.

the point at which it is anchored. This action builds up a tremendous friction, giving great braking power without the use of excessive pedal pressures. When such brakes are not properly designed and adjusted, and when the brake facings do not have the proper coefficient of friction, the braking action may be too sensitive. Several types of automotive brakes, all of which are self-energizing, are shown in Figs. 153 to 155.

The shoe of the brake shown in Fig. 154 is assumed to be of rigid construction. When the shoe tends to rotate about the anchor pin A, the pressure will compress the brake lining by an amount that is proportional to the distance from A and in a direction perpendicular to the radius from A. The pressure

on the contact surface is proportional to the amount of compression and hence is also proportional to the radius from A. Thus the unit pressure at any point C on the braking surface, measured perpendicular to the radius e, is some constant K times the radius e. Normal pressure p_n is this pressure Ke times the $\sin \beta$, or p_n equals $Ke \sin \beta$ equals $Ka \sin \phi$. The total normal pressure on the area $w\,ds$ is $p_n wr\,d\phi$ equal to $Kawr \sin \phi\,d\phi$.

From the conditions of equilibrium, it is known that the sum of the moments of the friction forces, the normal forces, and the operating force about the anchor pin must be zero, or

$$M_n - M_f - F_o b = 0$$

from which

$$F_o = \frac{M_n - M_f}{b} \tag{180}$$

The total moment of the normal forces is

$$\begin{aligned} M_n &= \int_{\theta_1}^{\theta_2} (Kawr \sin \phi\,d\phi)(a \sin \phi) \qquad (181)\\ &= \frac{Ka^2wr}{2}(\theta_2 - \theta_1 - \sin \theta_2 \cos \theta_2 + \sin \theta_1 \cos \theta_1)\\ &= \frac{Ka^2wr}{4}(2\theta - \sin 2\theta_2 + \sin 2\theta_1) \end{aligned}$$

The total moment of the friction forces is

$$\begin{aligned} M_f &= \int_{\theta_1}^{\theta_2} (fKawr \sin \phi\,d\phi)(r - a \cos \phi) \qquad (182)\\ &= f\frac{Kawr}{2}[2r(\cos \theta_1 - \cos \theta_2) - a(\cos^2 \theta_1 - \cos^2 \theta_2)] \end{aligned}$$

Examination of Eq. (180) shows that friction alone will lock the brake when the friction moment M_f exceeds the moment of the normal pressure M_n. The brake should be self-energizing, but not self-locking. The amount of self-energizing is measured by the ratio of the friction moment and normal pressure moment. Hence

$$E = \frac{M_f}{M_n} = \frac{2f}{a}\left[\frac{2r(\cos \theta_1 - \cos \theta_2) - a(\cos^2 \theta_1 - \cos^2 \theta_2)}{2(\theta_2 - \theta_1) - \sin 2\theta_2 + \sin 2\theta_1}\right] \tag{183}$$

When E is equal to or greater than unity, no force F_o is necessary to set the brake, which is therefore self-locking. When E

is between unity and zero, the brake will be self-energizing. A well-designed brake should be self-locking only when the coefficient of friction has increased about 0.20 above the highest coefficient that the brake lining will ever have.

The torque capacity of the brake is the moment of the friction forces about the center of the brake drum. Hence

$$\begin{aligned} T &= \int_{\theta_1}^{\theta_2} fKawr^2 \sin \phi \, d\phi \\ &= fKawr^2(\cos \theta_1 - \cos \theta_2) \end{aligned} \tag{184}$$

When the dimensions of the brake shoe are known, the equations for T, F_o, M_n, and M_f reduce to a constant times K. The value of K is determined from the braking torque, or from the maximum permissible pressure on the contact surface. It has been shown that

$$p_n = Ka \sin \phi$$

and this relation indicates that the maximum normal pressure will be on a line normal to the line OA, *i.e.*, when $\sin \phi$ is unity. Hence

$$K = \frac{p_{\max}}{a} \tag{185}$$

The brake shoe should, in general, be symmetrical about the line of maximum normal pressure, and when the required cam movement, lining clearances, pressure distribution, and wear are considered, the practical length of shoe is limited to about 120 deg.

There is a reaction between the shoe and the supporting pin A, the direction and magnitude of which can be determined by the conditions of equilibrium of the forces acting on the shoe. Figure 155 shows a brake shoe similar to the one just analyzed except that the pin A is carried on a link that in turn is pivoted at the point E. As long as the reaction at A acts along the center line of this link, the link will remain stationary and the shoe will act as though the pin A was fixed. If, however, the coefficient of friction increases above that required for this condition, the direction of the reaction will change, and the link will swing slightly. The direction of swing is such that the pressure on the shoe, and hence the friction, is slightly decreased. On the other hand, when the coefficient of friction decreases, the direction of

swing will be such that the pressure on the shoe and the friction are slightly increased. Thus, within limits, the articulating link tends to compensate for variations in the coefficient of friction and

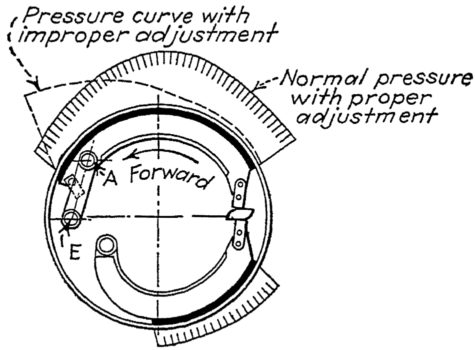


FIG. 155.—Pressure distribution on block brake with articulating link.

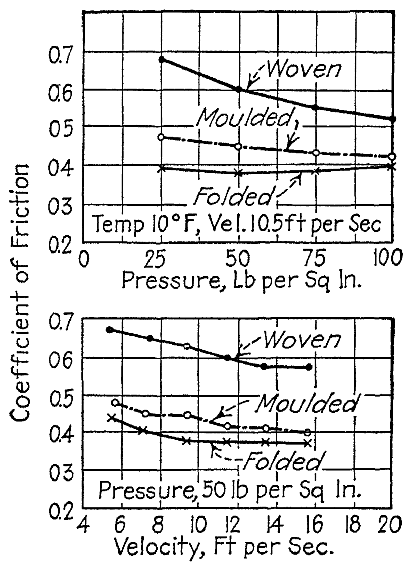


FIG. 156.—Typical variation in coefficient of friction of brake linings under variable operating conditions.

automatically keeps the energizing factor of the brake practically constant.

The pressure distribution on typical automotive brakes is shown in Figs. 151 and 155. These diagrams are based on

analysis similar to that made for the brake shown in Fig. 154. Note that when the brakes are properly adjusted, and when facings having the proper coefficient of friction are used, the maximum normal pressures range from 100 to 135 psi when the developed braking torque is sufficient to slide the tires. With improper coefficients of friction, the pressures may exceed 200 psi and the wear will be excessive. The effect on the operating pedal pressure should also be carefully considered. Well-designed automobile brakes require a force at the operating pedal of about 25 to 35 lb to overcome the linkage friction and the release-spring pressure, and about 125 lb to produce torque enough to slide the tires. Many brakes now in use are operated by auxiliary hydraulic booster cylinders and require less pedal pressure for the operation.

The brake shoes here considered have been of rigid construction. When the shoes are somewhat flexible, the pressure distribution will be slightly different from that already indicated. Also, it is evident that the heat of friction will raise the temperature of the outer surface of the shoe above that of the inner surface, causing the shoe to curl up. This action raises the pressure near the center of the shoe.

166. Design Factors for Brakes. The heat generated by the friction must be radiated to the atmosphere; hence the capacity of a brake may be limited by its ability to dissipate this heat. Stated in a different manner, the work done, which is proportional to the product of the unit pressure and the rubbing velocity, must be kept below certain maximum values that depend upon the type of brake, the efficiency of the radiating surfaces, and the continuity of service. Brakes operating very infrequently can be much smaller than those which operate almost continuously. The lower the unit pressure at the braking surface, the longer will be the life, and the less the danger of overheating. Hutte recommends the following for different types of service:

$pV = 55{,}000$ for intermittent operation with long rest periods, and poor heat radiation, as with wood blocks.

$pV = 28{,}000$ for continuous service with short rest periods, and with poor radiation.

$pV = 83{,}000$ for continuous operation and with good radiation, as with an oil bath.

where p = psi of projected area.
V = rubbing velocity, fpm.

One prominent manufacturer of mine hoists uses allowable pressures as shown in Table 41.

TABLE 41 —WORKING PRESSURES FOR BRAKE BLOCKS

Rubbing velocity, fpm	Pressure, psi		
	Wood blocks	Asbestos fabric	Asbestos blocks
200	80	100	160
400	65	80	150
600	50	60	130
800	35	40	100
1,000	25	30	70
2,000	25	30	40

Allowable pressures and coefficients of friction for several brake materials operating under ordinary conditions are given in Table 42.

TABLE 42.—DESIGN VALUES FOR BRAKE FACINGS

Facing Material	Design coefficient of friction	Permissible unit pressure psi	
		200 fpm	2,000 fpm
Cast iron on cast iron:			
Dry	0 20		
Oily	0 07		
Wood on cast iron	0 25–0 30	80–100	20–25
Leather on cast iron:			
Dry	0 40–0 50	8– 15	
Oily	0 15		
Asbestos fabric on metal:			
Dry	0 35–0.40	90–100	25–30
Oily	0 25		
Molded asbestos on metal, dry	0.30–0.35	150–175	30–40

167. Hydrodynamic Brakes. A brake utilizing fluid friction instead of mechanical surface friction has been introduced in the oil fields to handle the heavy loads incident to deep drilling. These brakes do not replace the regular mechanical brakes. They are used to provide the necessary brake effect

when lowering tools, drill pipe, and casing into the hole. The regular mechanical brake is required to stop the descending load and to hold it stationary when drilling operations make this necessary.

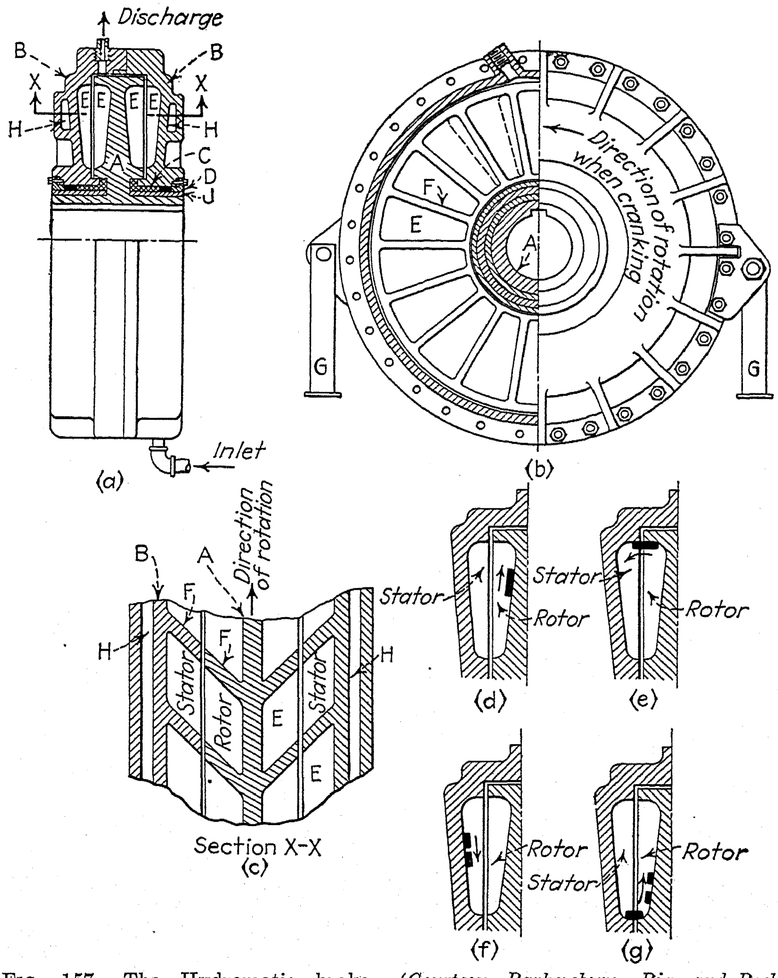


FIG. 157.—The Hydromatic brake. (*Courtesy Parkersburg Rig and Reel Company.*)

The Hydromatic brake is illustrated in Fig. 157. The rotor A is attached to the shaft, which is supported on bearings in the stator B, which is restrained from rotation. On both sides

of the stator and rotor are pockets E, opposed to each other and separated from each other by partitions F inclined to oppose the direction of rotation during the load lowering period. Cool liquid flows from the inlet pipe along the passages H and through nozzles into the pockets E. In Fig. 157d, a slug of water is shown traveling outward in the rotor pocket under the action of centrifugal force and absorbing kinetic energy. In Fig. 157e, the slug is moving from the rotor to the stator pocket and is being sheared off by the pocket edges. This action retards the rotor. In Fig. 157f, the sheared slug is passing inward along the stationary pocket and losing energy by friction against the walls. At the inner edge of the pocket the slug still retains enough energy to force it back into the rotor, the shearing action as it passes the pocket edges again retarding the rotor. The motion of this slug represents the cycle of operation of a continuously circulating stream of fluid. During the operation, the fluid absorbs all the heat generated by the braking action. The heated fluid is discharged at the top of the stator, passed through a cooler, and returned to the inlet pipe. The resistance, or braking effort, is controlled by varying the amount of fluid in the pockets. With any given amount of circulating fluid, the resisting torque of the brake increases faster than the applied load torque so that the brake acts as a governor limiting the lowering speed. When raising the load, the inclination of the pocket walls allows them to slip through the fluid rings without appreciable resistance. In practice, water is the usual operating fluid.

168. Heating of Brakes. Brakes (and many clutches) used continuously often present a more difficult problem in heat dissipation than in mechanical design. The temperature will increase until the rate of dissipation is equal to the rate of heat generation, and the braking area, the radiating surface, and the air circulation must be proportioned so that overheating will be prevented. The heat to be radiated is the heat due to the work of friction, and when expressed in Btu per minute is

$$H = \frac{fPV}{778} \tag{186}$$

For a lowering brake the heat to be radiated is, in Btu,

$$H = \frac{Wh}{778} \tag{187}$$

where W = weight lowered, lb.

h = total distance, ft.

The ability of the brake drum to absorb heat is proportional to the mass and to the specific heat of the material. Assuming that all heat generated is absorbed by the brake drum and its supporting flange, the temperature rise is

$$t_r = \frac{H}{W_r c} \tag{188}$$

where W_r = weight of the brake rim, lb.

c = specific heat of the material, Btu per lb (0 13 for cast iron and 0.116 for steel).

Some of the heat generated will be immediately radiated to the air, carried away by air currents, and to a slight degree conducted away through the contacting parts, so that it is impossible to compute the actual temperature attained by the rim. It is also impossible to compute the time required for the brake to cool, since cooling laws have not been well established, but it is possible to compute the rate of heat loss at a given temperature, and since the heat loss must equal the rate of heat generation, the horsepower capacity of the brake can be determined for any given maximum temperature. The temperature rise should be limited to a maximum of 500 F (350 for average conditions) for the best grades of asbestos on cast iron, or 150 for wood or leather. The actual temperature of the drum after a single application of the brake is not so important as the assurance that the rate of cooling equals or exceeds the rate of heat generation.

Since 1 hp is equivalent to 42.4 Btu per min, the heat generated in Btu per min, is

$$H_g = 42.4q \text{ hp} \tag{189}$$

where q = a load factor or the ratio of the actual brake operating time to the total cycle of operation.

The rate of heat dissipation in Btu per min, is

$$H_d = Ct_rA_r \tag{190}$$

where C = radiation factor, Btu per sq in. per min per °F temperature difference.

A_r = radiating surface, sq in.

t_r = difference between the temperature of the radiating surface and the surrounding air.

The factor C, which increases with the temperature difference, may be taken from Table 43.

TABLE 43.—RADIATING FACTORS FOR BRAKES

Temperature difference t_r	Radiating factor C	Ct_r
100	0 00060	0.06
200	0 00075	0.15
300	0.00083	0.25
400	0 00090	0.36

Equating the heat generated and the heat dissipated, and solving for A_r, the radiating surface required is found to be

$$A_r = \frac{42.4q \text{ hp}}{Ct_r} \tag{191}$$

The radiating surface includes all the exposed surface of the brake drum not covered by the friction material, and the solid portion of the center web down to the center of the holes that are usually inserted to lighten the casting. Both sides of the web may be counted provided the air can circulate freely. Radiation may be increased by providing air passages through the drum and by using fins to increase the surface.

Example. Assume a hoisting engine to be equipped to lower a load of 6,000 lb by means of a band brake having a drum diameter of 48 in. and a width of 8 in. The band width is 6 in , and the arc of contact is 300 deg. The load is to be lowered 200 ft, the hoisting cycle being 1.5 min hoisting, 0.75 min lowering, and 0.3 min loading and unloading.

The generated heat, equivalent to the power developed, is

$$H_g = \frac{6{,}000 \times 200}{0.75 \times 33{,}000} = 48.5 \text{ hp}$$

The load factor is

$$q = \frac{0.75}{1.5 + 0.75 + 0.3} = 0.294$$

Allowing a temperature rise of 300°, the radiating surface required is

$$A_r = \frac{42.4 \times 0.294 \times 48.5}{.25} = 2{,}418 \text{ sq in.}$$

The radiating surface of the brake drum is

$$A_d = 2(\pi \times 48 \times 8) - (\pi \times 48 \times 6)\tfrac{300}{360} = 1{,}658 \text{ sq in.}$$

which leaves about 760 sq in. to be provided for in the flanges and web.

The actual temperature of the drum will vary slightly above and below the 300° rise assumed, since heat is radiated during the whole cycle but generated during only 29.4 per cent of the cycle This brake drum will weigh about 600 lb. Hence the temperature rise during the braking operation will be

$$\begin{aligned}\Delta t &= \frac{1}{778W_rc}(Wh - Ct_rA_rm \times 778)\\ &= \frac{1}{778 \times 600 \times 0.13}(6{,}000 \times 200 - 0.25 \times 2{,}418 \times 0.75 \times 778)\\ &= 14\end{aligned}$$

In this, m is the lowering time in min. This result indicates that the drum temperature will vary about 14°, or 7° above and below the average.

As already stated, the actual temperature attained by the brake drum and the time required for it to cool can not be accurately calculated, but the method just outlined may be used for preliminary computations. In the final design of a new brake, heating should be checked by a proportional comparison with a brake already known to give good performance in actual service.

An approximation of the time required for the brake to cool may be made by the formula

$$T_c = \frac{W_rc \log_e t_r}{KA_r} \tag{192}$$

where T_c = cooling time, min.

K = a constant varying from 0.4 to 0.8.

A_r = radiating surface, sq in.

The other symbols have the same meaning as before.

CHAPTER XII

SPRINGS

The problem of spring design is the application of the principle that a load applied to any member will produce a proportionate deformation. In most machine members, the deformation must be kept low, that is, the member must be kept stiff and rigid. In a spring, the reverse effect is desired, and the deformation must be relatively large, the spring being a machine member built to have a high degree of resilience.

Springs are used as cushions to absorb shock, as in machine supports, on automobile frames, and in airplane landing gear; as a source of power by storing up energy that is later delivered as driving power, as in clocks, trigger mechanisms, etc.; and to maintain contact between machine members by exerting a direct force, as in clutches, brakes, valve springs, and cam followers. Springs are also used as load-measuring devices, as in spring balances, power dynamometers, and in instruments such as gauges, meters, and engine indicators.

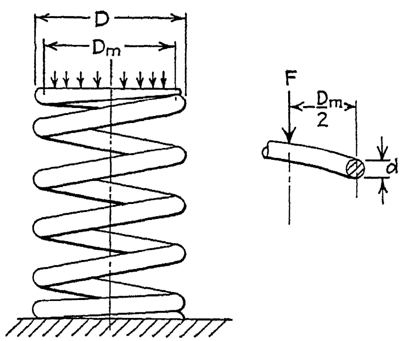


FIG. 158.

169. Stress in Coil Springs of Round Wire. The coil, or helical, spring consists of a wire or rod wound about a mandrel to form a helix and is primarily intended for axial direct compression or tension loads.

The action of the force F in Fig. 158 tends to rotate the wire, thereby causing torsional stresses in the wire. Also, bending, direct compression, and direct shearing stresses, which are neglected in the conventional spring equations, are set up in the wire. Considering only the

torsion and assuming a round wire, the stress is

$$s_s = \frac{Tc}{J} = \frac{FD_m}{2} \times \frac{16}{\pi d^3} = \frac{8FD_m}{\pi d^3} \tag{193}$$

where F = axial load, lb.
D_m = mean coil diameter, in.
d = wire diameter, in.

In this equation, the bending, direct shear, compression, and wire curvature are neglected. An analysis by A. M. Wahl* that considers these stresses indicates that the stress at the inner surface of the coiled wire may reach values 60 per cent higher than those given by Eq. (193), when the spring index is low. The spring index is the ratio of the mean coil diameter to the wire diameter. According to Wahl, the maximum stress in the spring wire is

$$s_s = K\frac{8FD_m}{\pi d^3} \tag{194}$$

where

$$K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C}$$

and C is the spring index D_m/d. To simplify the computations, the value of K may be taken from the curve in Fig. 159.

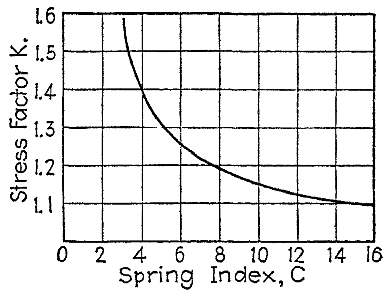


FIG. 159.—Stress factors for coil springs.

170. Stress in Coil Springs of Noncircular Wire. Coil springs made of square or rectangular wire are frequently used, since a stronger spring can be built into the same space required for a spring of round wire. The stress in a square-wire spring, based on St. Venant's torsion theory for noncircular bars, is

$$s_s = K\frac{FD_m}{0.416b^3} = \frac{2.4FD_m}{b^3}K \tag{195}$$

* WAHL, A. M , General Considerations in Designing Mechanical Springs, *Machine Design*, May to August, 1930, and January to April, 1938.

WAHL, A. M., *Trans. A.S.M.E.*, 1929, APM-51-17.

WAHL, A. M., Stresses in Heavy Coiled Helical Springs, *Trans. A.S.M.E* , Vols. 51 and 52.

See also LATSHAW, ELMER, Stresses in Heavy Helical Springs, *Jour. Franklin Inst.*, June, 1930.

For rectangular wires with the long dimension parallel to the axis, the stress is

$$s_s = K\frac{FD_m(3b + 1.8t)}{2b^2t^2} \tag{196}$$

where t = wire dimension perpendicular to the spring axis, in.
b = dimension parallel to the spring axis, in.

The spring index for coil springs of noncircular wire may be closely approximated by using the ratio of the mean coil diameter to the dimension of the wire perpendicular to the axis.

171. Deflection of Coil Springs. The axial deflection of the spring is found from the angular twist of the coiled wire. Then

$$\theta = \frac{TL}{JG} = \frac{FD_m}{2} \times \frac{\pi D_m n}{\cos\psi} \times \frac{32}{\pi d^4} \times \frac{1}{G} = \frac{16FD_m^2 n}{d^4 G \cos\psi} \tag{197}$$

where θ = angular twist, radians.
ψ = helix angle.
n = number of effective coils.
G = modulus of rigidity, psi.

In most springs, the helix angle is small, and $\cos\psi$ is practically unity. The movement of the point of load application is practically $\theta D_m/2$. Hence the deflection of a spring of round wire is

$$y = \frac{\theta D_m}{2} = \frac{8FD_m^3 n}{d^4 G} = \frac{8FC^3 n}{Gd} = \frac{n\pi D_m^2 s_s}{KdG} \tag{198}$$

where y = axial deflection, in., and the other symbols have the same meaning as before.

The deflection of a coil spring of square wire is

$$y = \frac{5.575FD_m^3 n}{b^4 G} = \frac{2.32D_m^2 n s_s}{KbG} \tag{199}$$

The deflection of a coil spring with rectangular wire is

$$y = \frac{2.45FD_m^3 n}{Gt^3(b - 0.56t)} = \frac{4.90b^2 D_m^2 s_s n}{KGt(b - 0.56t)(3b + 1.8t)} \tag{200}$$

172. Conical Coil Springs. Certain installations require a spring with increasing stiffness as the load increases, *i.e.*, a decreasing rate of deflection per unit load. This can be accomplished by winding the wire in a conical form so that the larger

coils, which have the greater deflection rate, will successively drop out of action by seating on the next smaller coil. In order to save space, some springs are wound so that the coils telescope into each other.

The maximum stress will generally be in the coil of largest diameter, but since the spring index decreases at the small end, the stress should be checked in the coil of least diameter. The stress equations already developed hold for conical springs.

The deflection of a conical spring of round wire is

$$y = \frac{2nF(D_1 + D_2)(D_1^2 + D_2^2)}{d^4G} \tag{201}$$

and for a conical spring of flat wire with the long dimension parallel to the axis

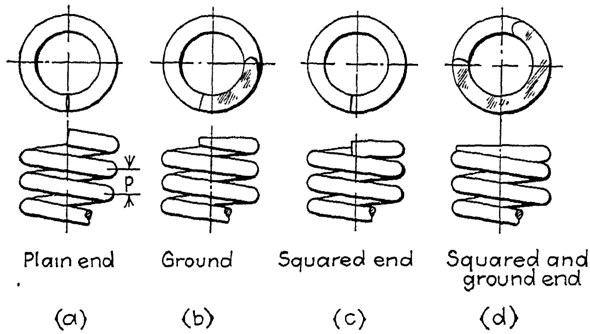
$$y = \frac{0.71nF(b^2 + t^2)(D_1 + D_2)(D_1^2 + D_2^2)}{b^3t^3G} \tag{202}$$

where D_1 and D_2 are the mean diameters of the smallest and largest coils, respectively.

173. Design of Compression and Tension Springs. By changing the mean diameter, the wire diameter, and the number of coils, any number of springs may be obtained to support a given load with a given deflection. However, there are certain limitations imposed by the use to which the spring is to be put. The usual procedure is to assume a mean diameter and a safe working stress, after which the wire diameter is found by substitution in the proper stress equation. The number of effective coils is then found from the deflection equation. Several trials are usually required before a suitable combination is obtained. The stress factor K depends on the spring index that must be assumed in the first trial solution for the wire diameter. For general industrial uses the spring index should be 8 to 10; for valve and clutch springs 5 is common; and 3 is a minimum value to be used only in extreme cases. Because of slight variations in the modulus of rigidity, variations in wire diameter, and other manufacturing tolerances, the deflection equations do not give extremely accurate values, and if extreme accuracy is required, the manufacturer should be consulted.

Compression springs are usually compressed solid when subjected to the maximum permissible stress, and the free

length will be the closed length plus the maximum deflection. The closed length depends upon the number of effective coils and the type of end provided. Different types of ends are illustrated in Fig. 160, together with equations to determine the free length. Note that the effective number of coils n is less than the actual number in the spring.



	Actual no. coils	Solid length	Free length
(a)	n	$(n+1)d$	$np + d$
(b)	n	nd	np
(c)	$n+2$	$(n+3)d$	$np + 3d$
(d)	$n+2$	$(n+2)d$	$np + 2d$

FIG. 160.—Compression springs.

A helical compression spring that is too long compared to its mean diameter may buckle at comparatively low axial loads since such a spring is a very flexible column. The critical axial load that will cause buckling is indicated by the formula*

$$F_{cr} = K_S K_L L_0 \tag{203}$$

where F_{cr} = axial load to produce buckling, lb.

K_S = spring constant, or load per inch of axial deflection.

K_L = factor depending on the ratio L_0/D_m, from Fig. 161.

L_0 = free, or open, length of spring, in.

D_m = mean diameter of coil, in.

In the selection of the value of K_L from Fig. 161, a hinged-end spring may be considered as one supported on pivots at both ends, and a built-in end as one in which a squared and ground-end spring is compressed between two rigid and parallel flat plates.

* WAHL, A. M., When Helical Springs Buckle, *Machine Design*, May, 1943

Tension springs are usually wound with the coils close and under an initial tension; *i.e.*, it is necessary to apply from 20 to 30 per cent of the maximum load before the coils begin to separate. When loops or end hooks are provided, the small radius

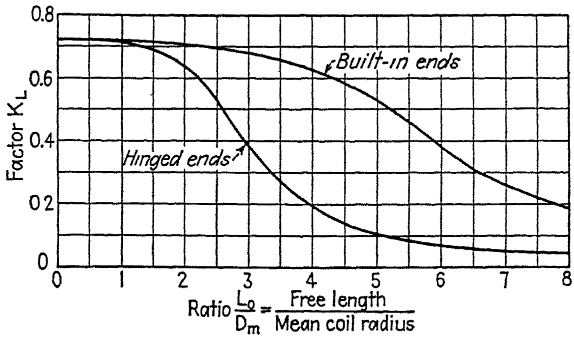


FIG. 161.—Buckling factor for helical compression springs.

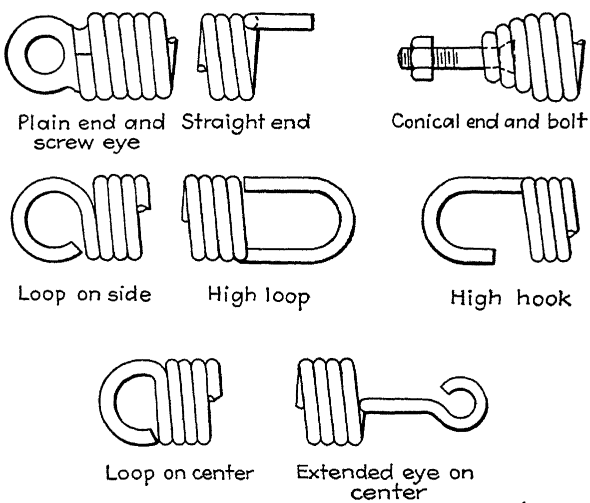


FIG. 162.—Extension spring ends.

where the hook joins the first coil is a region of high local stress. To provide for this local stress and for the greater possibility of excess deformation, the design stress for tension springs should not exceed 70 per cent of that used with compression springs.

174. Eccentric Loads on Springs. The requirements of the design may be such that the load line does not coincide with

the axis of the coils. This not only reduces the safe load for the spring but may affect the stiffness. When the load is offset a distance a from the spring axis, the safe load on the spring should be reduced by multiplying the safe load with axial loading by the factor $D_m/(2a + D_m)$.

175. Materials for Coiled Springs. The majority of coil springs are made of oil-tempered carbon-steel wire containing 0.60 to 0.70 per cent carbon and 0.60 to 1.00 per cent manganese.

TABLE 44.—STRENGTH OF OIL-TEMPERED STEEL WIRE

C = 0.60% Ph = 0.04% max
Mn = 0.80% S = 0.04% max
Si = 0.15%
E = 30,000,000 G = 11,600,000

W-M gauge, steel wire gauge	Diam., in. d	Ultimate strength, minimum, in tension s_t	Yield* stress, torsion s_{sy}	Diam., in. d	Ultimate strength, minimum, in tension s_t	Yield* stress, torsion s_{sy}
34	0 0104	300,000	135,000	$\frac{3}{32}$	234,000	105,000
24	0 0230	287,000	129,000	$\frac{1}{8}$	220,000	99,000
20	0 0348	274,000	123,000	$\frac{5}{32}$	209,000	94,000
18	0 0475	262,000	118,000	$\frac{3}{16}$	200,000	90,000
16	0.0625	251,000	113,000	$\frac{7}{32}$	193,000	87,000
14	0 080	240,000	108,000	$\frac{1}{4}$	189,000	85,000
12	0.1055	228,000	102,000	$\frac{9}{32}$	182,000	82,000
10	0.135	216,000	97,000	$\frac{5}{16}$	178,000	80,000
8	0.162	207,000	93,000	$\frac{11}{32}$	176,000	79,000
6	0.192	198,000	89,000	$\frac{3}{8}$	171,000	77,000
4	0.2253	191,000	86,000	$\frac{13}{32}$	169,000	76,000
2	0.2625	186,000	84,000	$\frac{7}{16}$	167,000	75,000
0	0 3065	180,000	81,000	$\frac{15}{32}$	165,000	74,000
00	0.331	176,000	79,000	$\frac{1}{2}$	162,000	73,000

Higher strengths may be obtained, but the ductility will be decreased.

* Stresses in the table are the maximum stresses that a compression spring will sustain without permanent set and should be used only when the maximum defection is mechanically limited and the load is virtually steady. The stress in springs subject to fluctuations should be decreased as indicated in the notes below Table 45.

Music wire is used for high-grade springs using wire less than $\frac{1}{8}$ in. diameter. Annealed wire containing 0.85 to 0.95 per cent carbon and 0.30 to 0.40 per cent manganese is used in the larger

TABLE 45.—PROPERTIES OF COMMON SPRING MATERIALS

Material	Type of spring	Type of stress	Maximum working stress, s_0 based on $d = \frac{1}{8}$ in	Modulus of elasticity	Stress at maximum temperature
Spring steel, oil-tempered, S A E 1360	Compression Extension Torsion	Torsion Torsion Flexure	100,000 70,000 120,000	E = 28,500,000 G = 11,600,000	
Spring steel, hard-drawn, S A.E 1360	Compression Extension Torsion	Torsion Torsion Flexure	70,000 50,000 85,000	E = 30,000,000 G = 11,600,000	
Music wire, S A E 1095	Compression Extension Torsion	Torsion Torsion Flexure	100,000 70,000 120,000	E = 30,000,000 G = 11,600,000	
Vanadium steel,	Compression Extension Torsion	Torsion Torsion Flexure	100,000 70,000 120,000	E = 30,000,000 G = 11,600,000	
Chrome-vanadium, tempered, S A.E 6150	Compression Extension Torsion	Torsion Torsion Flexure	100,000 70,000 120,000	E = 30,000,000 G = 11,600,000	60,000 at 450 F
Chrome-vanadium, hard-drawn, S.A.E. 6150	Compression Extension Torsion	Torsion Torsion Flexure	70,000 50,000 85,000	E = 28,000,000 G = 10,800,000	
*Stainless steel, tempered C, 0 35%; Cr, 18%; Ni, 8%	Compression Extension Torsion	Torsion Torsion Flexure	100,000 70,000 120,000	E = 30,000,000 G = 11,600,000	60,000 at 550 F
Stainless steel hard-drawn, Cr, 18%; Ni, 8%	Compression Extension Torsion	Torsion Torsion Flexure	70,000 50,000 85,000	E = 28,000,000 G = 10,800,000	42,500 at 450 F
Phosphor bronze, S.A.E. 81	Compression Extension Torsion	Torsion Torsion Flexure	45,000 31,500 54,000	E = 16,000,000 G = 6,000,000	
Brass, spring, S A.E. 80	Compression Extension Torsion	Torsion Torsion Flexure	40,000 30,000 48,000	E = 12,000,000 G = 5,000,000	
Monel metal, Cu, 28%; Mn, 2%; Ni, 67%; Fe, 3%	Compression Extension Torsion	Torsion Torsion Flexure	45,000 31,500 54,000	E = 23,000,000 G = 9,250,000	40,000 at 350 F
Everdur, Cu, 95%; Mn, 1%; Si, 4%	Compression Extension Torsion	Torsion Torsion Flexure	45,000 31,500 54,000	E = 16,000,000 G = 6,000,000	

Stresses tabulated are the maximum stresses without permanet set and should only be used where the spring load is constant with no fluctuation, and when the maximum spring deformation is mechanically limited.

Springs subject to infrequent fluctuations of variable amount from zero to the maximum or from an intermediant to the maximum should have maximum stresses 90 per cent of the tabulated values.

Springs subject to rapid fluctuations of variable amount from zero to the maximum or from an intermediant to the maximum should have maximum stresses 80 per cent of the tabulated values

Springs subjected to rapidly repeated and regular fluctuations from zero to the maximum or from an intermediant to the maximum should have maximum stresses 70 per cent of the tabulated values

* Chromium spring steels should not be used where subjected to low temperatures, since at 0 F, and below, these steels may become brittle under stress.

sizes that are coiled hot and hardened and drawn after coiling. Nickel steel, chromium steel (stainless steel), brass, Phosphor bronze, Monel metal, and other metals that can be hard drawn are used in special cases to increase fatigue resistance, corrosion resistance, and temperature resistance.

The strength of drawn steel wire increases with the reduction in size, and the hardness increases with a corresponding loss in ductility. The minimum tensile strength of steel spring wire is given very closely by the equation

$$s_u = \frac{C_1}{\sqrt[4]{d}} - \frac{C_2}{d} \tag{204}$$

where $C_1 = 177{,}500$ and $C_2 = 2{,}050$ for music wire
$C_1 = 138{,}000$ and $C_2 = 1{,}600$ for oil-tempered 0.60 C steel wire
$C_1 = 124{,}000$ and $C_2 = 1{,}450$ for hard-drawn and stainless-steel wire

The yield stress increases from 75 to 85 per cent of the ultimate strength as the diameter decreases. Since coil springs are in torsion, the design stress must be based on the torsional shear strength. Tests indicate that the yield stress in torsion is approximately 45 per cent of the ultimate strength in tension for oil-tempered wire, and 35 per cent for hard-drawn wire.

TABLE 46.—VARIATION OF THE MODULUS OF RIGIDITY WITH TEMPERATURE

Temperature, F	Modulus of rigidity, G		
	Spring steel	Stainless steel	Brass
−200	11,700,000		
100	11,600,000	11,600,000	5,000,000
200	11,500,000	11,600,000	4,900,000
400	11,200,000	11,450,000	4,300,000
600	10,800,000	11,200,000	
800	10,200,000	10,800,000	
1000	9,500,000	10,200,000	

Safe working stresses should not exceed 80 per cent of the yield stress in torsion, and if the spring is continuously subjected to rapid load fluctuations, the working stress must be reduced

from 25 to 50 per cent. A method of estimating suitable working stresses was developed in Art. 80.

When springs are to be used in high temperatures, the modulus of rigidity should be taken from Table 46. This table is only approximate since the modulus of rigidity varies with temperature, wire size, analysis, and heat-treatment.

176. Critical Frequency of Coiled Springs. Any spring having periodical applications of load may be subject to surging. Thus, at certain critical speeds, the valves of internal-combustion engines may surge or vibrate after the valve has closed, permitting the valve to flutter on its seat and interfere with the proper operation of the engine. Since the surges require time to travel from coil to coil and return, a periodic force may cause surges to be superimposed upon each other; and if this occurs, high stresses that have been known to cause spring breakage are induced.

The speeds at which surging will occur correspond to the natural frequency of the spring, which depends upon the wire diameter, the coil diameter, the number of effective coils, and the elastic properties of the spring wire. The natural frequency is expressed by the equation

$$f = \frac{d}{2\pi D_m^2 n}\sqrt{\frac{6Gg}{w}} \tag{205}$$

where f = frequency, cycles per sec.
w = weight of the wire, lb per cu in.
g = acceleration due to gravity = 32.2 ft per sec^2.

Experimental results* indicate that the actual frequency of the spring is from 10 to 15 per cent below that given by the equation. According to W. M. Griffith,† the critical frequency of the spring should be at least twenty times the frequency of application of a periodic load, in order to avoid resonance with all harmonic frequencies up to the twentieth order.

177. Torsion Springs. Torsion springs are used as cushions on flexible drives transmitting rotary motion or torque and as

* KENT, C. H., Don't Overlook Surge in Designing Springs, *Machine Design*, October, 1935.

† GRIFFITH, W. M., Engineering Standards for the Design of Springs, *Prod. Eng.*, July, 1933.

sources of power for driving such mechanisms as clocks. The springs may be coiled in a helix or in a spiral.

In the helical torsion spring, the wire is subjected to flexural stresses. The wire forms a curved beam, the stresses in which may be found by Eq. (454). Starting with the curved beam

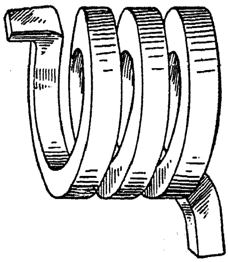


FIG. 163*A*.—Helical torsion spring.

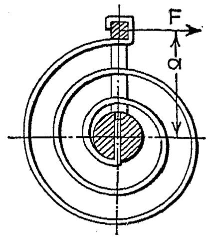


FIG. 163*B*.—Spiral torsion spring.

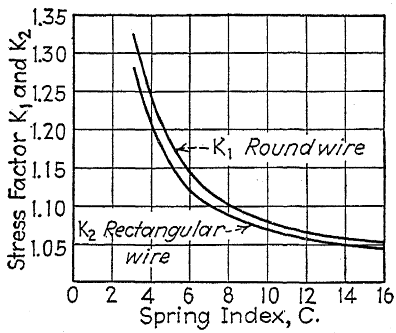


FIG. 164.—Stress factors for torsion springs.

equations and making certain simplifications, A. M. Wahl shows that the stress in a torsion spring of round wire is

$$s_f = \frac{32M}{\pi d^3}\left[\frac{4C^2 - C - 1}{4C(C-1)}\right] = K_1\frac{32M}{\pi d^3} = K_1\frac{32Fa}{\pi d^3} \qquad (206)$$

where M = bending moment, lb-in.
K_1 = stress factor from Fig. 164.
F = load, lb.
a = moment arm of load or distance from load line to spring axis, in.

For rectangular wire, the stress is

$$s_f = \frac{6M}{bt^2}\left[\frac{3C^2 - C - 0.8}{3C(C-1)}\right] = K_2\frac{6M}{bt^2} = K_2\frac{6Fa}{bt^2} \qquad (207)$$

where b = wire dimension parallel to axis, in.
t = dimension perpendicular to axis, in.

The deflections of helical torsion springs are given by the equations

$$y = \frac{64FD_m a^2 n}{Ed^4} \quad \text{and} \quad \theta = \frac{3{,}665FD_m an}{Ed^4} \quad \text{for round wire} \tag{208}$$

and

$$y = \frac{12\pi FD_m a^2 n}{Ebt^3} \text{ and } \theta = \frac{2{,}160FD_m an}{Ebt^3} \text{ for rectangular wire} \tag{209}$$

where y = movement of point of load application, *i.e.*, at distance a from the spring axis, in.
θ = angular movement, deg.
a = moment arm, in.

The stress in spiral torsion springs is given by Eqs. (206) and (207). The deflections are given by

$$y = \frac{64FLa^2}{\pi Ed^4} \quad \text{and} \quad \theta = \frac{1{,}168FLa}{Ed^4} \quad \text{for round wire} \tag{210}$$

and

$$y = \frac{12FLa^2}{Ebt^3} \quad \text{and} \quad \theta = \frac{687.6FLa}{Ebt^3} \quad \text{for rectangular wire} \tag{211}$$

where L = length of wire in the spiral, in.

For all practical purposes, the length may be taken as πnD_m.

Deflection formulas for torsion springs give values slightly less than the actual deflections, since the equations make no allowance for the decrease in diameter as the coils wind up under load. All torsion springs should be installed so that the applied load will wind up the wire, reducing the diameter, and clearance must be provided when the spring operates around a mandrel. The springs should also be wound with a small clearance between adjacent coils to prevent sliding friction.

178. Protective Coatings. Before forming the springs, the wire may be coated with copper or tin and immediately drawn through a die to obtain a high burnish. The coating is thin and is not an efficient corrosion resistant. After forming, the springs may be electroplated with cadmium, chromium, nickel, brass, or

copper. The acid cleaning before plating may cause embrittlement of the material unless carefully done. The formed springs may be dipped in or sprayed with japan which has a good resistance to rusting. Music-wire springs are usually not protected, and the high chromium or stainless steels need no protection. To prevent clashing of the coils and injury to the coatings, the clearance between the coils when the spring is fully compressed should not be less than 10 per cent of the wire diameter.

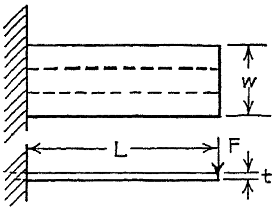


FIG. 165.

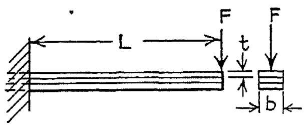


FIG. 166.

179. Leaf Springs. A single thin plate supported at one end and loaded at the other may be used as a spring. The stress in such a spring, shown in Fig. 165 is

$$s = \frac{Mc}{I} = \frac{6FL}{wt^2} \tag{212}$$

and the deflection is

$$y = \frac{1}{3}\frac{FL^3}{EI} = \frac{4FL^3}{wt^3E} = \frac{2sL^2}{3\,tE} \tag{213}$$

If the plate is cut into a series of n strips of width b, and these are placed as shown in Fig. 166, the above equations become

$$s = \frac{6FL}{nbt^2} \tag{214}$$

and

$$y = \frac{4FL^3}{nbt^3E} = \frac{2sL^2}{3Et} \tag{215}$$

which gives the stress and deflection in a leaf spring of uniform cross section.

The stress in such a spring is maximum at the support. If a triangular plate is used, as in Fig. 167, the stress will be uniform throughout. If this triangular plate is cut into strips

and placed as in Fig. 168 to form a graduated leaf spring, then

$$s = \frac{6FL}{nbt^2} \tag{216}$$

and

$$y = \frac{6FL^3}{nbt^3E} \tag{217}$$

If bending stress alone is considered, the graduated spring may have zero width at the loaded end, but sufficient metal must

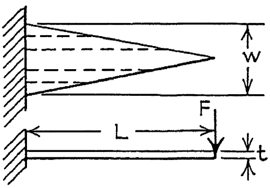


FIG. 167.

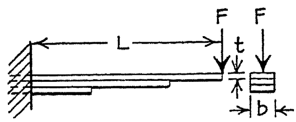


FIG. 168

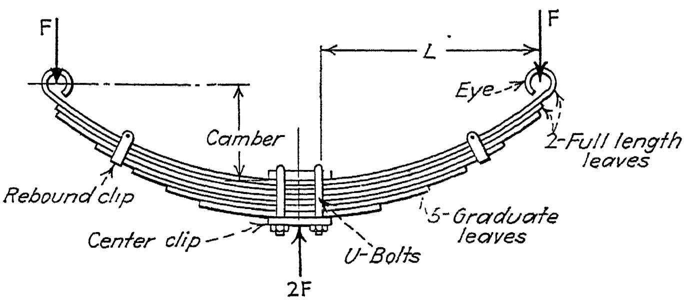


FIG. 169.—Typical semielliptic leaf spring.

be provided to support the shear. Hence it is necessary to have one or more leaves of uniform cross section extending clear to the end. Examination of the deflection equations indicates that, for the same deflection, the stress in the uniform-section leaves is 50 per cent greater than in the graduated leaves. If the subscripts f and g are used to indicate the full-length (uniform section) and graduated leaves, then

$$s_f = \tfrac{3}{2}s_g$$

and

$$\frac{6F_fL}{n_fbt^2} = \frac{3}{2}\left(\frac{6F_gL}{n_gbt^2}\right)$$

from which

$$F_g = \frac{2n_g F_f}{3n_f} = \frac{2n_g}{2n_g + 3n_f} F \tag{218}$$

and

$$F_f = \frac{3n_f}{2n_g + 3n_f} F \tag{219}$$

From Eq. (214)

$$s_f = \frac{6FL}{n_f bt^2}\left(\frac{3n_f}{2n_g + 3n_f}\right) = \frac{18FL}{bt^2(2n_g + 3n_f)} \tag{220}$$

which is the relation between the maximum stress and the load applied at the end of the spring when all leaves have the same thickness.

The deflection of the spring is

$$y = \frac{12FL^3}{bt^3E(2n_g + 3n_f)} \tag{221}$$

180. Equalized Stress in Spring Leaves. In the spring just discussed, the stress in the full-length leaves was found to be 50 per cent greater than the stress in the graduated leaves. In order to utilize the material to the best advantage, all leaves should be stressed the same. This condition can be obtained if the full-length leaves are given a greater radius of curvature than that used in the graduated leaves, before the leaves are assembled to form the spring. As shown in Fig. 170, this will leave a gap or clearance space between the leaves. When the center spring clip is drawn up tight, the upper leaf will bend back and have an initial stress opposite to that produced by the normal loading, and the lower leaf will have an initial stress of the same nature as that produced by the normal loading. As the normal load is applied, the stress in the upper leaf is relieved and then increased in the normal way. The initial gap between the leaves can be adjusted so that under maximum-load conditions the stress in all leaves will be the same; or, if desired, the top leaves may have the lower stress. The latter case is desirable in automobile springs, since the top leaf must

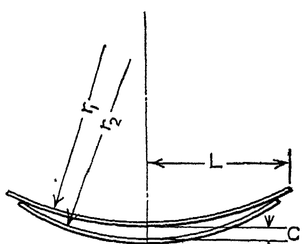


FIG. 170

carry additional loads caused by the swaying of the car, twisting, and in some cases compression due to driving the car through the rear springs.

Consider the case of equal stress in all leaves at the maximum load. Then at maximum load the total deflection of the graduated leaves will exceed the deflection of the full-length leaves by an amount equal to the initial gap c. Hence

$$c = \frac{6F_gL^3}{n_gbt^3E} - \frac{4F_fL^3}{n_fbt^3E}$$

But, since the stresses are equal,

$$F_g = \frac{n_g}{n_f}F_f = \frac{n_g}{n}F$$

and

$$F_f = \frac{n_f}{n}F$$

When the proper substitutions are made,

$$c = \frac{6FL^3}{nbt^3E} - \frac{4FL^3}{nbt^3E} = \frac{2FL^3}{nbt^3E} \tag{222}$$

The load on the clip bolts F_b required to close the gap is determined by the fact that the gap is equal to the sum of the initial deflections of the full and graduated leaves. Hence

$$c = \frac{2FL^3}{nbt^3E} = \left(\frac{4L^3}{n_fbt^3E}\right)\frac{F_b}{2} + \frac{6L^3}{n_gbt^3E}\,\frac{F_b}{2}$$

from which

$$F_b = \frac{2n_gn_fF}{n(2n_g + 3n_f)} \tag{223}$$

The final stress in the spring leaves will be the stress in the full-length leaves due to the applied load less the initial stress. Then

$$\begin{aligned} s &= \frac{6L}{n_fbt^2}\left(F_f - \frac{F_b}{2}\right) \\ &= \frac{6FL}{n_fbt^2}\left(\frac{3n_f}{2n_g + 3n_f} - \frac{n_fn_g}{n(2n_g + 3n_f)}\right) \end{aligned}$$

from which

$$s = \frac{6FL}{nbt^2} \tag{224}$$

The deflection caused by the applied load is the same as in the spring without initial stress.

181. Materials for Leaf Springs. Plain carbon steel of 0.90 to 1.00 per cent carbon, properly heat-treated, is commonly used. Chrome-vanadium and silico-manganese steels are used for the better-grade springs. The alloy steels do not have strengths greatly in excess of the carbon steels, but have greater toughness and a higher endurance limit and are better suited to springs subjected to rapidly fluctuating loads.

TABLE 47.—PROPERTIES OF LEAF SPRING MATERIALS

Material	Yield stress s_y	Endurance limit s_e
Spring steel:		
S.A.E. 1095	175,000	100,000
Chrome-vanadium:		
S.A.E. 6140	200,000	110,000
S.A.E. 6150	200,000	110,000
Silico-manganese:		
S.A.E. 9250	190,000	115,000
S.A.E. 9260	200,000	120,000

Spring-steel plate is rolled to the Birmingham wire gauge (B.W.G.).

The properties of common spring materials are given in Table 47. The selection of the proper working stress is complicated by the fact that the spring is essentially a shock-absorbing device. Hence, the working stress should be based on the endurance limit and, of course, should never exceed the yield stress. The endurance limit in reversed bending varies from 40 to 50 per cent of the ultimate strength in tension. In order to provide for surface defects in the rolled steel from which leaf springs are made, and for large variations in stress when in service, the springs should be designed for a working stress of about one-half the endurance limit when at the resting load. Where the load variation can be definitely predicted, suitable working stresses may be determined by the method outlined in Art. 80.

182. Design of Leaf Springs. The stress and deflection as determined by the usual spring equations are slightly altered

by several items that must be considered before the final design is accepted. In all important designs, a sample spring should be tested before being put into service.

In the development of the equations it was assumed that the leaves were uniform in thickness and that the ends of the graduated leaves were pointed. Heavy machinery, truck, and railway springs are usually designed in this way. Automobile and similar springs are usually rounded and thinned at the ends, and this condition slightly changes the deflection.

Friction between the leaves tends to reduce the deflection and make the springs stiffer. To reduce wear and to obtain uniform spring action, some designers are specifying a lubricant incorporated in the spring assembly and are providing spring coverings to retain the lubricant and to exclude grit.

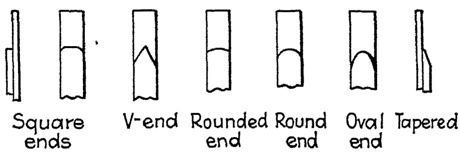


FIG. 171.—Types of spring-leaf ends.

The spring leaves are held together by a center bolt, a bolted clip, or a band shrunk on. The center bolt reduces the metal area at the section of maximum stress and also causes highly localized stresses near the hole. This is poor design and such springs usually break through the center hole. Shrunk bands are superior construction, but are used only on heavy springs. With spring clips, the bolts must be drawn up tightly or bending of the leaves will occur under the clip, increasing the effective spring length and hence the deflection and stress. The effective length of a spring with bolted center clips should be measured to a point about one-third the distance from the clip edge to the center.

As the load is applied to the spring, the curvature and the effective length change, thus altering the deflection rate. By properly choosing the initial camber, the spring may be made to soften or stiffen as the load is increased.

If the leaves are all given the same curvature before assembly, they will separate during the rebound, and dirt and grit may enter between them. This condition may be prevented by

rebound clips, or the springs may be nipped, *i.e.*, the shorter leaves are given a slightly greater initial curvature, thus maintaining contact at all times.

To protect the spring from excessive stress during the rebound, short rebound leaves may be placed above the master leaves. The rebound clips also aid in distributing the load to all the leaves during the rebound.

Some springs are made with the top leaf thinner than the others. Equation (214) shows that as t is decreased, the stress for the same deflection is decreased. Hence, when the top leaf is subjected to twisting or compression loads, it is desirable to use a thin top leaf to increase the margin of safety.

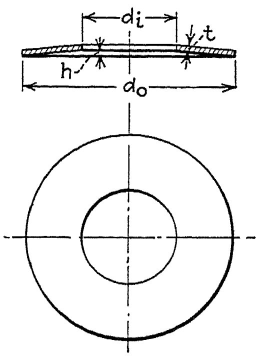


FIG. 172.—Disk spring.

183. Disk Springs.* Disk or "Belleville" springs are used where space limitations require high capacity units. Each element consists of an annular disk, initially dished to a conical shape as shown in Fig. 172.

When the load is applied uniformly around the edge, the relation between the applied load and the axial deflection is given by the equation

$$F = \frac{4Ey}{(1 - m^2)Md_o^2}\left[(h - y)\left(h - \frac{y}{2}\right)t + t^3\right] \qquad (225)$$

where y = axial deflection of each disk, in.
t = thickness, in.
d_o = outside diameter, in.
m = Poisson's ratio.
h = free height of truncated cone, in.
M = a constant depending upon ratio of outside to inside diameter, see Fig. 173.

* For more complete discussions see:

ALMEN, J. O., and LAZZLO, A , Disc Springs Facilitate Compactness, *Machine Design*, June, 1936.

BOYD, W. W., Deflection and Capacity of Belleville Springs, *Prod. Eng.*, September, 1932.

BOYD, W. W., Radially Tapered Disc Springs, *Prod. Eng* , February, 1933.

BRECHT, W. A., and WAHL, A. M., Radially Tapered Disc Springs, *Trans. A.S.M.E.*, Vol. 52, APM 52-4, 1930.

The maximum stresses occurring at the edges are given by the equations

$$s = \frac{4Ey}{(1 - m^2)Md_o^2}\left[C_1\left(h - \frac{y}{2}\right) + C_2t\right] \qquad \text{at the inner edge}$$

and (226)

$$s = \frac{4Ey}{(1 - m^2)Md_o^2}\left[C_1\left(h - \frac{y}{2}\right) - C_2t\right] \qquad \text{at the outer edge} \tag{227}$$

where C_1 and C_2 are constants depending upon the ratio of the outside to the inside diameter. The constants can be taken from Fig. 173.

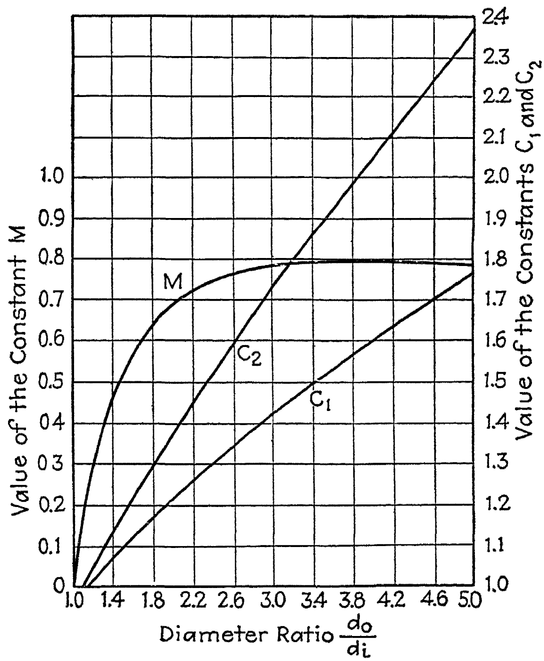


FIG. 173.—Disk-spring constants to be used with Eqs. (225), (226), and (227). (*Machine Design*, June, 1936.)

The true stresses are unknown but experience with this type of spring indicates that, under static conditions, the stress as given by these equations may be as high as 220,000 psi for steel having

a yield stress of 120,000 psi. Fatigue tests indicate that the maximum stress in these equations may be as high as 180,000 psi The fatigue life is greatly increased when the corners of the disk edges are rounded off.

Manipulation of these equations involves a large amount of mathematical work, and space is not available for a complete discussion. However, some properties of such springs are indicated in the curves of Fig. 174. With a flat spring ($h = 0$), the load increases rapidly as the deflection increases. When h is approximately $t\sqrt{2}$ (in this case 0.141 in.) there is a period

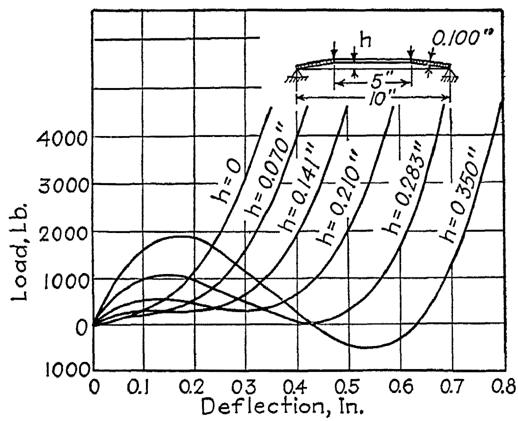


FIG 174.—Curves showing variation of load-deflection rate of disk spring identical except for the height h. (*Machine Design, June*, 1936.)

of considerable increase in deflection with constant load. Such springs may be used to support bearings and other members where temperature changes must be accommodated without appreciable variation in load. When $h > t\sqrt{2}$, the load increases with the deflection for a short period, followed by a period during which the load actually decreases with increase in deflection.

The load capacity of disk springs may be increased by moving the point of load application outward from the inner edge. The load capacity varies approximately as the ratio of the distances from the outer edge to the point of load application. The deflection varies approximately as this distance ratio. Initially flat springs have a maximum flexibility when the ratio of outside to inside diameters is approximately 2. In general the ratio d_o/d_i should be between $1\frac{1}{2}$ and 5.

CHAPTER XIII

SLIDING BEARINGS

When one member of a machine is supported by a second member and there is relative motion between them at the surfaces of contact, the pair constitutes a bearing. The supporting member is usually designated as the bearing, and the supported member may be a journal, thrust collar, or slipper. Bearings fall into three general classes: radial bearings, supporting rotating shafts or journals; thrust bearings, supporting the axial load on rotating members; slipper or guide bearings, guiding the moving part in a straight path. Bearings may also be classed as plain bearings, ball bearings, and roller bearings. Ball and roller bearings will be considered in the succeeding chapter.

Since there is relative motion between the contact surfaces, a certain amount of power must be absorbed in overcoming friction; and, if the surfaces actually touch, there will be more or less rapid wear. It is necessary to provide a lubricant to reduce the friction and to eliminate the wear as much as possible.

184. Lubricants. A lubricant is any substance that will form a film between the rubbing surfaces, preventing, to some degree, the actual contact of the surfaces. Oils and greases are the most common lubricants, although water is used in the thrust bearings and foot bearings of some types of vertical waterwheels where oil lubrication is difficult. Since water tends to corrode the bearings, water-lubricated bearings are usually made of rubber or lignum vitae.

A few solids (such as graphite, mica, soapstone, talc, and other greasy nonabrasive materials) are used as bearing lubricants. When properly used, they fill the cavities, smooth out the irregularities in the surfaces, and reduce the friction and wear below that produced between dry surfaces. Prelubricated, or oil-less, bearings consist of a metal alloy or compressed wood, mixed with these materials. In some cases the lubricating materials are pressed into cavities or spiral grooves in the bearing.

Such bearings will operate without oils; but as the friction and operating temperatures are fairly high, they should be used only when the bearings are located where proper oil lubrication can not be assured.

185. Oil-feeding Devices. When oil is used as the lubricant, some of the oil is squeezed along the journal and it passes out

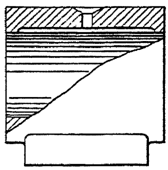


FIG. 175.

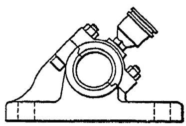


FIG. 176.

at the ends of the bearing; additional oil must be supplied to compensate for this loss. The simplest oiling device consists of a cavity cast in the bearing with a small hole extending into the clearance space, as in Fig. 175. An axial groove cut into the bearing serves to distribute the oil along the journal. This type of oiling is suitable only for slow speeds and intermittent service, since it requires constant attention. When the cavity is first filled with oil, it supplies too much oil to the bearing; and after a short interval, there is an insufficient supply. Drop-feed, bottle, wick and capillary oilers are used to provide a more regular oil supply from a reservoir which contains sufficient oil for several hours' operation. These oilers may have means of regulating the rate of oil supply and of shutting off the supply when the machinery is not operating. Bottle and drop-feed oilers are commonly used on high-grade machinery and give good service if the reservoirs are not allowed to run dry. They have the objectionable feature that the rate of oil delivery varies with the head of oil in the reservoirs and with the temperature.

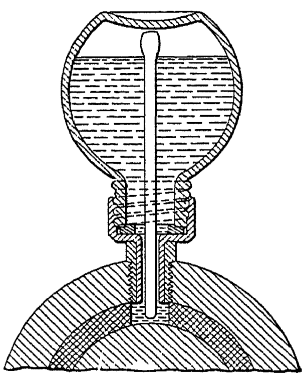


FIG. 177.—Typical bottle oiler, showing vertical pumping spindle. (*Courtesy Pure Oil Company.*)

Ring oilers (Fig. 178) consist of rings, approximately twice the shaft diameter, resting on the upper side of the journal and dipping into an oil reservoir below the bearing. Rotation of the journal causes the rings to rotate and carry oil to the top of the journal where axial grooves distribute it. This type of

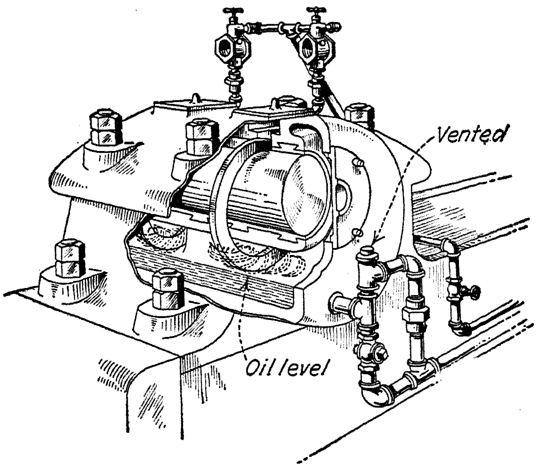


FIG. 178.—Ideal ring-oiled bearing. A continuous stream of clean, cool oil enters top of bearing to replace dirty oil which is drained continuously from reservoir, where a constant level is automatically maintained. (*Courtesy Pure Oil Company.*)

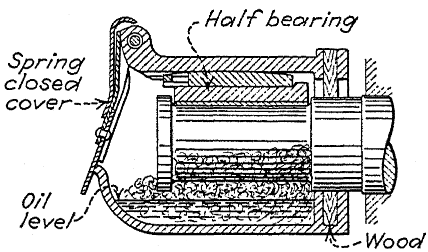


FIG. 179.—A Master Car Builders' railroad-car bearing. This is the most common type of half bearing. Note the oil reservoir and lubrication by wool waste. (*Courtesy Pure Oil Company.*)

oiling gives excellent service where each ring supplies not more than four inches of bearing on each side of the ring. With low speeds or viscous oils, the rings may not start readily; and if the speed becomes too high, the rings will slip. Chains may be substituted for the rings on large diameter journals operating at slow speeds.

Collar oilers act like ring oilers, and there is no trouble from slippage at the higher speeds. Scrapers may be provided at the top to direct the oil into the distributing grooves. The collar also serves as a means of locating the shaft axially.

Bath lubrication is used when it is possible to allow all or part of the rotating journal to be completely submerged in an oil reservoir. This type is especially useful when the load is carried on the upper side of the bearing. A modification of this type of bearing, applied to railway axles, is shown in Fig. 179. A bath-oiled spindle is shown in Fig. 180.

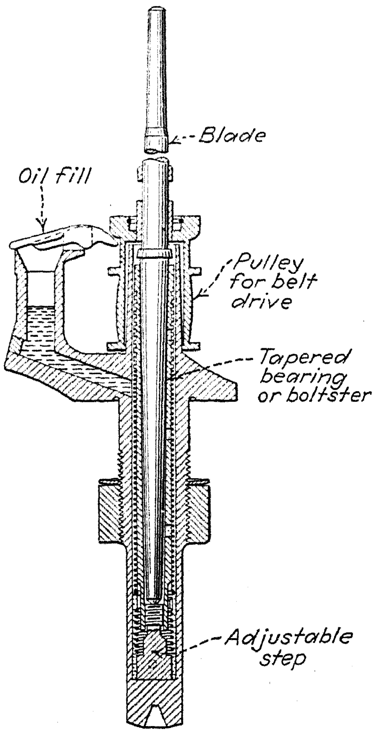


FIG. 180.—Textile-mill vertical spindle bearing, showing adjustable step. Oil reservoir surrounds tapered bolster provided with communicating oil holes. Entire tapered surface operates in an oil bath. (*Courtesy Pure Oil Company.*)

Splash oiling is used when cranks or other members can dip into an oil reservoir so that oil will be picked up and thrown onto the surfaces to be lubricated. This is a common method of lubricating the crankpin, piston pin, and cylinder walls of vertical internal-combustion engines, air compressors, and similar machines.

In order to insure a continuous supply of oil when the journal is rotating and to prevent oil waste when the journal is stationary, various types of mechanically operated feed pumps are used. In some types the oil going to any bearing may be metered, and since it is measured mechanically, the feed rate is not materially affected by temperature changes.

Pressure-feed lubrication is used on heavily loaded bearings where a positive feed is required. Oil is forced into the clearance space by a gravity head or by means of a circulating pump, and film lubrication is easily maintained. Considerable oil

is forced out of the bearing; and to prevent waste this should be collected in a suitable reservoir from which it may be recirculated through the pump and bearing. A filter or settling tank should be included in the system to prevent the pumping of metal particles, dust, or water through the bearings.

186. Grease Lubricants. Semisolid lubricants or greases may be used either in cavities in the bearing or in cups screwed into the bearing. Greases are used in cement mills, collieries, steel mills, bakeries, exposed automobile parts, and other places where dust or grit is liable to enter the bearings, and where slow motion and large clearances make it impossible to retain oil in the bearings. Since grease fills the clearance space and forms a shield around the bearing ends, it prevents the grit from entering, and in machines where excess oil dropping from the bearings might cause damage, as in weaving machinery, grease is used to advantage. Grease, however, provides very little lubrication until the bearing temperature rises sufficiently to melt the grease. Pressure guns, which force the grease into all parts of the clearance space, provide better lubrication than hand-feed or automatic grease cups.

187. Bearing Lubrication. The lubrication of bearings ranges from perfect film lubrication, through the stages of partial lubrication, down to the condition of practically dry metal-to-metal contact. With the perfect film lubrication, a complete and continuous film of oil is maintained between the sliding surfaces; and the metals do not come into contact; hence, there is no wear, and the friction depends entirely upon the properties of the lubricant. The metals used for the journal and bearing have no effect except at starting and stopping, when the oil film is not maintained. Many industrial bearings are only partially lubricated; *i.e.*, some oil that is introduced adheres to the journal relieving part of the metal-to-metal contact but not providing a complete separate film. Some bearings are so poorly lubricated that the operating conditions approach those of dry contact. In the latter two kinds of lubrication, it is extremely important that suitable metals be used in the journal and bearing.

188. Mechanism of Film Lubrication. The maintenance of a complete separating film of oil is desirable but not always possible; however, a knowledge of the formation of this film

aids in the proper design of any bearing. A bearing and journal, with the clearance greatly exaggerated, is shown in Fig. 181. Assume that this clearance space is kept completely filled with oil and that the load is acting down. When at rest, the journal

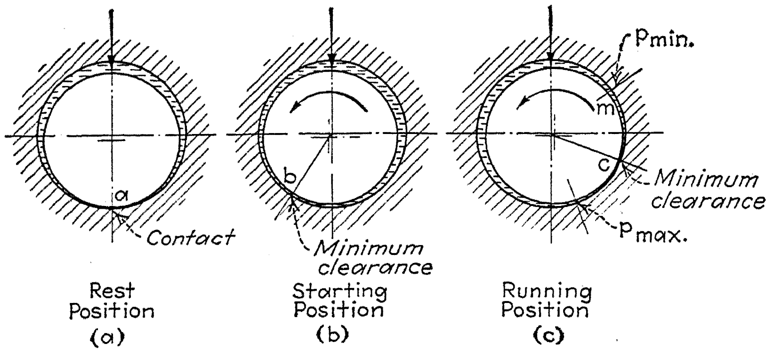


FIG. 181.

settles down and contacts with the bearing at a. As the journal begins to rotate, it rolls up the left side of the bearing, moving the point of nearest contact to the left. There is then a thin film of oil between the contact surfaces, and fluid friction is substituted for the metal-to-metal contact. The journal slides and begins to rotate, dragging more oil between the surfaces forming a thicker film and raising the journal.

As the speed of rotation increases, the oil drawn under the journal builds up a pressure that forces the journal up and to the right until a condition of equilibrium is reached. The net result is that the position of minimum clearance is at a point c, in Fig. 181c; the exact position depending on the original clearance, the speed of rotation, the oil viscosity, and the load on the journal. Increasing the load will cause the journal to settle lower in the bearing. Decreasing the viscosity or decreasing the speed has the same effect. After the journal reaches the equilibrium position, oil moving in at the left builds up a pressure that reaches a maximum some distance to the left of the point of minimum clearance c (Fig. 181c) and then decreases along the right side of the bearing, reaching a minimum in the region m.

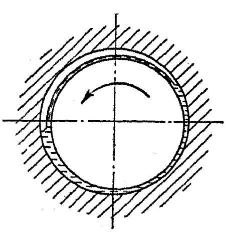


FIG. 182.

When the oil supply is insufficient to fill the clearance space completely, there is a condition similar to that shown in Fig. 182, with oil being carried over the top by adhesion. When the oil supply is still further reduced, a condition is reached where the oil wedge under the journal is no longer formed; and the film breaks down. The bearing will then be only partially film lubricated.

189. Viscosity. The term Z is the absolute viscosity* of the lubricant, in centipoises. The standard method of measuring viscosity consists of reading the time required for a given quantity of the liquid to flow through a standard tube. In this country, the viscosity test is usually based on the Saybolt Universal Viscosimeter, and the viscosity recorded in "seconds Saybolt." The viscosity is converted from seconds Saybolt to centipoises by the formula

$$Z = \rho\left(0.22S - \frac{180}{S}\right) \tag{228}$$

where S = Saybolt reading, sec.
ρ = specific gravity of liquid.

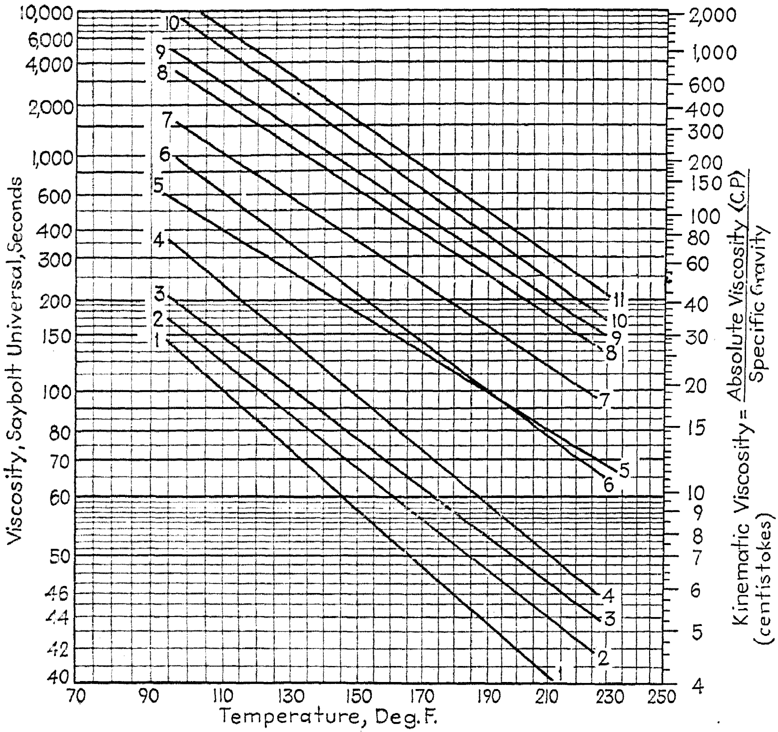
Since viscosity decreases as the temperature is increased, the oil must be selected on the basis of its viscosity at the operating temperature. The viscosity of a fluid is given by the equation

$$Z = A + Bt + Ct^2, \tag{229}$$

where A, B, and C are experimentally determined constants, and t is the fluid temperature. When the proper constants are inserted, this equation can be plotted to scales such that a straight-line relation between viscosity and temperature results. Plots of this kind are given in Fig. 183, from which the viscosities of common oils may be determined for any given temperature.

The absolute viscosity of any oil varies with its specific gravity, which also changes with temperature. The specific gravity of a

* Absolute viscosity is the force required to shear a fluid of unit area and unit thickness at a uniform rate In cgs units a poise is the force, in dynes, required to move one face of a 1-cm cube of liquid at the rate of 1 cm per sec relative to the opposite face. A centipoise is $\frac{1}{100}$ of a poise. Absolute viscosity, measured in centipoises, is the specific gravity times the kinematic viscosity, measured in centistokes. Absolute viscosity in English units (lb sec per sq in.) is expressed in reyns (μ) and $\mu = 145Z/10^9$.



Curve number	Type of oil	Specific gravity at 60 F
1	Light oil for light service and high speeds	0.875
2	Turbine oil (oil rings) for light service and high speeds	0.88
3	Turbine oil (oil rings) for light service and high speeds	0.89
4	Extra-light motor oil for ring-oiled bearings, transmission shafting, small generators, motors, and high-speed engines	0.935
5	S.A.E. 20—light transmission oil for gears	0.925
6	S.A.E. 40—medium transmission oil for large generators, motors, steam turbines, high-speed gears, heavy motor oil	0.93
7	Airplane 100 G—light cylinder oil	0.89
8	S.A.E. 110—light steam cylinder oil; heavy duty gears	0.93
9	Medium cylinder oil; slow-speed worm gears	0.91
10	S.A.E. 160—heavy cylinder oil; heavy-duty slow-speed gearing	0.935
11	Heavy steam cylinder oil	0.93

FIG. 183.—Viscosity-temperature chart.

liquid is given by the equation

$$\rho = \rho_0 + C_1(t - t_0) + C_2(t - t_0)^2 \qquad (230)$$

where the subscript 0 refers to standard temperature conditions. Bearce and Peffer* show that, for lubricating oils, the constant C_1 varies from -0.00035 to -0.00038 and the constant C_2 is practically zero. Hence, the specific gravity of an average lubricating oil at any temperature is given by the equation

$$\rho = \rho_{60} - 0.000365(t - 60) \qquad (231)$$

where ρ_{60} is the specific gravity at 60 F and t is the temperature of the oil film. The specific gravity of American lubricating oils varies from 0.86 to 0.95 at 60 F.

190. The Bearing Modulus. The design of modern bearings with film lubrication is based on the original experiments of Beauchamp Tower,† and the theoretical development of the hydrodynamic theory as applied to the lubrication of bearings by Osborne Reynolds‡ and N. P. Petroff.§ Later investigators‖ have extended the theory, and simplified it for application to bearing design. Space is not available for a complete theoretical analysis, and the present discussion will be limited to the general results of research investigations.

It has already been shown that, as the oil film builds up under the journal, the center of the journal moves and the position of the minimum clearance is at a point some distance from the load line. In a 360-deg bearing with the clearance space filled with oil, the positions of maximum and minimum pressures are approximately as shown in Fig. 187, and the pressure distri-

* BEARCE, H. W., and PEFFER, E. L., Density and Thermal Expansion of American Petroleum Oils, *Bur. Standards, Tech. Paper* 77, Aug. 26, 1926.

† TOWER, B., *Trans. Inst. Civil Eng.*, November, 1883.

‡ REYNOLDS, O., Theory of Lubrication, *Phil. Trans., Roy. Soc. London*, Vol. 177, 1886.

§ PETROFF, N. P., *Bull. Inst. Tech., St. Petersburg*, 1885–1886

‖ WILSON, R. E., and BARNARD, D. P., 4th, The Mechanism of Lubrication, *J. Soc. Automotive Eng.*, July, 1922.

HARRISON, W. J., The Hydrodynamical Theory of Lubrication with Special Reference to Air as a Lubricant, *Trans. Cambridge Phil. Soc.*, Vol. 22, 1913.

For a comprehensive list of references, see Progress in Lubrication Research, *Trans. A.S.M.E.*, Vols. 49–50, 1927–1928.

bution across the bearing is approximately as shown in Fig. 184, the maximum unit pressure reaching a value about twice the average pressure on the projected area of the bearing.

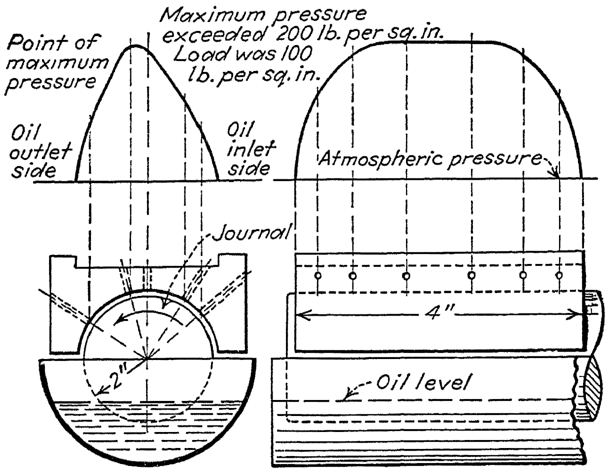


FIG. 184.—Beauchamp Tower's experiment, with curve showing maximum oil-film pressure of 200 psi on outlet side, slightly beyond vertical axis, although bearing load was only 100 psi. Longitudinally, the pressure rose sharply from atmospheric at bearing ends to maximum. When load on brass was increased or decreased, proportional changes in oil-film pressure were observed. (*Courtesy Pure Oil Company.*)

The average unit pressure on the projected area (bearing length times the diameter) is given by the equation

$$p = \frac{6Zvd}{2c^2} k \tag{232}$$

where Z = absolute viscosity of lubricant at bearing temperature, centipoises.

v = surface velocity of journal, fps.

d = journal diameter, in.

c = diametral clearance between journal and bearing, in.

k = a factor depending on bearing construction and ratio of its length to journal diameter.

When the velocity is in terms of the journal speed in rpm, the equation becomes

$$p = \frac{6Z\pi Nd^2k}{2c^2 \times 12 \times 60} = \frac{ZNd^2}{c^2}\frac{1}{K}$$

from which

$$\frac{ZN}{p}\left(\frac{d}{c}\right)^2 = K \tag{233}$$

In this equation, c/d is the clearance per inch of shaft diameter, usually about 0.001 in. per in. for plastic bearings and 0.0015 in. for hard bearings. The term ZN/p is of special interest. When the coefficient of friction for any bearing having film lubrication is plotted against ZN/p, a curve similar to Fig. 185 is obtained. Changes in the clearance will shift the curve and change the slope; but if the coefficient of friction is plotted against ZN/p times (d/c), all points will fall on the same curve, within reasonable experimental errors. Changes in the bearing materials or in the "oiliness" of the lubricant will alter the value of ZN/p for minimum friction and will change the abruptness of the break in the curve. For any bearing, there is a combination of Z, N, and p that results in minimum friction, indicated by C on the curve. Values of ZN/p greater than C indicate that the bearing may operate with complete film lubrication. At values less than C, the rapid rise in friction indicates that the oil film has broken down and that there is metal-to-metal

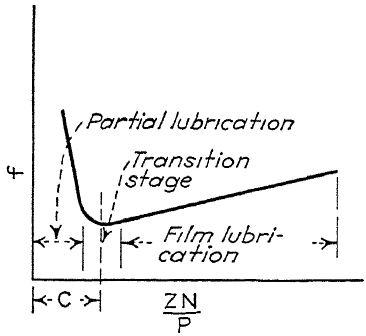


FIG 185.

TABLE 48.—VALUE OF C FOR VARIOUS COMBINATIONS OF JOURNAL AND BEARING MATERIALS

Shaft	Bearing	Bearing modulus $C = \frac{ZN}{p}$
Hardened and ground steel	Babbitt	20
Machined, soft steel	Babbitt	25
Hardened and ground steel	Plastic bronze	30
Machined, soft steel	Plastic bronze	35
Hardened and ground steel	Rigid bronze	40
Machined, soft steel	Rigid bronze	50

From LEIS, C. H., Sleeve-type Bearings, *Prod. Eng.*, September, 1934.

TABLE 49.—APPROXIMATE LOADING CONDITIONS FOR BEARINGS

Bearing type	Unit pressure on the projected area, psi p	Rpm N	Rubbing velocity, fpm V	Maximum pressure-velocity relation pV	Viscosity Z	ZN/p	Clearance ratio c/d	Length ratio L/d	Lubricant
Axles, locomotive	550	250	200– 450	120,000	100	30–50	0.001 max		Heavy machine oil
Railway car	300– 450	300	200– 450	120,000	100	50–100	0.001 max		Heavy machine oil
Crankpins, aircraft	650–1,800	1,500–3,000	1,300– 2,200	2,500,000					
Automobile	1,500–2,500	1,500–3,000			40		0.001 max	1.2 –1.4	Medium machine oil
Diesel	1,500–4,000	60–1,200	150– 300			2– 5	0.001	1.0 –1.5	Heavy machine oil
Gas	1,200–1,800	250– 800	125– 250	400,000	50	15– 20	0.001	1.0 –1.5	Medium machine oil
Steam, H. S	400–1,500	300– 400	140– 250	400,000	30	6– 15	0.001 max	1.0	Machine oil
Steam, L. S	800–1,300	40– 100	125– 200		80	6– 8	0.001 max	1.0 –1.25	Heavy machine oil
Shears, punches	5,000–8,000	10– 100							
Crankshaft and main bearings									
Aircraft	600–1,800	1,500–3,000	1,500– 2,500	2,000,000	7–8	15– 25	0.001 max		
Automobile	300–1,800	1,500–3,000			7–8	20– 30	0.001 max	1.0 –1.75	Medium machine oil
Diesel	350–1,200	60–1,200	200– 800	1,000,000	30	15– 20	0.001 max	1.0 –1.50	Heavy engine oil
Gas	500–1,000	250– 800	150– 300		30	20– 30	0.001 max	2.0 –2.5	Medium machine oil
Steam, H. S	60– 500	300– 400	150– 400		15–30	25– 30	0.001 max	2.0 –3.0	Engine oil
Steam, L. S	80– 400	40– 100	150– 500		70	20– 30	0.001 max	1.75–2.25	Heavy machine oil
Shears, punches	2,000–3,000								
Crosshead and wrist pins									
Aircraft	2,000–4,000	1,500–3,000					0.001 max		
Automobile	1,500–4,000	1,500–3,000					0.001 max		
Diesel	1,200–1,800	60–1,200					0.001 max	1.2 –1.7	Heavy engine oil
Gas	1,200–2,000	250– 800			40	10	0.001 max	1.2 –1.5	Medium machine oil
Steam, H. S	1,500–1,800	300– 400			25	5	0.001	1.4 –1.6	Engine oil
Steam, L. S	1,000–1,500	40– 100			70	5	0.001	1.2 –1.5	Heavy engine oil
Generators, motors	30– 150	150– 750	400– 1,600	50,000	25	200	0.001	2.0 –3.0	Engine oil
Hoisting machinery	70– 90							1.5 –2.0	
Line shafts	15– 150	250–1,000	250– 500	25,000				2.5 –3.0	Machine oil
Reducing gears	80– 250	25–1,000	1,000–10,000	100,000				2–4	Heavy engine oil
Machine tools	50– 300	50–1,000	50– 600	10,000				2–3	Machine oil
Steam turbines	75– 300	500–5,000	1,000–10,000	1,000,000			0.002	2–3	Light machine oil

contact, with the consequent higher friction and wear. The value of ZN/p at the breakdown point may be termed the bearing modulus. When the bearing is operated at or near this value, slight decreases in speed or increases in pressure may be accompanied by large increases in friction, heating, and wear. To prevent this, the bearing should operate at values of ZN/p at least three times the minimum value of C; and if the load is subject to large fluctuations and heavy impacts, values as high as $15C$ may be required. Operating values of ZN/p for common types of bearing service are suggested in Table 49.

191. The Coefficient of Friction. The coefficient of friction cannot be accurately determined, since the exact degree of lubrication is not generally known. It can, however, be approximated with sufficient accuracy for checking purposes.

For complete film lubrication, we have by definition:

$$f = \frac{F_f}{Lpd}, \qquad \text{and} \qquad Z' = \frac{F_f t}{Av}$$

from which

$$f = \frac{Z'Av}{Lpdt} \tag{234}$$

where F_f is the tangential friction force and Z' is in poises.

In this equation, t is the film thickness, which is practically $c/2$, v is $\pi dN/60$, and $A = \pi Ld$. When these values are substituted in Eq. (234) and when all units, except Z', are changed to English units and Z' is changed to Z

$$f = \frac{1}{20{,}983{,}000}\left(\frac{ZN}{p}\right)\left(\frac{d}{c}\right) \tag{235}$$

where Z = viscosity, centipoises.
N = rpm of journal.
p = psi of projected area.
c/d = clearance, in. per in. of diameter.

Messrs McKee,* in a series of tests on small journals, found the coefficient of friction to be determined by the equation

$$f = \frac{473}{10^{10}}\left(\frac{ZN}{p}\right)\left(\frac{d}{c}\right) + k \tag{236}$$

* McKee, S. A., and McKee, T. R., Friction of Journal Bearings as Influenced by Clearance and Length, *Trans. A.S.M.E.*, Vol. 51, APM-51-15, 1929.

The value of k varies with the ratio of the bearing length to its diameter, as shown in Fig. 186. When the ratio L/d

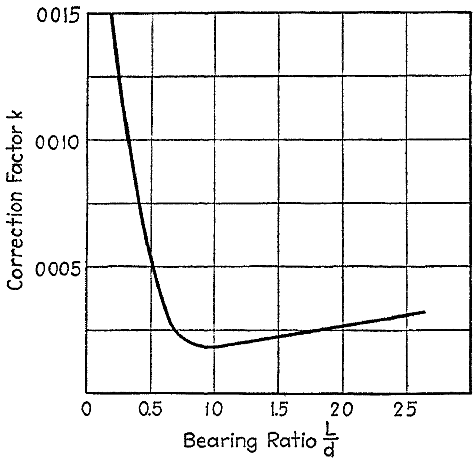


FIG. 186.—Correction factor for Eq. (236).

is greater than 0.75 and below 2.8 (the highest ratio used in the tests), the value of k varies only slightly and may be taken as 0.002.

The majority of bearings are only partially lubricated, and a complete separating film is not maintained. With this type of

TABLE 50.—VALUES OF THE CIRCUMSTANCE CONSTANT C_1 FOR EQ (237)

Lubrication	Workmanship	Attendance	Location	Constant
Oil bath or flooded	High-grade	First class	Clean and protected	1
Oil, free drop (constant feed)	Good	Fairly good	Favorable (ordinary conditions)	2
Oil cup or grease (intermittent feed)	Fair	Poor	Exposed to dirt, grit, or other unfavorable conditions	4

lubrication, there is no simple formula suitable for use in bearing design. Rough estimates of the coefficient of friction may be made by use of the following formula suggested by Louis Illmer.*

* ILLMER, LOUIS, High Pressure Bearing Research, *Trans. A.S.M.E.*, Vol. 46, 1924.

$$f = \frac{C_1 C_2}{250} \sqrt[4]{\frac{p_a}{V}} \tag{237}$$

where C_1 and C_2 are constants taken from Tables 50 and 51, and V is the rubbing velocity in ft per min. The term p_a is the average pressure in psi of projected area, but must never be assumed to be less than one-half of the maximum pressure imposed during a complete revolution.

TABLE 51.—VALUES OF THE TYPE CONSTANT C_2 FOR EQ. (237)

Type of Bearing	Constant
Rotating journals, such as rigid bearings and crankpins.	1
Oscillating journals, such as rigid wrist pins and pintle blocks . .	1
Rotating bearings lacking ample rigidity, such as eccentrics and the like	2
Rotating flat surfaces lubricated from the center to the circumference, such as annular step or pivot bearings.	2
Sliding flat surfaces wiping over the guide ends, such as reciprocating crosshead shoes. Use 2 for relatively long guides and 3 for short guides	2–3
Sliding or wiping surfaces lubricated from the periphery or outer wiping edge, such as marine thrust bearings and worm gears	3–4
Long power-screw nuts and similar wiping parts over which it is difficult to effect a uniform distribution of lubricant or load .	4–6

192. Minimum Oil-film Thickness.* In any bearing with perfect fluid lubrication, the minimum oil-film thickness must be sufficient to prevent contact between peaks on the journal and bearing surfaces. Surface finish is measured by profilometers as the average height of the peaks and valleys in root-mean-square microinches. A microinch is 0.000001 in., designated as 1 μin. The peaks may be three to five times the average roughness; hence, to provide a safety film at least equal to the sum of the peaks on both journal and bearing, the minimum oil-film thickness (h in Fig. 187) must be at least equal to $2 \times 2 \times 5 \times$ roughness, or 20 times the average roughness in root-mean-square microinches.

The significance of surface finish lies in the fact that it determines the required oil-film thickness; the rougher the surface, the thicker the required film. Thicker films mean higher coefficients of friction, higher power loss, and more heating.

* Part of the material that follows is abstracted from Bruno Sachs, Sleeve Bearings, *Prod. Eng.*, May, June, and July, 1942.

Experience indicates that the minimum oil-film thickness should be at least 0.00075 in. for ordinary babbitted bearings, such as those used in small medium-speed electric motors. It may be reduced to 0.0001 in. for fine-finished bronze bearings, such as those used in automotive and aeronautic engines. With large steel shafts of fans, turbogenerators, and the like, the minimum oil-film thickness may be 0.003 to 0 005 in.

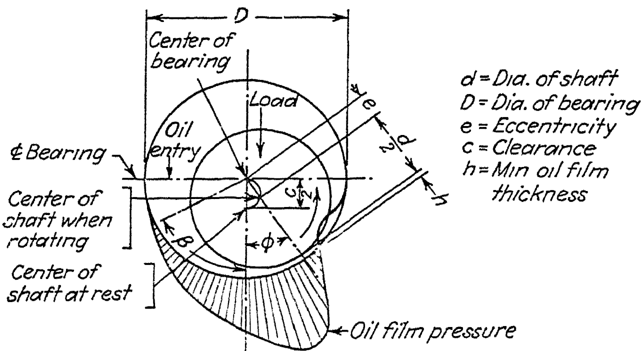


FIG 187 —Operating condition of a loaded journal.

Examination of Fig. 187 shows that the minimum oil-film thickness depends upon the position of the shaft center under operating conditions. The center position depends upon the unit pressure p, the clearance ratio c/d, the oil viscosity Z, the speed of rotation N, and the angle β between the load line and the beginning of the supporting oil wedge. The beginning of the oil wedge is the position where the oil pressure is equal to the atmospheric pressure and in many cases is at an oil groove. The angle β may be taken as 60 deg for a complete 360 deg bearing if no oil groove is located within 60 deg of the load line. The hydrodynamic theory of film lubrication indicates that the shaft center moves along a semicircular arc of diameter $c/2$. The ratio of the journal eccentricity to the radial clearance is called the eccentricity ratio, or C_e. From Fig. 187 we have

$$C_e = \frac{e}{c/2} = \sin\phi, \qquad \text{or} \qquad e = \frac{cC_e}{2}$$

and

$$h = \frac{c}{2} - e = \frac{c}{2}(1 - C_e)$$

from which

$$C_e = 1 - \frac{2h}{c} \tag{238}$$

Values of $(1 - C_e)$ and, hence, the minimum oil-film thickness can be taken from Fig. 188 when the bearing properties are

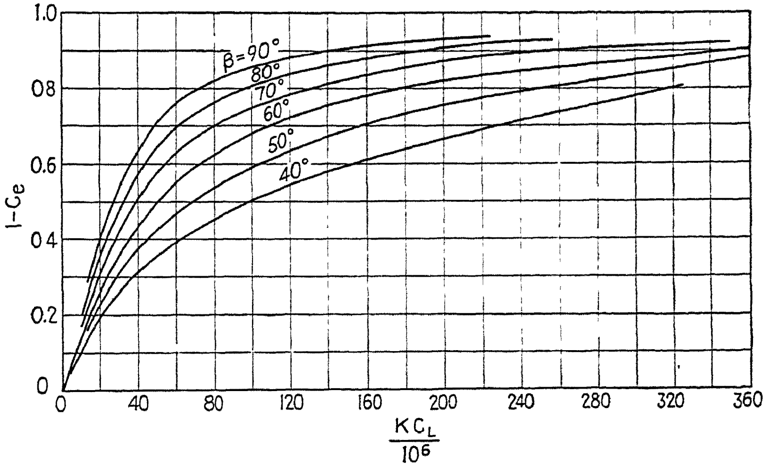


FIG. 188.—Variation of journal eccentricity coefficient with KC_L.

known. In this figure, K is determined from Eq. (233), and C_L from Fig. 189. The factor C_L depends upon the L/d ratio for the bearing and corrects for side leakage of oil which reduces the average oil pressure between the bearing surfaces.

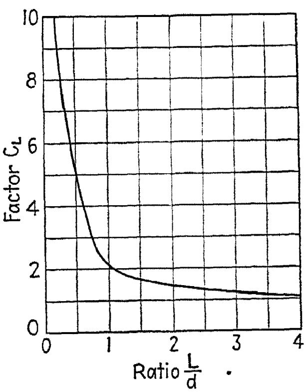


FIG. 189.

193. Journal Deflection and Clearance. Clearance should be such that when loaded the ends of the journal will not rub against the bearing surface. The required clearance, indicated by c_m in Fig. 190, may be calculated as follows. For a shaft supported as a simple beam as shown in Fig. 191,

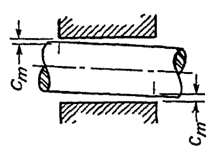


FIG. 190.

$$c_m = 2.55\,\frac{pL}{E}\left(\frac{L}{d}\right)^3 + 3.4\,\frac{pdL^2}{ED_s^4}\,(2C - a)a \qquad (239)$$

where the first member of the equation represents the deflection of the journal and the second member the shaft deflection.

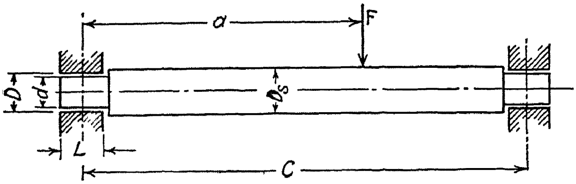


FIG. 191.

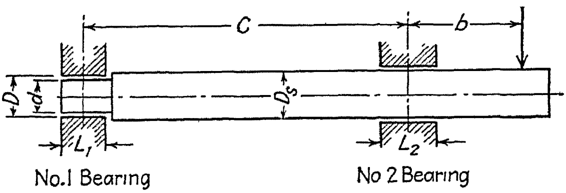


FIG 192.

When the load overhangs one bearing, as in Fig. 192, the minimum clearance is

For bearing No. 1

$$c_{1m} = 2.55\,\frac{p_1L_1}{E}\left(\frac{L_1}{d_1}\right)^3 + 3.4\,\frac{p_1d_1L_1^2}{ED_s^4}\,C^2 \qquad (240)$$

For bearing No. 2

$$c_{2m} = 2.55\,\frac{p_2L_2}{E}\left(\frac{L_2}{d_2}\right)^3 + 6.8\,\frac{p_2d_2L_2^2}{ED_s^4}\left(\frac{C^2b}{C + b}\right) \qquad (241)$$

where p, p_1, p_2 = bearing pressure, psi.
D_s = shaft diameter, in.
d, d_1, d_2 = journal diameter, in.
C = distance between bearings, in.
L, L_1, L_2 = bearing length, in.
a, b = distance of load from bearing, in.
E = modulus of elasticity of shaft material, psi.

When the journal deflection cannot be readily calculated, a rule of thumb is to allow 0.001 in. per in. of journal diameter. This value can be decreased 50 per cent for soft babbitts, and must be increased 50 per cent for bronzes.

194. Operating Pressures. The pressure at which the oil film breaks down so that metal-to-metal contact and imperfect lubrication begin, is called the critical pressure. This pressure depends on the materials used in the bearing and, to a large extent, on the degree of smoothness of the contact surfaces. No reliable formula for determining critical pressure is known.

For film lubrication, Victor Tatarinoff* gives the following equation for the safe operating load

$$F = \frac{ZNd^4}{127 \times 10^6 hc} \frac{(L/d)^2}{\left(1 + \frac{L}{d}\right)} \tag{242}$$

The average oil-film thickness h is approximately $c/4$, and the projected area of the bearing is Ld. When these values are substituted in the equation, the following relation for the permissible unit pressure is obtained.

$$p = \frac{ZN}{3175 \times 10^4} \left(\frac{d}{c}\right)^2 \frac{L}{d + L} \tag{243}$$

The permissible unit pressures with imperfect lubrication depend upon the amount of use, the effect of the wear on the proper functioning of the machine, and the cost of repairs. Pressures generally encountered in continuously loaded bearings range from 50 to 300 psi. Very high pressures of short duration can be successfully used. For example, 4,000 psi and over is used on punches, shears, and presses where the machines are intended for intermittent use, as in structural steel plants, but much lower pressures are used in the same class of machines when they are intended for high production manufacturing as in automobile plants. The load is carried on the bearing for a short portion of the cycle (0.005 to 0.02 sec), and the oil film has a chance to build up during the low-pressure period. These bearings must be carefully finished and ground, and they must be short so that the deflection within the bearing is very slight. Pressures of 1,800 psi are common with hardened and ground steel wrist pins on phosphor bronze when there is a cyclic relief of the pressure; and 2,400 psi has been successfully

* TATARINOFF, VICTOR, Safe Loads for Journal Bearings, *Prod. Eng.*, December, 1934.

carried with lapped bearings and forced lubrication. Oscillating bearings may be loaded to 2,400 psi if the average pressure is low and the minimum pressure during each cycle practically zero.

In general, crankpins and journal bearings, subjected to variable cyclic loading where the pressure during half the cycle is reduced to less than half the maximum pressure, may be considered as permanent if the pressure is less than 1,000 psi, and as commercially good if the pressure is 1.500 psi.

TABLE 52.—ABRASION PRESSURES FOR BEARINGS

Materials in contact	Pressure, psi	Remarks
Hardened tool steel on lumen or phosphor bronze	10,000	Value applies to rigid, polished, and accurately fitted rubbing surfaces
0.50C machine steel on lumen or phosphor bronze	8,000	When not worn to a fit or well lubricated reduce to 6,000
Hardened tool steel on hardened tool steel	7,000	
0.50C machine steel or wrought iron on genuine hard babbitt	6,000	
Cast iron on cast iron (close-grained or chilled)	4,500	
Casehardened machine steel on casehardened machine steel	4.000	
0.30C machine steel on cast iron (close-grained)	3,500	
0.40C machine steel on soft common babbitt	3,000	
Soft machine steel on machine steel (not casehardened)	2,000	
Machine steel on lignum vitae (water lubricated)	1,500	

Experience with certain classes of machinery leads to the use of empirical relations in the bearing design. The most common of these is that the product of the unit pressure and the rubbing velocity is a constant. The constant varies widely with different classes of machines and with the amount of wear that is permissible. Although this product may be taken as a

constant for similar bearings operating under similar conditions, it cannot be used to compare bearings operating under different service conditions. As this method does not consider the viscosity of the oil, it often leads to the design of unsatisfactory bearings when used by a designer lacking experience in particular service fields.

With high-pressure low-speed bearings, the maximum or peak load during a revolution should be well within the pressure at which abrasion will occur. The abrasion pressures for typical combinations of bearing materials at minimum rubbing velocities are given in Table 52. These values are approximately four times the usual pressure allowance for a lubricated bearing.

195. Partial Bearings. Inspection of Fig. 187 will show that the portions of the bearing extending about 45 deg from the normal to the load line have little load-supporting capacity, since the pressure components parallel to the load are small. Partial bearings with about 120 deg of contact are sometimes used without loss in load-carrying capacity; in fact, at high speeds such bearings have higher load capacity, since the area of close clearances, where rapid shearing of the oil film generates heat, are reduced. Space is not available for a complete discussion of partial bearing design and the reader is referred to the works* of H. A. S. Howarth, and S. J. Needs.

196. Heating of Bearings. The power lost in friction in the bearing is converted to heat and must be radiated from the housing without producing excessive temperatures. At high temperatures, the viscosity of the lubricant decreases permitting the lubricant to squeeze out so that the lubrication becomes negligible and the bearing may seize. Bearings usually operate at temperatures from 80 to 140 F, although 160 is common and some turbine bearings operate at 200. On the other hand, in refrigerating and in some air-driven tools, the bearing temperatures may be extremely low. The oil temperature is somewhat higher than the bearing temperature and may be approximated from the bearing temperature by using the curve in Fig. 193.

* HOWARTH, H. A. S., The Loading and Friction of Thrust and Journal Bearing with Perfect Lubrication, *Trans. A.S.M.E.*, Vol. 57, 1935

NEEDS, S J., Effects of Side Leakage in 120° Centrally Supported Journal Bearings, *Trans. A.S.M.E.*, Vol. 56, 1934.

The heat to be radiated is

$$H = fpdLV \text{ ft-lb per min} \tag{244}$$

where p = average pressure of projected area, psi.

The radiating capacity of the bearing depends on the temperature difference, on the form of the radiating surface, on the mass of the adjacent members, and on the air flow around the bearing. The heat-radiating capacity may be expressed by the equation

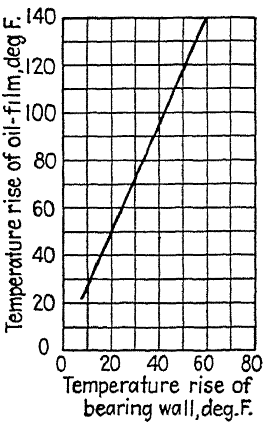


FIG. 193.

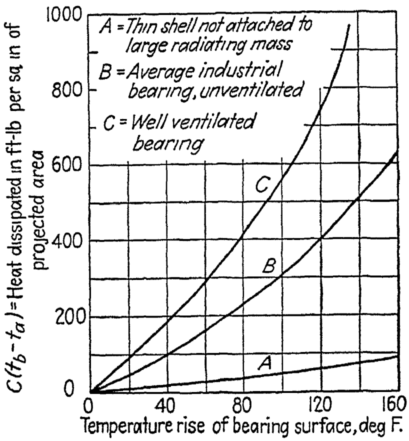


FIG. 194.

$$H = CA(t_b - t_a) \tag{245}$$

where t_b = temperature of bearing, °F

t_a = temperature of surrounding air, °F

A = area of radiating surface or bearing housing, sq in.

C = radiating capacity, ft-lb per sq in. per min per °F temperature difference.

Tests made by G. B. Karelitz* indicate that the constant C is 0.184 when the bearing is located in still air and 0.516 when the air velocity over the bearing is 500 ft. per min. The area A includes all the area to a distance of 4 in. below the bottom of the oil reservoir if the bearing pedestal extends that far. The Karelitz tests were made on an oil-ring bearing.

* KARELITZ, G. B., Performance of Oil-ring Bearings, *Trans. A.S.M.E.*, APM-52-5, Vol. 52, 1930.

The results of tests by O. Lasche indicate that $C(t_b - t_a)$ has the values indicated in Fig. 194 when the area A is the projected area of the journal

197. Example of Bearing Design. Design a bearing and journal to support a load of 1,000 lb at 600 rpm using a hardened steel journal and a bronze-backed babbitt bearing An abundance of oil is supplied by means of oiling rings The oil viscosity is 250 sec Saybolt at 100 F and the specific gravity is 0 90 at 60 F. The bearing is relieved for 20 deg from the normal to the load line.

Solution. If the oil temperature is limited to 180 F, or a temperature rise of 110 F above room temperature, the oil density as given by Eq. (231) will be

$$\rho = 0.90 - 0.000365(180 - 60) = 0.856$$

From Fig 183, the oil viscosity at 180 F is approximately 60 sec Saybolt. From Eq. (228), the absolute viscosity is

$$Z = 0.856(0.22 \times 60 - \tfrac{180}{60}) = 8.73 \text{ centipoises}$$

From Table 48, the bearing modulus for this type of bearing is about 20, and since ZN/p should be about three times the modulus,

$$\frac{ZN}{p} = 3 \times 20 = 60$$

and

$$p = \frac{ZN}{60} = \frac{8.73 \times 600}{60} = 87.3 \text{ psi}$$

The required projected area is 1,000/87.3 or 11.45 sq in. If the bearing diameter is not limited by other considerations, assume an L/d ratio of 1.5 and

$$A = Ld = 1.5d^2 = 11.45$$
$$d = 2.76 \text{ in., say } 2\tfrac{3}{4} \text{ in.}$$
$$L = \frac{11.45}{2.75} = 4.16 \text{ in., say } 4\tfrac{1}{4} \text{ in.}$$

Then the area is $2\frac{3}{4} \times 4\frac{1}{4}$, or 11.6875 sq in., and p is 1,000/11.6875, or 85.6 psi.

Adjustments in bearing dimensions may be required after considering permissible pressures, minimum oil-film thickness, and heating.

The permissible pressure for film lubrication, as given by Eq. (243) is

$$p = \frac{8.73 \times 600}{3{,}175 \times 10^4}\left(\frac{d}{c}\right)^2 \frac{4.25}{2.75 + 4.25} = 0.0001\left(\frac{d}{c}\right)^2$$

The clearance ratio c/d may be taken as 0.001, and

$$p = 0.0001 \times 1{,}000^2 = 100 \text{ psi}$$

which is larger than the required pressure of 85.6 psi and is satisfactory unless the bearing is subjected to severe overloads for short periods.

From Eq. (233),

$$K = \frac{ZN}{p}\left(\frac{d}{c}\right)^2 = \frac{8.73 \times 600}{85.6}(1{,}000)^2 = 61{,}200{,}000$$

From Fig 189 and $L/d = 4.25/2.75 = 1.55$, the factor C_L is found to be 1.6, and KC_L is 97,920,000. Since the bearing is relieved for 20 deg, the angle β is 70 deg and from Fig 188 the value of $(1 - C_e)$ is 0.74. Then from Eq (238)

$$h = \frac{c}{2}(1 - C_e) = \frac{2.75 \times 0.74}{2 \times 1{,}000}$$
$$= 0.001018 \text{ in.} = 1{,}018 \ \mu\text{in.}$$

The minimum oil-film thickness should be at least twenty times the surface roughness reading in root-mean-square microinches, and in this case the surface roughness should be less than 50.9, which is a value easily obtained.

To check for heating in the bearing, find the approximate coefficient of friction from Eq. (236)

$$f = \frac{473}{10^{10}}\left(\frac{ZN}{p}\right)\left(\frac{d}{c}\right) + 0.002$$
$$= \frac{473}{10^{10}}\left(\frac{8.73 \times 600}{85.6}\right)(1{,}000) + 0.002 = 0.00489$$

The heat generated is

$$H = pL\,dfV = 1{,}000 \times 0.00489 \times 432 = 2{,}112 \text{ ft-lb per min}$$

The oil temperature rise is 110 F, and Fig. 193 indicates that the temperature rise of the bearing surface will be approximately 46 F. The heat-radiating capacity is found from Eq. (245) with $C(t_b - t_a)$ equal to 115 (from Fig. 194) for an average commercial bearing, unventilated.

$$H = CLd(t_b - t_a) = 2.75 \times 4.25 \times 115$$
$$= 1{,}343 \text{ ft-lb per min}$$

Since the heat-radiating capacity is less than the heat generated, the bearing must be redesigned or provision made for ventilating it. In a well-ventilated bearing the radiating capacity may be increased to 2,555 ft-lb per min, which is greater than the heat generated.

If the bearing is pressure-lubricated, the difference between the oil temperature and the bearing wall temperature will drop to approximately 25 F, and the wall temperature will be approximately 155 F. This bearing would have a radiating capacity of over 2,900 ft-lb per min which is greater than the heat generated and the bearing would operate at an oil temperature less than 180 F.

The student should solve this problem using a different value for ZN/p, for the clearance ratio, and for the minimum oil-film thickness and compare the results obtained.

198. Thrust Bearings. Thrust bearings are used to take the end thrust or unbalanced axial loads on horizontal shafts and to support the suspended weight of vertical shafts.

The simplest type of thrust bearing for horizontal shafts consists of one or more collars arranged as shown in Fig. 195.

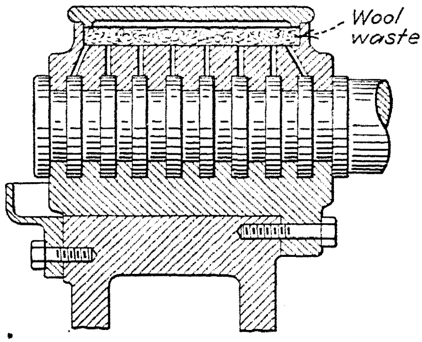


FIG. 195.—Collar-type thrust bearing. Note the oil reservoir and wool waste that acts as a filter and assures a more uniform oil feed. (*Courtesy Pure Oil Company.*)

Such bearings may be oiled by reservoirs in the top of the bearings, by wick oilers, or through hollow shafts. Automatic oil-feeding devices are preferable, since collar thrust bearings are usually operated at high speeds and under heavy loads.

A simple pedestal or vertical thrust bearing is shown in Fig. 196. Usually two or more washers are provided, with or without spherical faces to provide for self-alignment. The oil level should be high enough to completely submerge the thrust washers, and openings should be provided so that the oil can enter at the center and move outward between the bearing surfaces.

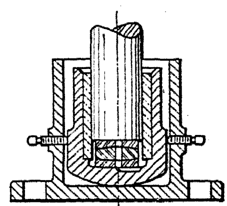


FIG. 196.

A suspension bearing, of the type used in the upper end of vertical hydraulic turbines, is shown in Fig. 197. This bearing is completely enclosed in a housing that holds sufficient oil to completely submerge the bearing surfaces and provide oil-bath lubrication. Radial grooves on the upper side of the bearing shoe distribute the oil over the bearing surfaces. When the edges of the grooves are carefully scraped away to form a thin wedge-shaped space

for oil, a very good supporting film is obtained between the bearing surfaces, and high unit pressures can be carried.

Collar thrust bearings have a coefficient of friction ranging from 0.03 to 0.05, and the ordinary type of step bearings has a coefficient of friction ranging from 0.01 to 0.02.

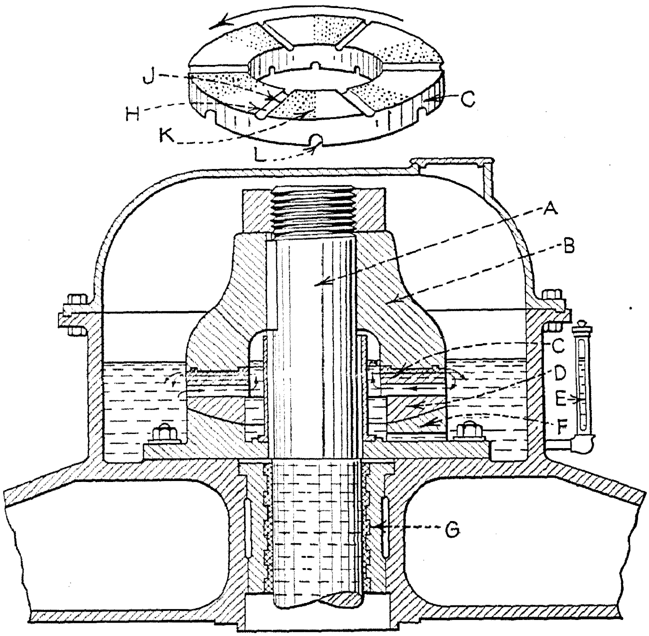


FIG. 197.—Suspension bearing for hydraulic turbine. (*Courtesy Vacuum Oil Company.*)

199. Kingsbury Thrust Bearing. This bearing, shown in Fig. 198, consists of a plain collar supported by a number of pivoted segments, or shoes; the pivoted shoes being mounted so that they are free to tilt radially and tangentially. When the thrust collar begins to move, oil adhering to it is carried between the collar and shoes building up a pressure that causes the shoes to tilt slightly, as shown in Fig. 198*b*. When the bearing surfaces are completely submerged in oil, this wedge-shaped oil film is continuously maintained; the bearing surfaces are completely separated, and true film lubrication is maintained. The amount of the tilting of the shoes, although very small, varies with the oil viscosity, the thrust load, and the speed

so that the oil film automatically adjusts itself to operating conditions, and minimum friction is assured.

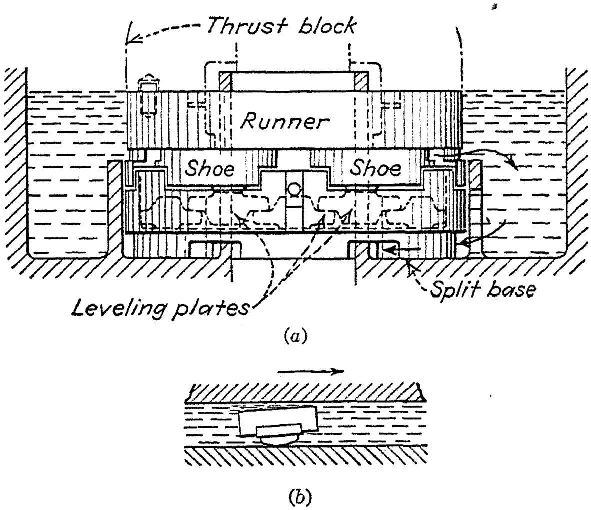


FIG. 198.—Kingsbury thrust bearing; (a) section of six-shoe vertical thrust bearing, showing oil circulation and main features of mounting; (b) diagram of tilted shoe.

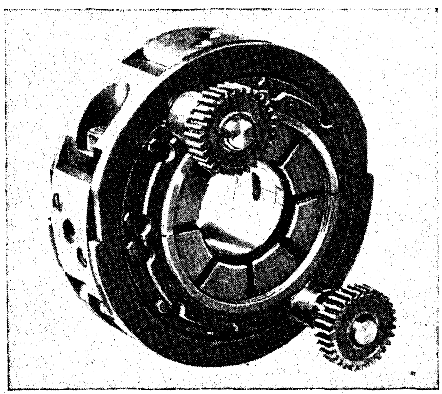


FIG. 199.—Combination journal and Kingsbury thrust bearing. (*Courtesy Allis-Chalmers Manufacturing Company.*)

200. Coefficient of Friction for Kingsbury Thrust Bearings. Thrust bearings of the Kingsbury type sustain loads of 300 to 500 psi with the coefficient of friction ranging from 0.001 to 0.003 after running conditions are reached. During the first revolution when starting, the coefficient is much higher.

201. Distributing the Lubricant. The oil for any bearing should be introduced into the clearance space in the region of minimum pressure. To distribute the oil to all parts of the journal, a groove should be cut in the bearing parallel to the axis and extending not closer than $\frac{3}{16}$ in. from the bearing end. When the bearing is more than 8 in. long, two oil holes should be provided. In general, no other grooves should be provided; and in no case should grooves be cut in the high-pressure region. When the bearings have large clearance, the oil may be squeezed out at the bearing ends before an oil film can be formed. In this case, spiral grooves may be provided to lead the oil toward

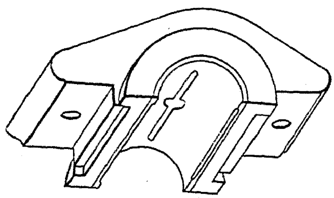


FIG. 200.—Bearing cap, illustrating longitudinal groove through oil inlet. (*Courtesy Vacuum Oil Company.*)

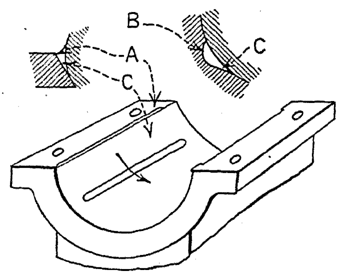
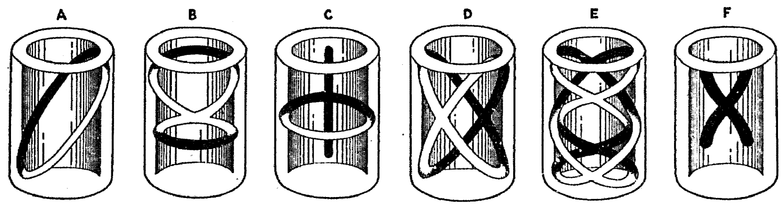


FIG. 201.—Special grooving of bearing for slow speeds and heavy pressures; also illustrating correct form of chamfers and grooves. (*Courtesy Vacuum Oil Company.*)

the bearing center. Such grooves, when used, should be located approximately 40 deg ahead of the highest-pressure region.

An excellent method of lubricating is the use of a circumferential groove. The Reynolds theory indicates that this in effect shortens the L/d ratio by forming two bearings and that the end leakage will increase and the load-carrying capacity will decrease 30 to 50 per cent. Tests indicate that this is not true and that the load capacity actually increases, probably owing to an increase in oil flow with consequent lower oil temperature and less reduction in oil viscosity.

All oil grooves should have the sharp edges removed, and the trailing edge should be tapered off to prevent oil from being scraped from the journal and to assist in forming the oil wedge and film. When the bearings are split, the edges should be treated in the same manner as the grooves. Correctly shaped oil grooves are illustrated in Figs. 200, 201, and 202.



FUNDAMENTAL TYPES OF OIL GROOVING

Application	Type of bearing	Bearing support	Direction of		Method of grooving
			Load	Rotation	
General purpose	Solid or two-part	Fixed in stationary housing	Unidirectional	Unidirectional	Straight groove, stopping short of each end
General purpose	Solid or two-part. Lubrication provided through shaft	Fixed in stationary housing	Unidirectional	Unidirectional or changing	Circular groove to coincide with oil exit in shaft and straight groove stopping short of each end. (*C*)
General purpose	Solid	Fixed in moving housing (pulleys, gears, etc.)	Unidirectional or changing	Unidirectional or changing	Oval type groove. (*A*)
General purpose	Solid or two-part	Fixed stationary housing	Unidirectional	Changing	Double spiral groove short of bearing ends, (*F*)
General purpose	Solid	Fixed in stationary housing	Unidirectional	Changing	Complete figure eight type of groove (*B*)
General purpose	Two-part	Fixed in stationary or rotating housing	Unidirectional or changing	Unidirectional or changing	Longitudinal or circular grooves and/or chamfers on each side at parting line.
Slow speed, grease lubricated					Grooved as shown in (*D*) and (*E*). Recommended only for grease or graphite lubrication

Courtesy of Bruno Sachs and *Product Engineering*, May–July, 1942.

FIG. 202.

In general, bearings should have lengths from 1 to 2.25 times their diameter. Shorter bearings are subject to excessive end leakage, and it is difficult to maintain a film of lubricant; the result being that partial lubrication, with a correspondingly high coefficient of friction, exists in most of the bearing.

202. Bearing Materials. When a complete oil film is maintained between the journal and bearing surfaces, the materials used have little effect on the power loss and wear. With imperfect lubrication and during the starting and stopping periods, the

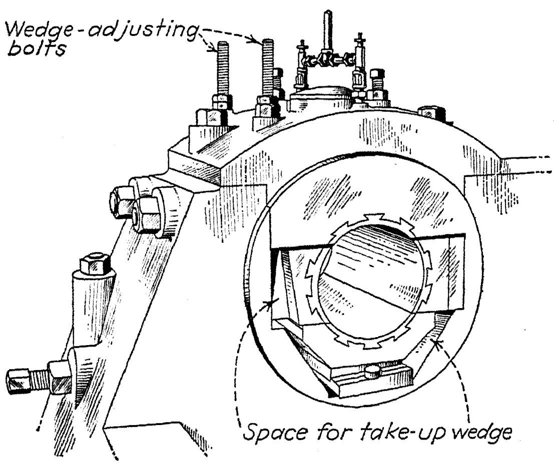


FIG. 203.—A large engine-type quarter-block shaft bearing. Accurate alignment of the shaft is secured by adjusting the wedges back of the quartered bearing shell. (*Courtesy Pure Oil Company.*)

surfaces come into contact; and the materials must, therefore, be selected to resist wear and to provide a low coefficient of friction. Some bearing materials are desirable because they absorb some oil, which provides lubrication during the starting period. Others are desirable because they are plastic enough to conform to slight irregularities of the journal. In general, it is claimed that unlike materials for the journal and bearing give the best results. However, hardened steel on hardened steel, and cast iron on cast iron for light pressures give excellent results. Good bearing materials must have sufficient strength so that they will not crush under the load, must be good conductors of heat, uniform in structure, resistant to abrasion, and have a low coefficient of friction when dry or slightly greasy.

Babbitt metal is probably the most common bearing material. Babbitt is a composition of 90 parts of tin, 5 parts of antimony, and 1 part of copper, although many of the babbitts now in use have lead substituted for the tin. Babbitts are lead or tin base depending upon which metal is present in the larger quantity. The S.A.E. Handbook contains standard specifications for typical babbitts.

For pressures above 1,000 psi, babbitt should not be used. For these higher pressures, bronze linings are commonly used. Bronze is expensive and is, therefore, usually used as a bushing pressed into place or as a split bushing, held in place in the bearing and cap by means of dowels. Bronze bearings should have a thickness of $\frac{3}{32}$ to $\frac{1}{8}$ in. per in. of journal diameter.

Bronze is a copper-tin alloy made in many different compositions that give good service with hardened steel journals. In the newer bearing bronzes, the percentages of copper and tin have been decreased, and varying amounts of lead have been added. In these bronzes, the copper-tin combination forms a strong matrix that supports the softer and more plastic materials; the strong matrix supports the heavy loads, and the plastic materials permit adjustment to the shaft.

Bearing bronzes usually contain from 5 to 15 per cent tin, 0 to 25 per cent lead, and the remainder copper. Zinc is sometimes included, and in a few cases, nickel is added. In general, the higher tin percentages give bronzes having high crushing strength, resistance to pounding, high wear resistance, and a relatively high coefficient of friction. The low-tin, high-lead bronzes have lower coefficients of friction, and, as they are more plastic, conform to the shaft better than the high-tin alloys, but do not wear so well. The low-tin, high-lead bronzes are suitable for high-speed low-pressure service when the load is not of an impact nature.

The increased speed used in aeronautic and automotive engines of recent design has brought about research for bearings able to withstand higher operating temperatures. The first step in this direction was the introduction of very thin (0.015 in.) babbitt linings on steel or bronze backings. The second step was the introduction of copper and lead mixtures (not alloys). Although bronze bearings can sustain high pressures at high temperatures and speeds, the machining cost is high and the journals must be

very hard. The copper-lead mixtures, first developed for aeronautic use, have good heat conductivity, a moderately low coefficient of friction, and ability to operate safely at high temperatures. However, many lubricating oils attack the free lead in these bearings.

A recent development (1935) is the introduction of the cadmium-nickel-copper and the cadmium-silver-copper alloys, which allow much higher operating temperatures than are safe with babbitt and do not require the extremely hard journals necessary with bronze. Cadmium fuses directly to the steel backing; and, since no solder or tin is used to make the bond, the bearing can be operated at temperatures approaching the softening point of the alloy. Like the copper-lead bearing materials, these alloys are attacked by some lubricating oils. Cadmium-silver and pure silver bearings are used in some airplane engines.

Cast-iron bearings with hardened steel journals are used where high precision and freedom from wear are the chief requirements. Good lubrication must be provided or the hard cast iron will score the journal.

Lignum-vitae and bakelite-composition bearings have been used successfully in heavy-duty installations such as roll-neck bearings in steel mills, where pressures of 4,500 psi are encountered.

Rubber bearings, consisting of a rubber lining vulcanized into a metal backing, are used for pump bearings, marine propeller-shaft bearings, and similar installations where the presence of water renders oil lubrication difficult. They must be flooded with a copious supply of water and must never be allowed to run dry, even during the short starting period. Spiral grooves are usually provided to insure good water distribution. Close fits must be avoided; temperatures must be kept below 150 F; pressures should be from 30 to 50 psi; surface speeds should be greater than 100 fpm, and oil and grease should be excluded. With copious water lubrication, pressures as high as 1,000 psi have been used at high speeds. Small bearings have been operated at 4,300 fpm surface speed. The coefficient of friction varies from 0.010 to 0.005 as the speed increases.

Rubber bearings are desirable where sand, grit, and other abrasive particles, which would cut an ordinary metallic bearing, are encountered. The particles will embed themselves in the relatively soft rubber without harming the metallic journal.

Fabric bearings are made of special woven duck impregnated with rubber or certain phenolic resins hot-molded under high pressures. They have proved very successful for heavy roll-neck bearings and other bearings subjected to repeated heavy blows. Water, grease, or oil lubrication may be used, depending upon the molding material. The coefficient of friction is about 0.007 and pressures as high as 4,000 psi have been used. Surface speeds up to 2,000 fpm have been used.

203. Bearing Caps. When split bearings are used, the cap does not generally support the applied load but may be subjected to considerable pressure. When split connecting-rod ends are used in double-acting engines, the cap must carry the full load. In single-acting engines, the cap is subjected to heavy inertia loads under starting conditions.

The cap should be checked for strength by assuming it to be a simple beam loaded at the center and supported at the bolt centers. In many cases, the requirements of forging or casting require greater thicknesses than does strength. For the bearing cap

$$s = \frac{Mc}{I} = \frac{3Fa}{2Lt^2} \tag{246}$$

from which

$$t = \sqrt{\frac{3Fa}{2Ls}} \tag{247}$$

where a = distance between bolt centers, in.
t = cap thickness, in.

When oil holes are provided in the cap, the length is the bearing cap length less the diameter or length of the oil hole.

The deflection of the cap is

$$y = \frac{1}{48}\frac{Fa^3}{EI} = \frac{Fa^3}{4ELt^3} \tag{248}$$

from which

$$t = 0.63a\sqrt[3]{\frac{F}{ELy}} \tag{249}$$

The deflection should be limited to 0.001 in. The bolts holding the cap in place are usually designed to carry $1\frac{1}{3}$ times their proportionate share of the load.

CHAPTER XIV

ROLLER AND BALL BEARINGS

A bearing in which the journal or thrust collar is supported by rolling contact, *i.e.*, by rollers or balls, is commonly referred to as an antifriction bearing. Such conceptions of low friction have, in many cases, been overemphasized and it should be noted that a well-designed sliding bearing having true film lubrication has a frictional resistance nearly as low as that of a roller or ball bearing. If the same care is observed in the design and grit protection of sliding bearings that is customary with rolling bearings, the sliding bearings will give excellent service. However, the use of ball and roller bearings is justified where maximum continuity of service is desired, where the bearing is located so that proper attention to lubrication is difficult, where the loads are heavy and the speeds low, and where the power loss in bearings is a major part of the power used and where low starting torque is essential.

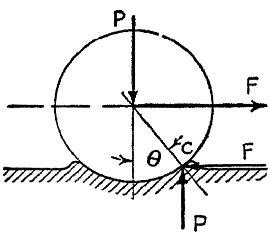


FIG. 204.

204. Rolling Friction. The effect of a roller pressed against a flat supporting surface is shown in exaggerated form in Fig. 204. When the roller is pulled forward, it must climb out of the groove formed. When moments are taken about the point of contact c it is found that

$$Fr \cos \theta = Pr \sin \theta$$

and

$$Fr = Pr \tan \theta \tag{250}$$

In sliding friction, the coefficient of friction is sometimes defined as the tangent of the friction angle. Similarly, in rolling contact, the term $r \tan \theta$ may be called the coefficient of friction. Equation (250) may then be written in the form

$$F = \frac{f_r P}{r} \tag{251}$$

where F = frictional resistance, lb.
P = normal force, lb.
r = radius of roller, in.
f_r = coefficient of rolling friction, in.

The coefficient of rolling friction decreases as the hardness and rigidity of the materials increase and apparently is independ-

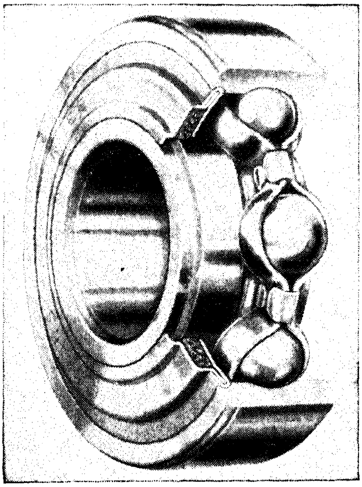


FIG. 205.—(*Courtesy New Departure Manufacturing Company.*)

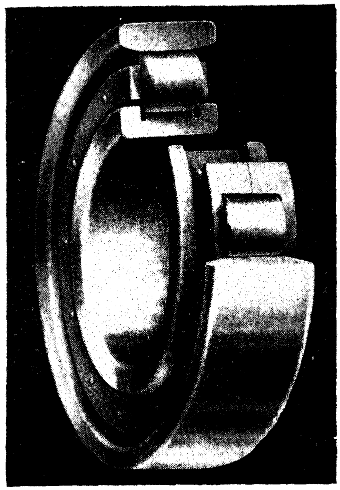


FIG. 206.—(*Courtesy SKF Industries, Inc.*)

ent of the velocity and temperature. Average values expressed in inches are as follows:

Lignum vitae rollers on oak	0.020
Elm rollers on oak	0.032
Soft wood rollers on wood	0.060
Iron rollers, on wood	0.060 –0.200
on asphalt	0.145
on granite	0.085
Cast iron rollers on iron	0.018 –0.065
Railroad wheels (cast steel) on steel	0.020 –0.025
Commercial radial ball bearings	0.0008–0.0012
Commercial ball thrust bearings	0.0032–0.0036
Commercial roller bearings	0.0010–0.0015

205. Theory of Bearing Capacity. When loaded, the balls, rollers, and races deform, and the pressures are distributed over small areas as shown in Fig. 207 and there is more than merely

point contact for balls and line contact for rollers. The general solution of the pressure distribution between two elastic bodies was presented by H. Herz.* From the general equations, Timoshenko† determines the following equations for the maxi-

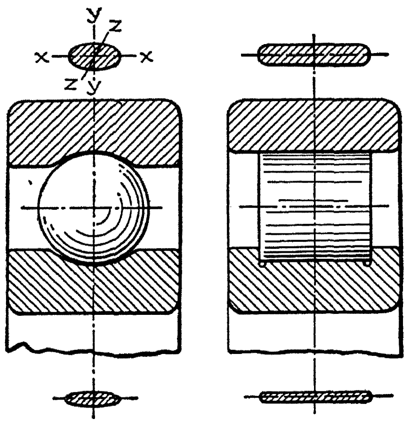


FIG. 207.—Typical contact areas of roller and ball bearings. Note "spot," not "point," contact of ball.

mum contact pressure between cylinders and spheres of steel (Poisson's ratio equal to 0.30).

$$p_{max} = 0.418 \sqrt{\frac{FE(r_1 + r_2)}{Lr_1r_2}} \quad \text{for two cylinders} \qquad (252)$$

$$p_{max} = 0.418 \sqrt{\frac{FE}{Lr}} \quad \text{for a cylinder and plane} \qquad (253)$$

$$p_{max} = 0.388 \sqrt[3]{FE^2 \frac{(r_1 + r_2)^2}{r_1r_2}} \quad \text{for two spheres} \qquad (254)$$

$$p_{max} = 0.388 \sqrt[3]{\frac{FE^2}{r^2}} \quad \text{for a sphere and plane} \qquad (255)$$

where p_{max} = maximum unit pressure on the contact surface, psi.
E = modulus of elasticity, psi.
L = length of cylinder contact, in.
r_1, r_2 = cylinder or sphere radii, in.
F = load applied, lb.

* HERZ, H., *J. Math.* (*Crelle's J.*), Vol. 92, 1881.
HERZ, H., *Gesammelte Werke*, Vol. L, p. 155, 1895, Leipzig.

† TIMOSHENKO, S., "Theory of Elasticity," 1st ed., Chap. XI, McGraw-Hill Book Company, Inc., 1934.

Timoshenko also shows that the maximum shear stress, which is the critical stress for ductile materials, is below the contact surface and is equal to

$$s_{s\,\max} = 0.31p_{\max} \qquad \text{for spheres} \tag{256}$$

and

$$s_{s\,\max} = 0.304p_{\max} \qquad \text{for cylinders} \tag{257}$$

The fundamental theory applying to roller and ball bearings is based on the work of Striebeck,* who developed a mathematical treatment based on his own experiments and the earlier ones of Herz. From static tests, Striebeck found that the strength of a single ball in compression is expressed by the formula

$$F_c = kd^2 \tag{258}$$

where k is a constant varying with the material and with the shape of the supporting races. At the breaking load, k is about 100,000 for carbon-steel balls, and 125,000 for hardened alloy-steel balls. A factor of safety of at least 10 should be used with these values.

For the average radial bearing containing n balls, the maximum load per ball is

$$F_c = \frac{4.37}{n}C \tag{259}$$

where C = radial load on bearing.

Hence the capacity of a radial ball bearing is

$$C = \frac{F_c n}{4.37} = \frac{knd^2}{4.37} \qquad \text{or approximately} \qquad \frac{knd^2}{5} \tag{260}$$

Safe working values of k for average bearing life, are 550 for unhardened steel, 700 for hardened carbon steel, and 1,000 for hardened alloy steel on flat races, 1,500 for hardened carbon steel and 2,000 for hardened alloy steel on grooved races having a radius equal to $0.67d$. Modern commercial bearings, using the higher strength alloy steels and races having nearly the same curvature as the balls, have much higher values of k. These values of k are for a life of 3,000 hr at 100 rpm.

* STRIEBECK, *Kugellager für beliebige Belastung*, Vol. 45, 1901, p. 75; Vol. 29, 1902, p. 1421, D.I.V. Translation by H. Hess, *Trans. A.S.M.E.*, Vol. 29, p. 367, 1907.

TABLE 53.—S.A.E. AND INTERNATIONAL STANDARD DIMENSIONS FOR BALL AND ROLLER BEARINGS

S.A.E No	Bore all series		Series 200		Series 300		Series 400	
	mm	in.	O D.	Width	O.D.	Width	O.D.	Width
200	10	0 3937	1 1811	0 354	1 3780	0 433		
201	12	0 4724	1 2598	0 394	1 4567	0 472		
202	15	0 5906	1 3780	0 433	1 6535	0 512		
203	17	0 6693	1 5748	0 472	1 8504	0 551	2 4409	0 669
204	20	0 7874	1 8504	0 551	2 0472	0 591	2 8346	0 748
205	25	0 9843	2 0472	0 591	2 4409	0 669	3 1496	0 827
206	30	1.1811	2 4409	0 630	2.8346	0 748	3 5433	0 906
207	35	1.3780	2.8346	0.669	3 1496	0 827	3 9370	0 984
208	40	1.5748	3.1496	0 709	3.5433	0.906	4.3307	1 063
209	45	1.7717	3 3465	0.748	3 9370	0 984	4 7244	1 142
210	50	1.9685	3 5433	0 787	4 3307	1 063	5 1181	1 220
211	55	2.1654	3 9370	0 827	4.7244	1.142	5 5118	1 299
212	60	2 3622	4 3307	0 866	5 1181	1 220	5 9055	1 378
213	65	2.5591	4.7244	0 906	5 5118	1 299	6 2992	1 457
214	70	2.7559	4.9213	0 945	5 9055	1 378	7 0866	1 654
215	75	2 9528	5 1181	0.984	6 2992	1 457	7 4803	1 772
216	80	3.1496	5 5118	1.024	6 6929	1 535	7 8740	1 890
217	85	3 3465	5 9055	1 102	7 0866	1 614	8 2677	2 047
218	90	3 5433	6 2992	1 181	7 4803	1 693		
219	95	3.7402	6 6929	1 260	7 8740	1 772		
220	100	3.9370	7.0866	1 339	8 4646	1 850		
221	105	4 1339	7.4803	1 417	8 8583	1 929		
222	110	4.3307	7.8740	1 496	9 4488	1 969		
224	120	4 7244	8 4646	1 574	10 2362	2 165		
226	130	5.1181	9.0551	1 574	11 0236	2 284		
228	140	5 5118	9.8425	1.654	11.8110	2 441		

Light series 200, numbered as 201, 202, etc.
Medium series 300, numbered as 301, 302, etc.
Heavy series 400, numbered as 401, 402, etc.

For roller bearings, Eq. (260) takes the form

$$C = \frac{knLd}{5} \tag{261}$$

where L is the roller length, in., and d is the roller diameter, in.

The value of k is 7,000 for hardened carbon steel and 10,000 for hardened alloy steel.

206. Types of Roller Bearings. Common types of roller bearings are shown in Fig. 208. The simplest roller bearing consists of an inner race, an outer race, and a set of rollers without any retainer. The loose rollers tend to twist in the races and this action increases friction, wears the rollers cigar-shaped, and causes a bending action which may break the rollers. To over-

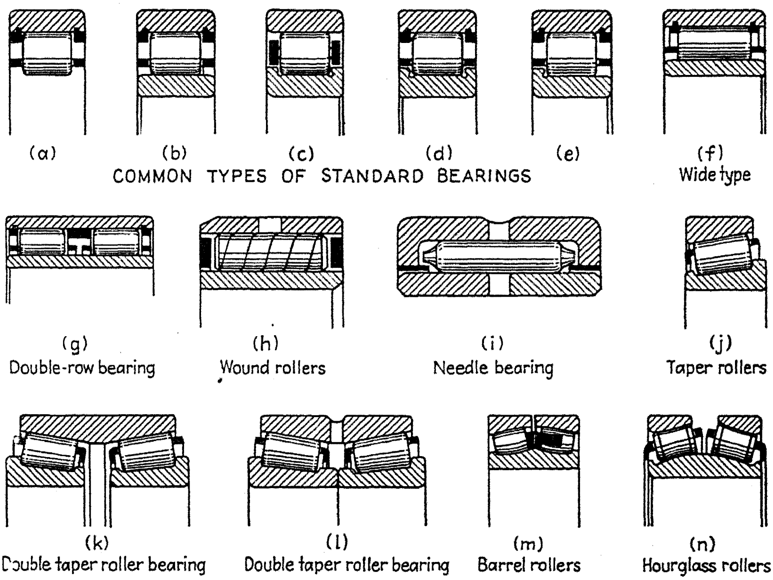


FIG. 208.—Types of roller bearings.

come this, retainer lips on the races and retaining cages, as in Figs. 208b to 208h, are used to confine, space, and guide the rollers. The small shoulders on the races provide a small thrust capacity for locating the shaft. One form of Hyatt bearing (208h) is distinguished by rollers formed of helically wound steel strips which makes these bearings especially suitable for shock loads. Since adjacent rollers are wound in opposite directions, they aid in sweeping the lubricant across the bearing surface.

In conical roller bearings (Figs. 208j, k, and l) the rollers and races are all truncated cones having a common apex on the shaft center to ensure true rolling contact. The taper must be small

(6 or 7 deg) to prevent the imposed radial load from including excessive end thrust. These bearings are capable of carrying some thrust load.

TABLE 54.—RADIAL CAPACITY OF ROLLER BEARINGS BASED ON AN AVERAGE LIFE OF 10,000 HR

International Standard No	Bore		Load capacity at 500 rpm.		
	mm	in.	Series 200	Series 300	Series 400
00	10	0 3937	370	615	
01	12	0 4724	415	795	
02	15	0 5905	460	895	
03	17	0 6693	595	1,090	2,040
04	20	0 7874	745	1,190	2,680
05	25	0 9843	895	1,760	3,370
06	30	1 1811	1,320	2,450	4,150
07	35	1 3780	1,540	2,660	5,000
08	40	1 5748	1,840	3,220	6,000
09	45	1 7717	2,150	4,050	7,070
10	50	1 9685	2,450	4,980	8,240
11	55	2 1654	2,610	5,390	9,480
12	60	2 3622	2,760	6,000	10,810
13	65	2 5591	3,220	7,000	12,200
14	70	2 7559	3,620	7,500	13,650
15	75	2 9528	4,050	9,610	15,170
16	80	3 1496	4,550	10,300	16,770
17	85	3 3465	5,600	12,610	18,430
18	90	3 5433	5,910	13,510	20,180
19	95	3.7402	6,750	14,420	21,960
20	100	3.9370	7,500	15,920	23,840
21	105	4 1339	8,240	18,190	
22	110	4 3307	9,270	19,560	
24	120	4 7244	12,160	22,350	
26	130	5.1181	23,720	25,210	
28	140	5 5118	16,120	29,800	

Light series 200, numbered as 201, 202, etc
Medium series 300, numbered as 301, 302, etc.
Heavy series 400, numbered as 403, 404, etc.
These values are for comparison only. For the capacity of any specific bearing, consult the manufacturer's catalog.

The barrel and hour-glass types of roller bearings (Figs. 208*m* and *n*) are made with one spherical race to provide self-alignment with the shaft. Because of the angular contact, these bearings have a thrust capacity equal to approximately 25 per cent of the radial capacity.

207. Capacity of Commercial Roller Bearings. The load-carrying capacity of all rolling-contact bearings is determined by the fatigue life of the bearing materials, which in turn is determined by the imposed stress and the number of stress applications. The useful life is usually ended by pitting or spalling of the rolling surfaces, a typical fatigue failure. Equation (261) indicates that the capacity of any roller bearing depends upon the number, length, and diameter of the rollers, together with the materials used. The capacity also depends upon the expected life in hours, the speed of rotation, and the type of application.

In Art. 74 it was shown that if the relation between imposed stress and the number of stress repetitions at failure is plotted to logarithmic scales, the curve is a straight line as long as the imposed stress is greater than the endurance limit.* This relation can be represented by the equation

$$N_f = ks^a$$

where N_f = number of stress repetitions at failure.
k = experimental material strength factor.
s = imposed stress, psi.
a = experimentally determined exponent.

For any given size and design of roller bearing, this reduces to

$$N_f = kC^{-b} \qquad \text{or} \qquad C = \frac{\sqrt[b]{k}}{\sqrt[b]{N_f}}$$

from which

$$C = \frac{K}{\sqrt[b]{60NH}} \tag{262}$$

where N_f = fatigue life, revolutions.
N = rpm.
C = radial load capacity of the bearing, lb.
H = fatigue life, hr.

* Unit stresses imposed on roller bearings are from 120,000 to 160,000 psi.

K = material constant.

b = experimentally determined exponent = 3*

Catalog ratings give the radial load capacity C at some predetermined speed N and life H † For any required radial capacity R at a speed N_R and life H_R, we have

$$R = \frac{K}{\sqrt[3]{60H_RN_R}}$$

Combining this with Eq. (262) and introducing an application factor K_A, we have

$$C = \sqrt[3]{\frac{H_RN_R}{HN}} \times K_AR = \sqrt[3]{\frac{H_R}{H}}\sqrt[3]{\frac{N_R}{N}} \times K_AR$$

or

$$C = K_LK_SK_AR \tag{263}$$

where C = required catalog rated capacity, lb.

R = required capacity under operating conditions as calculated, lb.

K_L = life factor = $\sqrt[3]{H_R/H}$, from Fig. 209.

K_S = speed factor = $\sqrt[3]{N_R/N}$, from Fig. 209.

K_A = application factor, from Table 55.

Speed of rotation does not affect the number of revolutions required to produce failure but the actual life in hours will vary inversely as the speed unless the imposed load is corrected as indicated.

* The exponent b is determined by each manufacturer from tests run on a large number of bearings. Bantam, Fafnir, Norma-Hoffman, New Departure, and SKF use $b = 3$; MRC uses 3.3; Timken uses 3.32; Tyson and Hyatt use 3.33.

† Bearing manufacturers do not agree as to the basis of determining expected life. Some use the minimum life expectancy, *i.e.*, 90 per cent of any large number of bearings tested will have a life greater than the rated life, whereas 10 per cent will fail before the rated life is reached. Other manufacturers use the average life, *i.e.*, 50 per cent will have a life longer than the rated life and 50 per cent will fail before the rated life is reached. Various manufacturers use 2,500, 3,000, 3,500, 5,000, 10,000, and 15,000 hr as the basic life rating on which catalog rated capacities are based. Catalog ratings are also based on different speeds in rpm. It is therefore important that when comparing the rated capacities of bearings made by different manufacturers, capacities are all reduced to the same life expectancy and speed.

TABLE 55.—APPLICATION FACTOR K_A FOR ROLLER AND BALL BEARINGS

Type of service	Multiply calculated load by following factors	
	Ball bearings	Roller bearings
Uniform and steady load	1 0	1.0
Light shock load	1.5	1.0
Moderate shock load	2.0	1.3
Heavy shock load	2 5	1.7
Extreme and indeterminate shock load	3 0	2.0

Courtesy Norma-Hoffman Bearings Corporation

When the housing and outer race rotate, the bearing should be selected for a rated speed of 1.6 times the actual operating speed.

In general, when the speed is less than 50 rpm, the rating at 50 rpm should be used. When the bearing is oscillating, the capacity is about 50 per cent more than that of a rotating bearing.

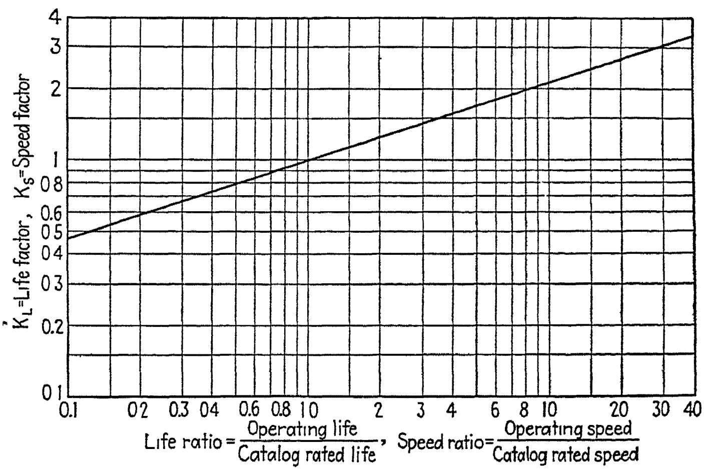


FIG. 209.—Life and speed factors for ball and roller bearings.

The permissible load that will give reasonable life is dependent upon the nature of the service, involving the effects of continuous or intermittent service, fluctuating load, shock, dirt, lubrication, and other items. The selection of proper life and application

factors is a complex problem based on experience with bearings operating under similar conditions. To assist in selecting the proper factors, Table 56 has been included. The factors in this table are products of K_L and K_A and are to be used with ratings from Table 54 and Eqs. (263) and (266).

TABLE 56.—COMBINED LIFE AND APPLICATION FACTORS $K_L \times K_A$

Type of equipment	$K_L \times K_A$
Agricultural equipment	0 65–1 00
Air compressors	1 50–2 00
Belt conveyors	1 00
Belt drives	1 50
Blowers and fans	1 00
Cars, industrial	1.00
mine and mill	1 50
Centrifugal extractors	1 00–1.40
Cranes and hoists,	
hand operated	0.80
power operated	1 00
powerhouse	0.80
steel mill	1 50
Crushers, pulverizers	1.40–1.8
Foundry equipment	1.00
Gear drives	1.75
Glassmaking equipment	1.00
Industrial locomotives	1.50–1.75
Machine tools, except spindles	1.50
Mining machinery	1.50–2.00
Oil-field equipment	2.00
Pumps, centrifugal	1.00
reciprocating	1 40–2.00
dredge, sludge, line	2 00
Refrigerating machinery	1 70
Road machinery	1 00
Rock crushers	2 00–3.00
Steam shovels	1 00
Textile machinery	1 00–1.50
Tractors, general	1 00
crawler tracks	3 00–4.00
crawler wheels	2.20
Transmission machinery	1.70
Turbines	2 00
Woodworking and sawmill equipment	1 00–1.50

208. Needle Bearings. Needle bearings, shown in Fig. 208*i*, are a type of roller bearing using a large number of long rollers

of small diameter ($\frac{3}{16}$ in. or smaller) without retaining cages. Their chief advantages are low cost, high capacity, and compactness. In many cases they can be used in the space occupied by an ordinary bronze bearing bushing.

The capacity of needle bearings at 3,000 hr average life is given by a form of Eq. (262). Thus

$$C = \frac{10{,}000nLd}{\sqrt[3]{N}} \tag{264}$$

When the load capacity is based on the projected area of the needles, we have

$$C = \frac{31{,}400(D + d)}{\sqrt[3]{N}} \tag{265}$$

where D is the outside diameter of the inner race.

When the bearing is oscillating, the capacity is 50 per cent more than that given by these equations.

Needle bearings may be used without special inner or outer races when the shaft or outer housing can be suitably hardened. The hardness should be 60 Rockwell C, and when lower hardness is used, the capacity is reduced according to the load factors given in Fig. 210.

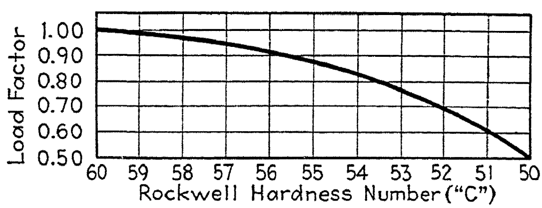


FIG. 210.—Curve of hardness load factors for needle bearings.

When using these bearings, the following items should be considered. Rollers should have lengths equal to 6 to 10 diameters, preferably 6 to 8. Spherical ends are preferable when the length is over 3 diameters, and square ends on shorter lengths. Roller lengths should not be less than one-eighth of the inner race diameter, and lengths of from one to two shaft diameters are preferred. The number of rollers should be less than 60, and the roller speed should not exceed 60,000 rpm. Circumferential clearance should be small, with 0.0001 in. per roller

minimum, increasing to a total of 0.5d for 20 rollers and 0.90d for 50 rollers with never more than one roller diameter total circumferential clearance.

209. Types of Ball Bearings. There are many types of ball bearings providing for different radial- and thrust-load characteristics and for different mounting methods. Single-row bearings providing the maximum radial-load capacity are provided with a filling notch in each race in order to insert the largest number and size of balls possible in a given bearing cross section. These bearings have maximum radial capacity, but because of the filling notch have very little thrust-load capacity.

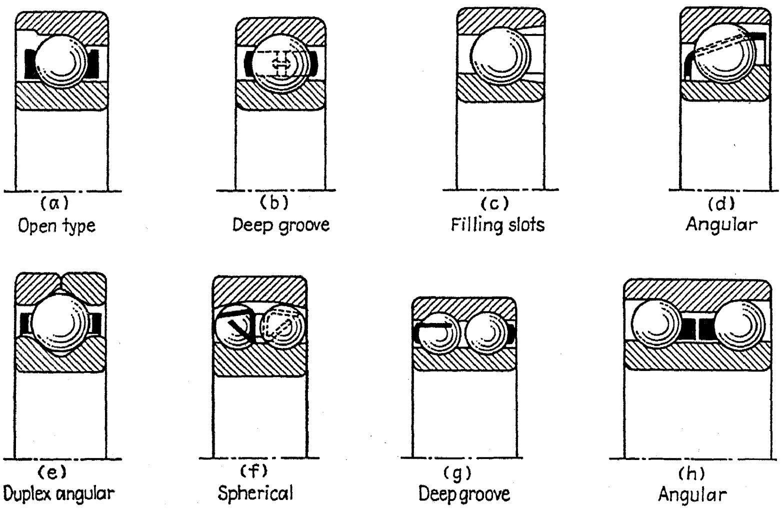


FIG. 211.—Types of ball bearings.

Radial bearings without filling notches contain as many balls as can be introduced by the eccentric displacement of the inner race. By reason of the smaller number of balls, their radial capacity is reduced, but since there is no filling notch they have a thrust capacity of about 75 per cent of their radial capacity.

Angular contact bearings have a high race shoulder on one side of both the inner and outer races and are open on the opposite side, except for a very small shoulder that serves to hold the bearing together after it is assembled. The outer race is assembled after expansion by heat. Some makes of angular contact

bearings have shoulders on both sides of the inner race and on one side of the outer race. The outer race is completed by a cylindrical race opposite the shoulder, which permits the outer race to be removed. The angular contact bearings are used for combined radial and thrust loads. The pure thrust capacity is 100, 200, or 300 per cent of the radial capacity depending upon the depth and the radius of curvature of the shoulders. They should be mounted in pairs, either side by side, or at opposite ends of the shaft in order to take thrust on the shaft in either direction.

Duplex bearings consist of two angular contact bearings mounted side by side. When so mounted the outer rings are ground with sufficient offset to provide a definite preloading when clamped in place.

Double-row bearings are simply two angular contact bearings built with a single inner and a single outer race. They will support twice the radial load of a similar angular contact bearing, and of course will take thrust loads in both directions.

Self-aligning bearings are made with the outer race a spherical surface centered on the axis of the shaft so that the bearing is free to adjust itself to angular displacement of the shaft due to deflection of the shaft or to misalignment of the bearing housings.

Commercial ball bearings are made in three standard proportions, known as the light, medium, and heavy series. The selection of the proper series depends upon the load, shaft size, and housing space limitations. Light-series bearings are used where loads are moderate and where the shaft sizes are relatively large (to obtain stiffness), where hollow shafts are used, and where housing space requires the smallest possible outside diameter. Medium-series bearings are wider and have a larger outside diameter than the light series; they are also about 30 per cent stronger. They are used when the loads are heavy when compared to the shaft size. Heavy-series bearings are 20 to 30 per cent stronger than the medium series. Since the medium-series bearings have a capacity equal to that of an ordinary steel shaft, the heavy series bearings are seldom used except for special installations and specially proportioned shafts.

210. Capacity of Commercial Ball Bearings. The theory of bearing capacity developed for roller bearings applies also to ball bearings, and the relation between the catalog rated load

capacity C and the imposed radial load is

$$C = K_L K_S K_A K_T R \tag{266}$$

where C = required catalog rated capacity, lb.
R = required capacity under operating conditions as calculated, lb.
K_L = life factor, from Fig. 209.
K_S = speed factor, from Fig. 209.
K_A = application factor, from Table 55.
K_T = thrust factor, from Table 57.

TABLE 57.—THRUST FACTORS FOR BALL BEARINGS

Ratio $\frac{\text{thrust load}}{\text{radial load}}$	Factor K_T for Eq. (266)		
	Single-row nonfilling groove type	Angular-contact type	Double-row bearings
0.10	1.00	1.05	1.10
0.15	1.00	1.08	1.15
0.20	1.00	1.10	1.20
0.30	1.00	1.15	1.30
0.40	1.10	1.20	1.40
0.50	1.25	1.25	1.50
0.60	1.40	1.30	1.60
0.70	1.55	1.35	1.70
0.80	1.70	1.40	1.80
0.90	1.85	1.45	1.90
1.00	2.00	1.50	2.00
1.25	2.38	1.63	2.25
1.50	2.75	1.75	2.50
1.75	3.13	1.88	2.75
2.00	3.50	2.00	3.00
3.00	5.00	2.50	4.00
4.00	6.50	3.00	5.00
5.00	8.00	3.50	6.00
10.00	15.00	6.00	11.00

This equation is the same as Eq. (263) with the factor K_T added to allow for any thrust load which the bearing must support. The radial load capacity and the thrust capacity of ball bearings depend upon the construction, the method of assembling the balls into the races, and the ratio of the radius of curvature

TABLE 58.—RADIAL CAPACITY OF BALL BEARINGS
(Based on an average life of 10,000 hr)

S.A.E. No	Bore		Radial load capacity at 500 rpm														
			Deep-groove type			Filling-slot type			Angular type*			Two-row spherical type			Two-row angular type		
			Series			Series			Series			Series			Series		
	Mm	In.	200	300	400	200	300	400	200	300	400	200	300	400	200	300	400
00	10	0 3937	190	330		.	.		265	330		205	310				
01	12	0 4724	220	380	....	. .			330	410		250	360	.	335		
02	15	0 5906	260	470	...				415	515		290	430		470		
03	17	0 6693	330	560	910	.			455	610	1,025	360	505	770	515	930	1,690
04	20	0 7874	400	650	1,380	.	970		615	735	1,550	495	640	1,050	770	1,020	2,080
05	25	0 9843	450	910	1,610		1,200		660	1,025	1,900	610	770	1,140	840	1,530	2,610
06	30	1 1811	610	1,190	1,945	940	1,460		1,020	1,335	2,290	760	1,050	1,370	1,160	2,020	3,090
07	35	1 3780	850	1,500	2,290	1,270	1,820		1,270	1,685	2,690	880	1,240	1,740	1,550	2,540	4,010
08	40	1.5748	1,000	1,840	2,630	1,560	2,200		1,675	2,070	3,285	1,000	1,460	1,960	1,660	3,120	4,980
09	45	1 7717	1,110	2,220	3,070	1,660	2,880		1,675	2,245	3,615	1,210	1,870	2,360	2,120	3,740	5,640
10	50	1 9685	1,220	2,490	3,510	1,840	3,350		1,950	2,905	4,270	1,300	2,120	2,870	2,120	4,450	6,780
11	55	2 1654	1,540	2,940	4,000	2,300	3,850		2,250	3,430	4,870	1,590	2,530	3,100	2,540	5,950	7,490
12	60	2 3622	1,540	3,050	4,510	..	.		2,520	4,000	5,595	1,850	2,900	3,390	3,110	6,320	8,770
13	65	2 5591	1,890	3,990	5,050		.		2,935	4,595	6,145	1,970	3,100	3,920	3,220	7,650	9,650
14	70	2 7559	2,070	4,500	6,210				2,605	5,240	[illegible],550	2,250	3,650	4,800	3,640	8,240	11,000
15	75	2 9528	2,280	4,500	6,820	. . .	. ....	. ..	3,740	5,610	8,305	2,360	3,920	5,080	3,980	8,700	12,400
16	80	3 1496	2,490	5,070	7,470	.	.	.	4,050	6,180	9,080	2,570	4,330	5,350	4,430	10,300	14,900
17	85	3 3465	2,940	5,680	8,130	.			3,150	7,090	9,895	2,880	4,780	6,700	5,150	10,500	16,000
18	90	3 5433	3,140	5,680	8,825	. .	. . .	.	5,140	7,560	10,740	3,340	5,480	7,980	5,900	12,400	16,600
19	95	3 7402	3,620	6,980	10,280		. .	. .	5,905	8,750	12,510	3,340	6,100	8,500	6,700	12,900	20,300
20	100	3 9370	4,120	6,980	11,825	. ..	.		6,740	9,290	14,390	4,100	7,100	10,600	7,160	16,100	22,700
21	105	4 1339	4,120	8,400	....	.... .	... ..	...	7,115	10,480	......	4,460	7,450	.	8,500	17,500	
22	110	4.3307	4,500	8,400	....	....	....	.	7,485	11,170	.....	4,800	7,970		9,500	18,900	

* Thrust capacity of angular-type bearings is twice the radial capacity.

of the race to the ball radius. Catalog capacities of several types of ball bearings are given in Table 58. For specific bearings the manufacturer's catalog should be consulted.

211. Combined Radial and Thrust Loads. A thrust load on a deep-groove type of radial bearing causes a small axial displacement of the balls, and the load line on the individual balls is inclined from the radial plane, the angle of inclination increasing with increased thrust load. Evidently the depth of the race groove limits the amount of this inclination and hence the thrust capacity. The maximum inclination for a deep groove bearing is about 25 deg, and the thrust capacity of the bearing is

$$T = nF_c \sin 25 = \frac{nF_c}{2.5}, \text{ approximately} \tag{267}$$

where F_c is the load capacity of a single ball. Comparing this with Eq. (260) shows that the thrust capacity of a deep-groove ball bearing is approximately 200 per cent of its radial capacity. At low speeds this thrust capacity is possible, but at higher speeds the thrust capacity is reduced by increased friction and by reduction of the angle of inclination due to lower loads at higher speeds.

When determining the size of a ball bearing that is to carry both radial and thrust loads, it is necessary to combine the loads into an equivalent radial load. Most manufacturers' catalogs give tables of correction factors.

The thrust factors in Table 57 apply only to one manufacturer's bearings, and catalogs of particular manufacturers should be consulted when using their make of bearings.

212. Correction for Rotating Outer Race. Catalog ratings are for installations in which the inner race is rotating. When the outer race is rotating the bearing should be selected for an equivalent speed

$$N_e = K_R N \tag{268}$$

where N = speed of outer race, rpm.
K_R = approximately 1.45 for light-series ball bearings, 1.6 for the medium series, and 1.75 for the heavy series.

213. Examples of Bearing Selection. Example 1. Select a roller bearing for the low-speed shaft of a gear-reduction unit operating at 65 rpm with an applied load of 2,850 lb. The shaft size has been determined and is $2\frac{1}{4}$ in.

Solution. The combined life and application factor, from Table 56, is 1.75. Table 54 gives speeds at 500 rpm only, and the speed ratio is $\frac{65}{500}$, or 0.13 For this speed ratio, Fig. 209 gives the speed factor $K_s = 0.51$. From Eq. (263), the required catalog capacity at 500 rpm is

$$C = K_L K_S K_A R = 1.75 \times 0.51 \times 2850 = 2{,}544 \text{ lb}$$

If the shaft can be turned down slightly at the bearing seat, bearing No. 211 with a bore of 2.1564 in. and a capacity of 2,610 lb at 500 rpm can be used. If the shaft cannot be reduced, then bearing No. 212 can be used.

Example 2. Determine the load that can be safely carried on the buckets of an aerial tramway, the cable trolleys of which are each equipped with a No. 306 roller bearing. The expected life of the bearings is about 5 yr with the tramway operating 6 hr per day, 6 days per week. The wheels revolve at 500 rpm, and there is little or no shock.

Solution. Since the outer race revolves, the capacity must be determined at 1.6 × 500 or 800 rpm. The speed ratio is $\frac{800}{500}$ or 1.6 and the speed factor from Fig. 209 is 1.17. The life ratio is 9,360/10,000, or 0.936, and the speed factor from Fig. 209 is 0.98. The application factor from Table 55 is 1.0. From Eq. (263), the permissible load at 500 rpm is

$$C = K_L K_S K_A R$$

$$2{,}450 = 0.98 \times 1.17 \times 1.0 \times R$$

$$R = \frac{2{,}450}{0.98 \times 1.17} = 2{,}215 \text{ lb}$$

for one wheel, or 4,430 lb per trolley.

Example 3. Select a ball bearing for a rock crusher to be used on building projects. The load is about 1,500 lb radial, and the shaft rotates at 300 rpm. The shaft size is $1\frac{7}{8}$ in.

Solution. The service will be intermittent; therefore, assume that the crusher will be in service the equivalent of 10 hr per day for two years, or 7,500 hr. The life ratio is 7,500/10,000, or 0.75, and the life factor from Fig 209 is 0.87. The speed ratio is $\frac{300}{500}$, or 0.60, and the speed factor from Fig. 209 is 0.81. Rock-crusher service would indicate severe and indeterminate shock loading, and the application factor, from Table 55, is 3. From Eq. (266), the required catalog capacity at 500 rpm is

$$C = K_L K_S K_A K_T R = 0.91 \times 0.81 \times 3 \times 1 \times 1{,}500 = 3{,}317 \text{ lb}$$

If the shaft can be slightly reduced at the bearing seat, bearings No 409 angular type or No. 309 two-row angular type may be used. Otherwise a sleeve and bearings No. 410 deep-groove type or No. 310 filling-slot type may be used.

214. Installation of Ball Bearings. Ball bearings are usually installed with the shaft and inner race revolving, but in some cases the outer race rotates. It is general practice to install the rotating race with a light press fit and to clamp it firmly

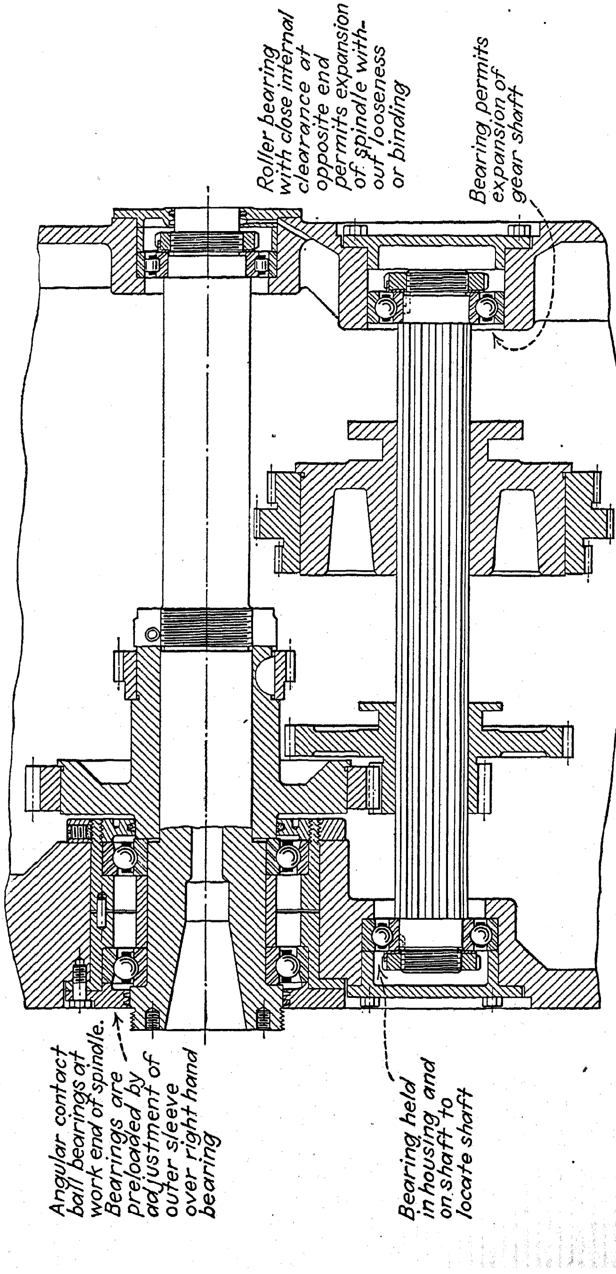


FIG. 212.—Typical bearing installation. Application of ball and roller bearings to a milling-machine drive. (*Courtesy Norma-Hoffman Bearings Corporation.*)

in position. The stationary race is installed with a close push fit (wringing fit). This method of mounting permits the stationary race to creep around gradually in its supporting housing, presenting new parts of the race to the load and distributing the wear. The rate of creep is very slow, and care should be exercised to prevent looseness and its consequent slippage and excessive wear on the supporting housing.

The stationary race of one bearing on a shaft should be clamped in position, with a slight axial clearance to permit the creeping action. All other bearings should have axial clearance provided to prevent cramping of the bearing due to inaccuracies in machining or due to temperature changes. Single-row bearings permit an axial movement of from 0.002 to 0.010 in., and, if all axial motion must be prevented, two opposed bearings with a slight preloading must be used. The proper method of mounting bearings is shown in Fig. 212. The heights of the retaining shoulders and the proper shaft dimensions can be found in tables included in the manufacturers' bulletins.

215. Ball and Roller Thrust Bearings. Several types of thrust bearings are shown in Fig. 213. Since all the balls or rollers support their share of the load, the thrust capacity of a pure thrust bearing is much greater than that of a pure radial bearing. However, at the higher speeds of rotation, centrifugal force throws the balls against the outer side of the race, causing a wedging action. For this reason, the speed limit for pure thrust bearings is much lower than for radial bearings. For the higher speeds, the deep-groove type of angular contact bearing should be used.

216. Preloading of Bearings. Ball bearings may be preloaded to reduce the axial and radial movement of the shaft when loaded. When the bearing is loaded it will be deformed, the relative deformation decreasing as the load is increased. When two angular contact bearings are mounted as shown in Fig. 214, the preloading nut N pulls the sleeve B to the right until a predetermined axial load is applied to both bearings, and the shaft has an initial tension. Each bearing has been deformed a definite amount. When an external thrust is applied to the shaft, there will be a slight increase in deformation in the front bearing but a decrease in the tension in the shaft, which in turn partially relieves the preload on both bearings. Hence, the

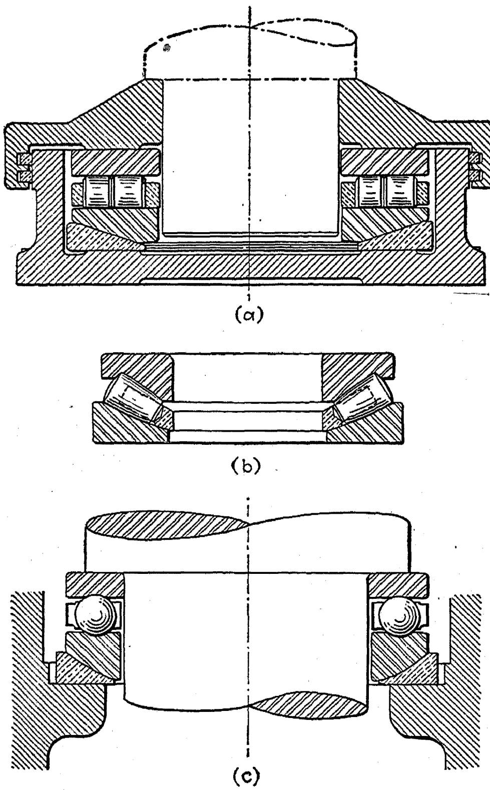


FIG 213.—Thrust bearings. [(*a*) *Courtesy Bantam Ball Bearing Company;* (*b*) *courtesy SKF Industries, Inc.*]

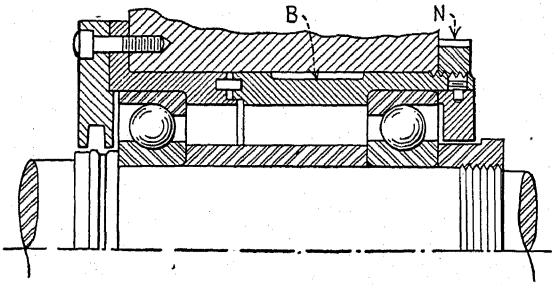


FIG. 214.—Bearings with provision for preloading. (*Courtesy New Departure Manufacturing Company.*)

load on the front bearing is less than the sum of the initial preload and the applied thrust, and the load on the rear bearing is less than the initial preload. Since the increase in the deformation of the front bearing is due to an increase in load smaller than the actual thrust load applied, the deformation caused by the thrust load is less than if there had been no preload. This principle is used when the bearings must be rigidly mounted to insure fixity, precision of operation, and permanence of shaft position. The applied thrust load required to relieve the preload completely may be as great as four times the initial preload. Since the axial preload also preloads the bearing in the radial direction, the radial movement of the shaft under load is also reduced.

Two single-row bearings, such as shown in Fig. 214, may be used together, placed back to back without the spacers. In this case the outer races can be ground slightly longer than the inner races, so that they will contact while the inner races are slightly separated. Then when the inner races are forced together, the balls will be in a preloaded condition. By this method of preloading, the applied preload can be very accurately controlled.

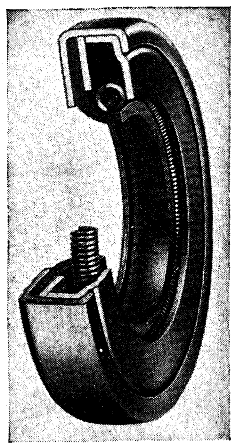


FIG. 215.—Oil and grease seal. (*Courtesy National Motor Bearing Company.*)

217. Lubrication of Ball and Roller Bearings. Rolling-contact bearings are essentially low-friction bearings. However, it is necessary to provide lubrication to reduce the frictional contact between the rolling members and the retaining cages, to reduce any friction occurring where contact between the races and the rolling elements is not pure rolling, to help dissipate the heat generated, and to protect the bearing elements from corrosion, grit, and dirt. Methods of providing lubrication are shown in the accompanying illustrations. Only light mineral oils and greases should be used, since animal oils develop acids that will corrode the balls and races.

218. Bearing Seals. Bearings and housings must be carefully sealed to exclude dirt, grit, and other foreign matter that would abrade, corrode, or clog the bearings and to retain the lubricant.

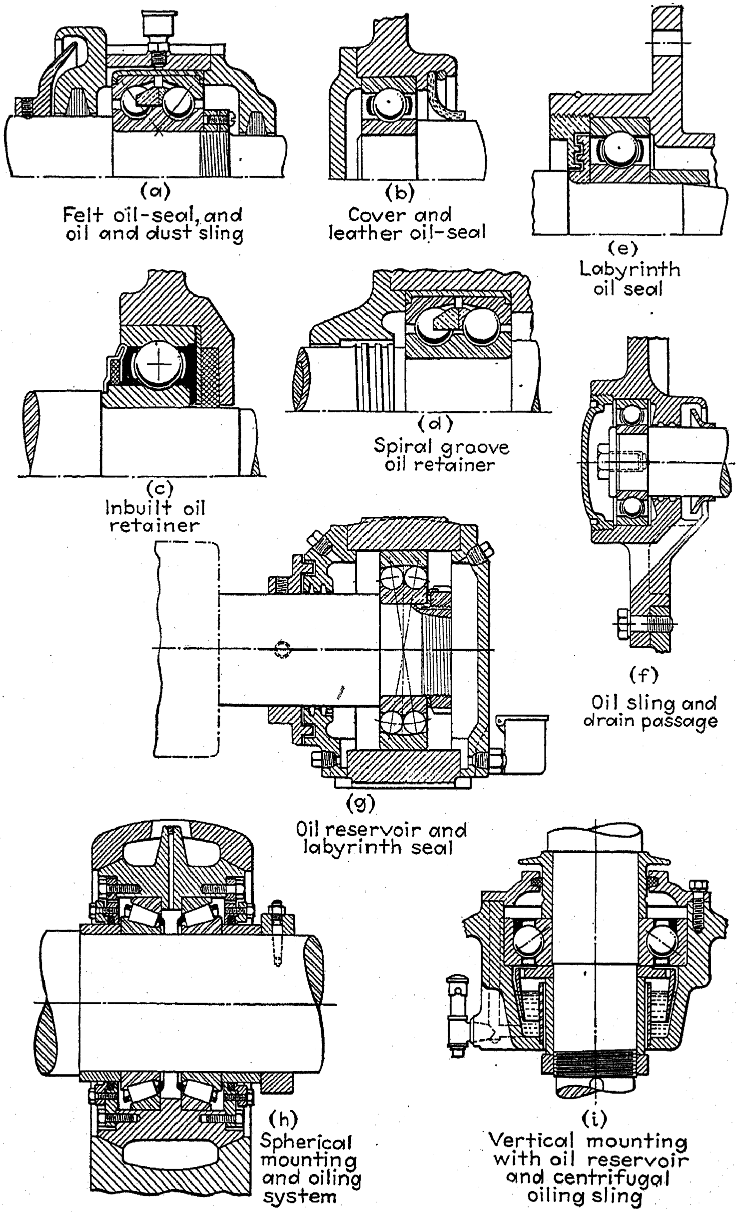


FIG. 216.—[*(a)–(f) and (i) courtesy New Departure Manufacturing Company; (g) courtesy SKF Industries, Inc.; (h) courtesy Timken Roller Bearing Company.*]

A very large percentage of ball bearing failures are due to dirt and loss of lubricant. Oil lubrication requires more elaborate sealing methods than does grease lubrication. The most common seal is the felt ring slightly compressed in a retaining groove.

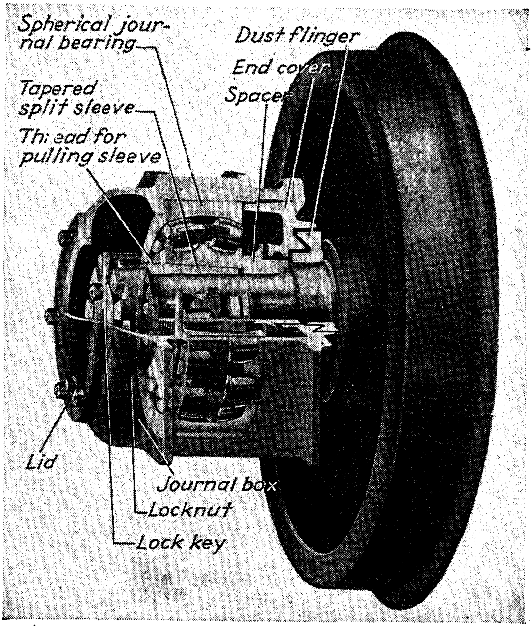


FIG. 217.—Application of roller bearing to a car axle. (*Courtesy SKF Industries Inc.*)

The rings are rectangular in cross section when cut, but should be used in tapered grooves to produce a light pressure against the rotating member. Other methods of sealing include the use of leather seals, grease grooves, spiral grooves in the rotating member to feed the lubricant inward, slingers, labyrinths, stuffing boxes, and seals built into the bearings themselves. Several designs of sealing elements are shown in Figs. 216 and 217.

CHAPTER XV

BELTS AND BELT CONVEYORS

The uses of belts may be grouped into three general classes: power transmission, conveyor service, and elevator service. Belts for power transmission have been in use for more than a century, competing formerly with rope drives and now with electric drives. Conveyor and elevator belts are of more recent development, and their use will undoubtedly increase in the future.

219. Belt Drives and Electric Drives. To choose between belt drives using line shafts driven by a single large motor, group drives using belts driven from short line shafts and several small motors, and individual motor drives is a problem in engineering economics that must be given consideration and solved for each individual plant layout. In general, one large motor is cheaper on a horsepower basis than several small motors. The power loss is high in installations with direct-connected electric drive, because the high motor speeds require the use of reducing gears. However, power losses in long line shafts and idling shafting may more than offset these gear losses. The convenience of locating individually driven machines at any desired place and at any desired distance from the original source of power, and the unsightly appearance of belts has caused the individual motor drive to become popular during recent years, but for economic reasons many plants are now adopting group drives.

220. Power-transmission Belts. Belts used for power transmission must be strong, flexible, and durable and must have a high coefficient of friction. The most common belt material is oak-tanned leather. Leathers tanned with chestnut bark, vegetable compounds, alum salts, and chrome salts are also used. Fabric, rubber, and balata belts are all commonly used. Steel belts are frequently used in Europe but are very rare in this country.

The steer hides from which belting is made vary in density and in strength, the fibers near the backbone being shorter but denser than the fibers farther down on the sides, and the belly fibers being the longest and least dense. The variation is of course gradual, but the difference between the backbone and the belly leather is very marked. The strength of the leather varies as shown in Fig. 218. Note that the backbone leather does not have the highest tensile strength, although it is the best belt material. The best leather is obtained from a strip extending about 15 in. on each side of the backbone and about 54 in. from the tail. Poorer grades of belting are made from strips taken outside, but adjacent to, this region. Short-lap belts are made from the backbone strip, and long-lap belts include the shoulder leather and are therefore inferior to short-lap belts. Double, or two-ply, belts are made by cementing two strips of leather together with the hair sides out.

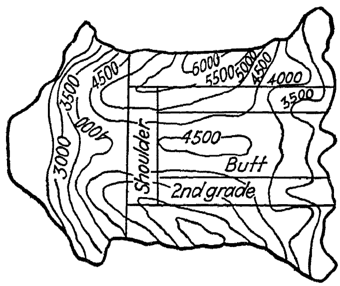


FIG. 218.

Oak-tanned leather, which is considered to be the standard belt material, is fairly stiff, whereas chrome leather is soft and pliable. The manufacturers of chrome leather claim that it grips the pulley better, is more pliable, will not crack when doubled upon itself, and that it is lighter, stronger, and longer fibered. On account of the flexibility of chrome leather, it is hard to shift with a belt shifter, hence two-ply belt with the inner ply of chrome leather and the outer ply of oak leather is used where shifting is necessary.

The best leather has an ultimate strength of about 4,000 psi. First-grade belts have a minimum strength of 3,000 psi and an average of 3,750 psi for single ply, and 3,500 psi for double ply.

221. Fabric and Canvas Belts. This type of belt is made from canvas or cotton duck folded to three or more plies (layers) and stitched together. Woven belts are made of cotton woven to any desired thickness in a loom. Fabric belts are usually impregnated with a filler, largely linseed oil, to make them waterproof and to prevent injury to the fibers. The filler makes the belts rather stiff. These belts are cheap and are used for inter-

mittent service, in hot dry places, and where little attention is given to their upkeep, as in farm machinery. Fabric belts are used to some extent for conveyors.

222. Rubber Belts. Rubber belting is made from folded canvas duck with layers of friction (rubber) between and surrounding the whole belt structure. The belt is vulcanized under heat and pressure. Such belts are commonly used where exposed to moisture or outside weather conditions, as in sawmills, oil fields, and paper mills.

The strength and pulling capacity of the belt are in the duck, the rubber acting only to protect the fibers from internal wear and moisture. For light high-speed service, a light weight duck, 24 and 26 oz,* is used. For heavy drives in centrifugals, oil fields, and main drives, high tensile strength is required and 36-oz duck is used, inner-stitched to prevent ply separation. Standard rubber belt is usually 32 oz.

Rubber belt is cheaper than leather belt, but it is affected by light, heat, and oil and deteriorates with age even when in service.

Balata belting is made like rubber belting except that balata gum is substituted for rubber, and it is not vulcanized. This material does not oxidize and age in the air, is waterproof, acidproof, and not affected by animal oils or alkalies, but is seriously affected by mineral oils. When heated, it becomes soft and sticky and should not be used where the belt temperature exceeds 100 to 120 F. Balata belt is about 25 per cent stronger than rubber belt.

223. Ratio of Belt Tensions. Referring to Fig. 219, consider the forces acting on a short section of belt, dl inches long. These forces are the belt pulls, F and $F + dF$, the pulley pressure P and the centrifugal force C.

From the conditions of equilibrium

$$P + C - F \sin \frac{d\phi}{2} - (F + dF) \sin \frac{d\phi}{2} = 0 \qquad (269)$$

and

$$(F + dF) \cos \frac{d\phi}{2} - F \cos \frac{d\phi}{2} - fP = 0 \qquad (270)$$

* Duck is graded by the weight of a strip 36 in. wide by 40 in. long.

from which

$$P = \frac{\cos \frac{d\phi}{2}\, dF}{f} \tag{271}$$

Also

$$C = Ma = \frac{12\rho btr\, d\phi}{g} \frac{v^2}{r} = F_c\, d\phi \tag{272}$$

where ρ = weight of the belt, lb per cu in.
b = belt width, in.
t = belt thickness, in.
r = pulley radius, ft.
v = belt velocity, fps.
g = 32.2 ft per sec.^{2}

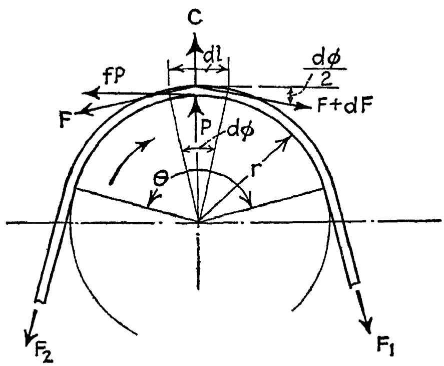


FIG. 219.

By substitution of the values of P and C in Eq. (269),

$$\cos \frac{d\phi}{2}\, dF - f(2F + dF) \sin \frac{d\phi}{2} + fF_c\, d\phi = 0$$

In the limit, $\cos \frac{d\phi}{2}$ is unity, and $\sin \frac{d\phi}{2}$ is equal to $d\phi/2$, nearly. By substitution, expansion, and elimination of the product $dF \times d\phi$, since it is negligible, this equation becomes

$$dF - fF\, d\phi + fF_e\, d\phi = 0$$

from which

$$\frac{dF}{F - F_c} = f\, d\phi$$

By integration over the entire arc of contact θ

$$\int_{F_2}^{F_1} \frac{dF}{F - F_c} = f \int_0^\theta d\phi$$

or

$$\log_e \frac{F_1 - F_c}{F_2 - F_c} = f\theta$$

from which

$$\frac{F_1 - F_c}{F_2 - F_c} = e^{f\theta} \tag{273}$$

The power transmitted is measured by the difference in tension on the tight and slack sides of the belt, and Eq. (273) is more useful in the form

$$F_1 - F_2 = (F_1 - F_c)\left(\frac{e^{f\theta} - 1}{e^{f\theta}}\right) \tag{274}$$

where F_1 = total tension on the tight side, lb.
F_2 = total tension on the slack side, lb.
$F_c = 12\rho btv^2/g$ = centrifugal tension, lb.
e = 2.718.
f = coefficient of friction.
ρ = belt weight, lb per cu in.: 0.035 for leather; 0.044 for canvas; 0.041 for rubber; 0.040 for balata; 0.042 for single and 0.045 for double woven belt.

The effective belt pull $F_1 - F_2$ is determined in practice by the initial tension, *i.e.*, the tension in the belt when the drive is standing idle. The necessary value of the initial tension F_i can be found approximately from the relation

$$F_i = \frac{(F_1^{1/2} + F_2^{1/2})}{4} \tag{275}$$

Initial tensions should range from 200 to 240 psi for leather belts and from 10 to 12 lb per ply per in. width for rubber belts.

224. Power Transmitted by Belts. The power transmitted by any belt depends on the arc of contact, difference in belt tensions, coefficient of friction, and center distance. The pulley having the lower value of $f\theta$ (usually the smaller pulley) governs the transmitting power. The general expression for

power transmitted is

$$\mathrm{hp} = \frac{(F_1 - F_2)v}{550} \tag{276}$$

where v = belt velocity, fps.

Combination of Eqs. (274) and (276) results in

$$\mathrm{hp} = \frac{(F_1 - F_c)v}{550}\left(\frac{e^{f\theta} - 1}{e^{f\theta}}\right) \tag{277}$$

The required cross-sectional area of the belt is found by substitution, in this equation, of $12\rho btv^2/g$ for F_c, and bts_w for F_1. Hence

$$bt = \frac{550\ \mathrm{hp}}{v\left(s_w - \dfrac{12\rho v^2}{g}\right)}\left(\frac{e^{f\theta}}{e^{\theta f} - 1}\right) \tag{278}$$

where v = belt speed, fps

s_w = maximum working stress in the belt, psi.

The thickness t and the standard widths of belts are given in Table 59. In the selection of the proper belt, it is not con-

TABLE 59.—LEATHER BELT THICKNESS AND MINIMUM PULLEY DIAMETERS

Weight, plies	Thickness	Tan	Width, in.	Pulley diam., in. Belt velocity, fpm 1,000	2,000	3,000 and over
Single	$\frac{5}{32}$	Chrome	to 8	2	$2\frac{1}{2}$	3
Light single	$\frac{8}{64}$–$\frac{10}{64}$	Oak	to 8	2	$2\frac{1}{2}$	3
Medium single	$\frac{10}{64}$–$\frac{12}{64}$	Oak	to 8	3	$3\frac{1}{2}$	4
Heavy single	$\frac{12}{64}$–$\frac{14}{64}$	Oak	to 8	4	5	6
Double	$\frac{5}{16}$	Chrome	to 12	6	7	8
Light double	$\frac{17}{64}$–$\frac{19}{64}$	Oak	to 12	4	5	6
Medium double	$\frac{22}{64}$–$\frac{24}{64}$	Oak	to 12	8	10	12
Heavy double	$\frac{33}{64}$–$\frac{35}{64}$	Oak	to 12	10	13	14
Medium triple	$\frac{1}{2}$	Chrome	to 24	18	24	36
Heavy triple	$\frac{17}{32}$	Oak	to 24	24	30	36

Note: Standard belt widths increase by $\frac{1}{8}$ in. from $\frac{1}{2}$ to 1 in.; by $\frac{1}{4}$ in. to 3 in.; by $\frac{1}{2}$ in. up to 6 in.; by 1 in. up to 10 in.; and by 2 in. up to 56 in.; and by 4 in. up to 72 in.

sidered good practice to use single-ply leather belts more than 8 in. wide.

225. Working Stress in Belts. A factor of safety of 10 is used with leather belts, making the maximum working stress 300 psi, so that F_1 is 300 bt. Belts operating at 350 psi have a very short life, whereas those operating at 250 psi have a very long life.

The strength of fabric and rubber belts depends upon the weight of duck used and the number of plies. The ultimate strength is about 300 lb per ply per in. width for 28-oz duck, 325 lb for 30- and 32-oz duck, and 360 lb for 36-oz duck. Approximate weights of rubber belts are 0.021 lb per ply per in. width per ft length for 28-oz duck, 0.024 lb for 32-oz duck, and 0.026 lb for 36-oz duck. Since the weight of the duck is usually unknown, the designer should refer to the manufacturer's horsepower tables before making the final selection. For preliminary computations, the values of effective tensions $F_1 - F_2$ given in Table 60 can be used, these values agreeing closely with average practice for an arc of contact of 180 deg.

TABLE 60.—EFFECTIVE TENSIONS PER INCH OF BELT WIDTH

Number of plies	Belt material				
	Rubber	Balata	Canvas	Woven cotton	Woven camel hair
3	30	48	30	Single 55	Single 60
4	40	65	40	Double 80	Double 85
5	50	80	50	Triple 110	Triple 125
6	60	95	60		
7	70	110	70		
8	80	130	80		
10	100	160	100		

The effective tensions should be increased or decreased ½ per cent for each degree of contact greater or less than 180 deg

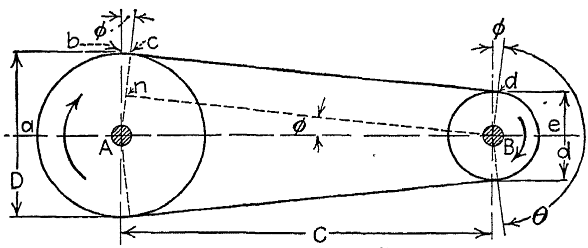
Effective tensions listed are for belts made of 28-oz duck For 30- and 32-oz duck multiply by 1.10. For 36-oz duck multiply by 1 20.

226. Belt Joints and Fasteners. Unless the belts are endless, some type of fastener is required at the joint. The joint is a weak place in the belt, and the permissible working stress should be multiplied by a joint factor from Table 61 when computing the power capacity of any belt.

TABLE 61.—CORRECTION FACTORS FOR BELT JOINTS

Joint	Factor
Cemented by beltmaker	1 00
Cemented	0 98
Wire laced by machine	0 90
Wire laced by hand	0 82
Rawhide laced	0 60
Metal belt hooks	0 35

227. Multiple-ply Leather Belts. Theoretical considerations indicate that if the thickness of the belt is doubled by making it two-ply, the horsepower capacity will also be doubled. However, laboratory tests and actual service experience indicate that a two-ply belt will have its capacity reduced to 85 per cent, and a triple-ply belt to 75 per cent, of the capacity computed on the basis of a single-ply belt of the same thickness.



$$\text{Belt length} = \frac{\pi}{2}(D + d) + 2C + \frac{(D - d)^2}{4C}$$

$$\text{Arc of contact} = \theta = 180 \mp 2 \sin^{-1} \frac{D - d}{2C} \text{ degrees}$$

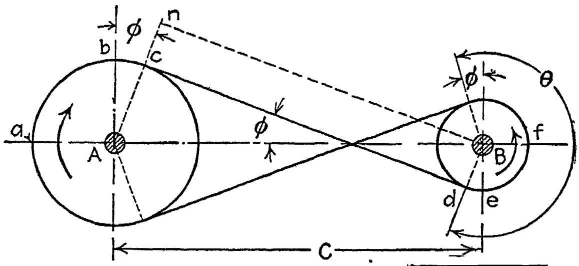
FIG. 220.—Open belt drive.

228. Inclined Belt Drives. Vertical drives are not so efficient as horizontal drives, since the belt does not cling to the lower pulley. The horsepower transmitted by any inclined belt should be reduced from that of an equivalent horizontal belt, by 1 per cent for each degree of inclination exceeding 60 deg.

229. Crossed Belts. Crossed belts wider than 8 in. should be avoided. Where wider belts are required, the reversing type of drive with idler pulleys should be used. The capacity of a crossed belt should be reduced to 75 per cent that of an open belt, and if the pulley ratio is 3:1 or more, the reduction should be 50 per cent.

230. Belt Slip and Creep. When a belt is transmitting power, there is always a small amount of slip between the belt and the

pulleys so that the actual velocity of the belt is slightly less than the surface speed of the driving pulley and slightly greater than that of the driven pulley. Belt slip up to about 3 per cent actually increases the coefficient of friction and under normal condition $1\frac{1}{2}$ to 2 per cent slip is present. Creep is a different type of movement of the belt on the pulley surface caused by the fact that any unit length of belt on the tight side decreases in length as it passes around the driving pulley to the slack side.



$$\text{Belt length} = \left(\frac{\pi}{2} + \phi\right)(D + d) + 2C\sqrt{1 - \frac{(D + d)^2}{4C^2}}$$

ϕ in radians

$$\text{Arc of contact} = 180 + 2\phi = 180 + 2\sin^{-1}\frac{D + d}{2C} \text{ degrees}$$

FIG. 221.—Crossed belt drive.

Hence, the belt velocity leaving the driving pulley is slightly less than that of the belt moving onto this pulley. This action adds to the actual belt slip on this pulley. On the driven pulley the creep action is reversed from that on the driving pulley. Percentage slip usually refers to the combined real slip and creep action.

231. Coefficient of Friction for Belts. The coefficient of friction depends on the belt material, the pulley-surface material, the belt slip, and the belt speed. For oak-tanned leather belts on cast-iron pulleys, C. G. Barth gives the following formula:

$$f = 0.54 - \frac{140}{500 + V} \tag{279}$$

where V = belt velocity, fpm.

This is the coefficient of friction when slipping impends, but most belts are operated below the slipping load. Average values of the coefficient as recommended for design purposes are given in Table 62.

TABLE 62.—COEFFICIENT OF FRICTION FOR BELTS

Belt material	Pulley material						
	Cast iron, steel			Wood	Compressed paper	Leather face	Rubber face
	Dry	Wet	Greasy				
Leather, oak tanned	0.25	0.20	0.15	0.30	0.33	0.38	0.40
Leather, chrome tanned	0.35	0 32	0.22	0.40	0.45	0.48	0.50
Canvas, stitched	0.20	0.15	0.12	0.23	0.25	0.27	0.30
Cotton, woven	0.22	0.15	0.12	0.25	0.28	0.27	0.30
Camel hair, woven	0.35	0.25	0.20	0.40	0.45	0.45	0.45
Rubber	0 30	0 18	...	0.32	0.35	0 40	0 42
Balata	0.32	0.20	...	0.35	0.38	0 40	0 42

232. Horsepower Rating Tables. The American Leather Belting Association* has adopted a standardized procedure for the determination of the capacity of oak-tanned leather belting. This method of belt selection considers such factors as service conditions, center distance, pulley size, belt speed, and belt thickness. Tables 63 to 65 give the factors, and the example illustrates the use of these tables.

Example. Find the width of a light, double-ply, oak-tanned, leather belt to be used on a shunt-wound, 25-hp, direct-current motor whose speed is 1,750 rpm and whose fiber pulley diameter is 8 in. The center distance is 15 ft, and the tight side is above. The service is continuous under normal conditions, and the angle of the center line is 70 deg from the horizontal.

Solution. For a belt speed of 3,660 fpm, Table 63 gives 8.4 hp per each inch of width. For an 8-in. pulley, a center distance of 15 ft, and tight side above, Table 64 modifies this value by the factor 0.72. The service correction factors from Table 65 are 1 0 for normal atmospheric conditions, 0.9 for 70-deg angle of center line, 1.2 for fiber pulley. 0.8 for continuous service, and 0.8 for direct-current motor.

$$\text{Width of belt} = \frac{25}{8.4 \times 0.72 \times 1.0 \times 0.9 \times 1.2 \times 0.8 \times 0.8}$$
$$= 5.98 \text{ in.} \quad \text{Use a standard width of 6 in.}$$

The student should compare this value with the value obtained from the use of Eq. (278).

* Horsepower ratings for oak-tanned flat leather belting adopted Dec. 7, 1938, by The American Leather Belting Association.

TABLE 63.—HORSEPOWER PER INCH OF WIDTH

Belt speed, fpm	Single-ply		Double-ply			Triple-ply	
	$\frac{11}{64}$ in.*	$\frac{13}{64}$ in.*	$\frac{18}{64}$ in.*	$\frac{20}{64}$ in.*	$\frac{23}{64}$ in.*	$\frac{30}{64}$ in.*	$\frac{34}{64}$ in.*
	Med.	Heavy	Light	Med	Heavy	Med.	Heavy
600	1 1	1 2	1 5	1 8	2 2	2 5	2 8
800	1.4	1 7	2 0	2 4	2 9	3 3	3 6
1,000	1 8	2 1	2 6	3 1	3 6	4 1	4 5
1,200	2 1	2 5	3 1	3 7	4 3	4 9	5 4
1,400	2 5	2 9	3 5	4 3	4 9	5.7	6 3
1,600	2 8	3 3	4 0	4 9	5 6	6 5	7 1
1,800	3 2	3.7	4 5	5 4	6 2	7.3	8 0
2,000	3 5	4.1	4 9	6 0	6 9	8.1	8 9
2,200	3 9	4 5	5 4	6 6	7.6	8 8	9 7
2,400	4 2	4.9	5 9	7 1	8 2	9 5	10 5
2,600	4 5	5 3	6 3	7 7	8 9	10.3	11 4
2,800	4 9	5.6	6 8	8 2	9 5	11 0	12 1
3,000	5 2	5.9	7 2	8 7	10.0	11 6	12 8
3,200	5 4	6 3	7 6	9 2	10 6	12 3	13 5
3,400	5 7	6.6	7 9	9 7	11 2	12 9	14 2
3,600	5 9	6 9	8 3	10 1	11 7	13 4	14 8
3,800	6 2	7 1	8 7	10 5	12 2	14 0	15 4
4,000	6.4	7 4	9 0	10.9	12 6	14 5	16 0
4,200	6.7	7.7	9 3	11 3	13.0	15 0	16 5
4,400	6.9	7 9	9 6	11 7	13 4	15.4	16 9
4,600	7.1	8 1	9 8	12 0	13 8	15 8	17 4
4,800	7.2	8 3	10.1	12 3	14 1	16 2	17 8
5,000	7.4	8 4	10 3	12 5	14 3	16 5	18 2
5,200	7.5	8.6	10 5	12.8	14 6	16 8	18 5
5,400	7.6	8 7	10.6	12 9	14 8	17 1	18 8
5,600	7.7	8 8	10 8	13 1	15 0	17 3	19 0
5,800	7.7	8 9	10 9	13 2	15 1	17 5	19 2
6,000	7 8	8 9	10 9	13 2	15 2	17 6	19 3

* Average thickness.

For pivoted-base drives, where the tight side of the belt is away from the pivot shaft, do not use these tables.

233. Operating Velocity of Belts. An examination of Eq. (278) indicates that at high velocities the power-transmitting capacity of the belt is seriously decreased by the centrifugal force; in fact at the higher velocities the power decreases until it is theoretically zero. This does not mean, as is sometimes claimed, that the belt will not transmit power. It simply

Table 64.—Correction Factor for Small Pulley Diameter

Diameter small pulley, in.	Center distance in feet							
	Up to 10 ft		15 ft		20 ft		25 ft and over	
	Tight side		Tight side		Tight side		Tight side	
	Above	Below	Above	Below	Above	Below	Above	Below
2	0 37	0 37	0 38	0 41	0.37	0 43	0 37	0 44
$2\frac{1}{2}$	0.41	0 41	0 43	0 46	0.41	0 48	0 42	0 49
3	0 45	0 45	0 48	0 52	0 48	0 54	0 48	0 55
$3\frac{1}{2}$	0 49	0 49	0 53	0 57	0 53	0 59	0 53	0 60
4	0 53	0 53	0.58	0 63	0.59	0 65	0 59	0 66
$4\frac{1}{2}$	0.56	0 56	0 61	0 66	0 62	0 68	0 62	0 70
5	0 59	0 59	0 65	0 70	0 66	0 72	0 66	0 74
$5\frac{1}{2}$	0.60	0 60	0 66	0 72	0 67	0 74	0.68	0 76
6	0.62	0 62	0 68	0 74	0 69	0 76	0 70	0 78
7	0 64	0 64	0 70	0 76	0 71	0 78	0.72	0 80
8	0 66	0 66	0 72	0 78	0 73	0 80	0.74	0 82
9	0.67	0 67	0 73	0 79	0 74	0 81	0.75	0 83
10	0 68	0 68	0 75	0 81	0 76	0 83	0 77	0 85
11	0 69	0 69	0 76	0 82	0 77	0 84	0 78	0 86
12	0 70	0 70	0 77	0 83	0 78	0 86	0 79	0 88
13	0 71	0 71	0 78	0 84	0 79	0 87	0.80	0 89
14	0 72	0 72	0 79	0 85	0.80	0 88	0.81	0 90
15	0 73	0 73	0 80	0 86	0 81	0 89	0.82	0 91
16	0 74	0 74	0 80	0 87	0 81	0 89	0 82	0 91
17	0 74	0 74	0 81	0 88	0 82	0 90	0.83	0 92
18	0 75	0 75	0 82	0 89	0 83	0 91	0 84	0 93
20	0.75	0.75	0 83	0 90	0 84	0 92	0.85	0 94
22	0 76	0.76	0 84	0 91	0 85	0 93	0.86	0 95
24	0 77	0.77	0 85	0 92	0 86	0 94	0.87	0 96
30	0 79	0 79	0 87	0 94	0 88	0 96	0.89	0 98
36	0.80	0.80	0.88	0.95	0.89	0 98	0 90	1 00

means that the assumed permissible stresses will be exceeded if power is transmitted. In practice, this decrease in capacity actually occurs but apparently not so rapidly as the equation indicates. In general factory practice with line shafting and machine belts, moderate velocities of 1,000 to 3,000 fpm are most satisfactory, higher speeds requiring the use of excessively large pulleys. For large power transmission with the pulley bearings mounted on solid foundations, velocities of

TABLE 65.—SERVICE CORRECTION FACTORS

Select the one appropriate factor from each of the five divisions

Atmospheric condition:	
Clean, scheduled maintenance on large drives	1.2
Normal	1.0
Oily, wet or dusty	0.7
Angle of center line:	
Horizontal to 60 deg from horizontal	1.0
60 to 75 deg from horizontal	0.9
75 to 90 deg from horizontal	0.8
Pulley material:	
Fiber on motor and small pulleys	1.2
Cast iron or steel	1.0
Service:	
Temporary or infrequent	1.2
Normal	1.0
Important or continuous	0.8
Peak Loads:	
Steady belt loads as obtained with steam engines, turbines, Diesel and multicylinder-gas engines, fans, centrifugal pumps, and steady line shaft loads	1.0
Jerky belt loads as obtained with large induction motors; compensator-started, shunt-wound, direct-current motors; single-cylinder gas engines; reciprocating machines; and machines developing series of peak loads, such as compressors, rock crushers, and punch presses	0.8
Shock and reversing belt loads on all motors 10 hp and under, all cross-the-line start motors, wound-rotor (slip-ring) motors, synchronous motors, and reversing loads such as printing presses, elevator service, and laundry washers	0.6

5,000 and 6,000 fpm are used, although high velocities always increase the belt troubles.

Belts, like other mechanical equipment, have critical speeds, which are indicated by the belt riding from side to side on the pulley face and by violent flapping of the slack side. The critical speed depends upon the belt tannage, thickness, and center distance and can be remedied by altering the center distance, the load, or the belt velocity.

234. Pulley Sizes. The minimum pulley diameters recommended for use with various belts are given in Table 59. The pulley face should be about 1 in. wider than the belt for belts up to 12 in.; 2 in. wider for 12- to 24-in. belts; and 3 in. wider for belts over 24 in. wide. The face should be crowned, *i.e.*, slightly larger in diameter at the center than at the edges, to assist in keeping the belt centered.

235. Short-center Drives. High speed ratios and short-center distances decrease the arc of contact on the smaller pulley until the power-transmitting capacity of the drive is seriously reduced. The arc of contact should never be less than 155 deg, and in practice it is found that arcs less than 165 deg require high belt tensions. The proper arc of contact may be obtained with short-center distances by the use of spring-loaded or gravity idlers, as shown in Figs. 222 and 223. The

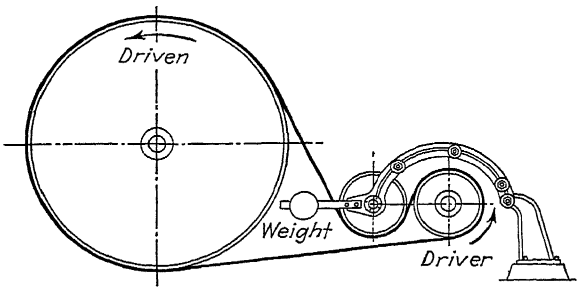


FIG. 222.—(*Courtesy Link-Belt Company.*)

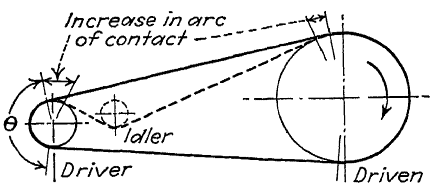


FIG. 223.—Effect of an idler on a short-center drive.

idler should be located so that the wrap on the small pulley is 225 to 245 deg, and the clearance between the idler and the pulley should be not more than $1\frac{1}{2}$ to 2 in. The idler should always be flat-faced, never crowned, and should always be located next to the small pulley (whether driving or driven) and on the slack side. A fixed idler is suited only to drives in which the small pulley is driven, and in which the drive is steady. Spring-loaded idlers are not so satisfactory as gravity idlers, since the increase in the slack on the belt when loaded relieves the spring tension when it is most needed.

With the use of properly designed and balanced idlers, it is possible, but not always advisable, to use extremely short center distances. Many operating men are prejudiced against

short drives and idlers in any form because of the troubles they have experienced, such as short belt life, insufficient slack in belt to permit the belt stretch to absorb shock loads, and the necessity of frequent adjustments. However, the decreased

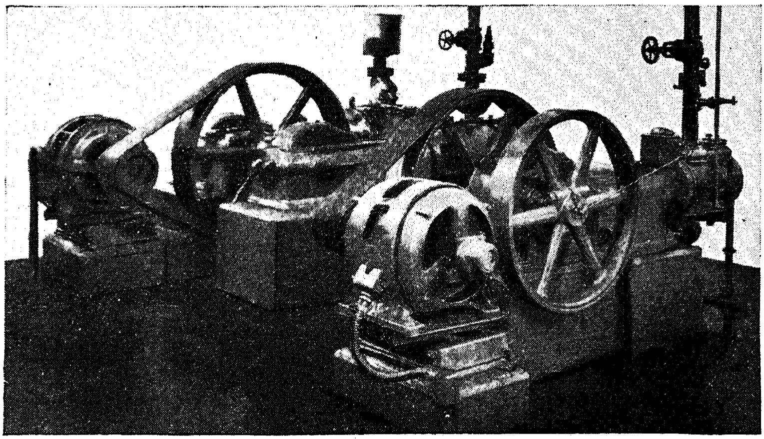


FIG. 224.—Rockwood short-center drive with pivoted motor mounting. (*Courtesy Rockwood Manufacturing Company.*)

cost of the original belt and the saving in space may often justify the use of these drives.

236. Rockwood Belt Drive. The latest development in short-center drives is the Rockwood drive, or pivoted mount, which lessens some of the objections common to idlers. In this drive, a part of the weight of the driving motor is balanced against the belt pulls as shown in Fig. 225. The leverage and the belt tensions may be adjusted to any desired value by shifting the motor along the hinged support. When the motor has been properly balanced to maintain the required belt pull at the heaviest peak loads, no further adjustment is necessary. Belt stretch, whether caused by effective belt tension or by centrifugal force, is taken up by the movement of the motor about the hinge point. An important feature of this drive is that the pressure

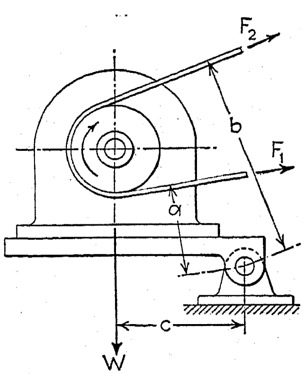


FIG. 225.

between the belt and pulley is fixed by the motor weight and its leverage, and the sum of the belt tensions decreases as the power load transmitted is decreased. Hence at all loads, except the extreme peak loads, the belt tension is much less than that required with an ordinary open belt drive, and the life of the belt is correspondingly increased.

Whenever possible, the tight or pulling side of the belt should pass between the motor pulley and the hinge point. This will reduce the effective moment of the belt pulls about the hinge point and permit the motor to be mounted closer to the hinge than would be possible if the slack side were placed nearest the hinge.

Once balanced, the Rockwood drive limits the maximum load that may be imposed on the belt and hence on the motor. At this load the belt will slip and protect the motor from further overloading. This slippage, of course, may shorten the life of the belt but may prevent burning out the motor. The action of the drive may be better indicated by an example.

Example. Assume that, at maximum load, F_1 is 300 lb, F_2 is 100 lb, and the effective tension 200 lb. The motor weight is 150 lb and the distances a and b are 3 and 10 in., respectively.

To determine the required motor moment arm, take moments about the hinge point when maximum power is being transmitted.

$$F_1a + F_2b = Wc$$

and

$$c = \frac{F_1a + F_2b}{W} = \frac{300 \times 3 + 100 \times 10}{150} = 12.67 \text{ in.}$$

When operating at one-half the maximum load,

$$F_1 - F_2 = 100$$

and

$$3F_1 + 10F_2 = 150 \times 12.67 = 1{,}900$$

from which

$$F_1 = 223 \text{ lb}$$

and

$$F_2 = 123 \text{ lb}$$

When operating under no load, the belt tensions will be practically equal. Hence

$$F_1 \doteq F_2$$

and

$$F_1(3 + 10) = 1{,}900$$

from which

$$F_1 = F_2 = 146 \text{ lb}$$

The total belt pull or load on the motor bearing ($F_1 + F_2$) will be 400 lb at maximum load, 346 lb at half load, and 292 lb when idling. With an open belt drive with fixed centers, the sum of the belt tensions will remain practically constant at all loads. Hence the bearing load will always be 400 lb, a value only reached with the Rockwood drive when the belt is operating at its maximum load.

237. V-belt Drives. The first V-belts were made of small leather stampings, hinged together to form an endless chain

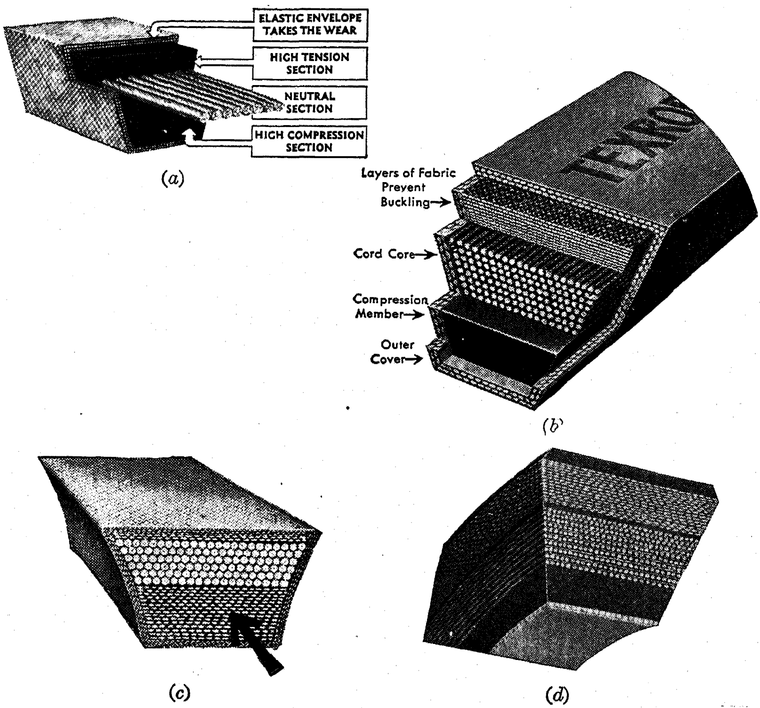


FIG. 226.—[(*a*) *Courtesy Goodyear Tire and Rubber Manufacturing Company;* (*b*) *courtesy Allis-Chalmers Manufacturing Company;* (*c*) *courtesy Gates Rubber Company;* (*d*) *courtesy Dayton Rubber Company.*]

running in grooved pulleys. Modern V-belts are made of fabric and cords molded in rubber and covered with fabric. Several types of these belts are shown in Fig. 226. The belt runs in wedge grooves whose sides usually have an included angle of 38 deg, with 34 or 30 deg being used for the small-sized belts. The tension on the belt forces it into these grooves, giving

increased frictional grip. From Fig. 227, the normal force on the groove face is

$$P_n = \frac{P}{2 \sin \beta} \tag{280}$$

Then the tractive force is

$$F = 2fP_n = \frac{2fP}{2 \sin \beta} = f_eP \tag{281}$$

where f = coefficient of friction.

f_e = equivalent or effective coefficient of friction $= \dfrac{f}{\sin \beta}$.

The design coefficient of friction on flat surfaces is taken as 0.13, making f_e equal to 0.50 for 30-deg, 0.45 for 34-deg, and 0.40 for 38-deg grooves.

The regular power-transmission equations for belts may be applied to V-belts when f_e is substituted for the regular coefficients of friction f. Since the construction used by different manufacturers* varies, one should consult catalog power tables when making the final selection of a V-belt. Examination of manufacturers' tables indicates that the belt properties are approximately as given in Table 66. The weight of the belt is approximately $0.235b^2$ lb per ft, and the maximum permissible working load $145b^2$ lb, where b is the belt width at the outer surface. Using these values, Eq. (274) becomes

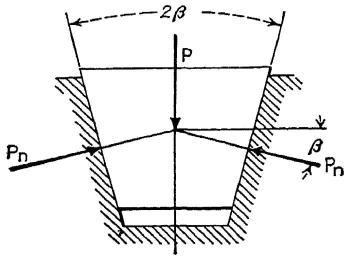


FIG. 227.

$$F_1 - F_2 = nb^2(145 - 0.0073v^2)\left(\frac{e^{f_e\theta} - 1}{e^{f_e\theta}}\right) \tag{282}$$
$$= \frac{33{,}000 \times 12 \times \text{hp } K_s}{\pi dN}$$

*Texrope, Allis-Chalmers Co., Milwaukee, Wis.; Cogbelt, Dayton Rubber Manufacturing Co., Dayton, Ohio; D-V, Dodge Manufacturing Co., Mishawaka, Ind.; Emerald V, Goodyear Tire and Rubber Co., Akron, Ohio; Vulco Rope, The Gates Rubber Co., Denver, Colo.

TABLE 66.—PROPERTIES OF V-BELTS

Symbol	Size, outside width × depth	Weight per ft, lb, w	Maximum working load, lb, F_1	Minimum recommended grooved pulley diam., in.	Minimum flat pulley diam., in.	Correction for pitch diam., in.	
						Grooved*	Flat†
A	$\frac{33}{64}$ by $\frac{9}{32}$	0.06	35	3	17	$\frac{3}{8}$	0.30
B	$\frac{11}{16}$ by $\frac{13}{32}$	0.106	55	5.5	24	$\frac{1}{2}$	0 40
C	$\frac{15}{16}$ by $\frac{9}{16}$	0.20	126	9	42	$\frac{3}{4}$	0.60
D	$1\frac{5}{16}$ by $\frac{3}{4}$	0.40	240	13	52	$\frac{7}{8}$	0.80
E	$1\frac{19}{32}$ by $\frac{29}{32}$	0.60	400	21 5	62	$1\frac{1}{8}$	0.95

Sizes and weights are approximate, as they vary with the manufacturer.
* Subtract from outside diameter of sheave to get pitch diameter.
† Add to outside diameter of flat pulley to get pitch diameter.

where n = number of individual belts in the drive.
d = pitch diameter of the small pulley, in.
N = rpm of the small pulley.
K_s = a service factor from Table 68.

When the driven pulley is comparatively large, a grooved surface may not be necessary. This will be true if $e^{f\theta}$ for the large flat-faced pulley exceeds $e^{f_e\theta}$ for the small grooved pulley.

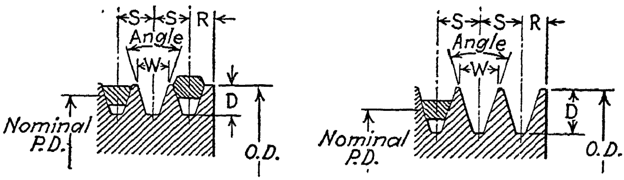
Since V-belts are made endless and in standard lengths, the pulley center distances must be arranged to suit the available belt lengths. The minimum center distance will be approximately equal to the diameter of the larger pulley. In computing belt lengths, the pitch diameters of the pulleys should be used. These may be obtained from the outside diameters of grooved or flat pulleys by making the corrections indicated in Table 67. Belt length is given by the equation

$$L = 2C + 1.57(D + d) + \frac{(D - d)^2}{4C} \tag{283}$$

where C = center distance, in.
D = pitch diameter of the large pulley, in.
d = pitch diameter of the small pulley, in.

The diameter of the small pulley should not be less than that given in Table 67.

TABLE 67.—GROOVE DIMENSIONS AND ANGLES



Standard groove dimensions

Belt section	W	S		R	D	Nominal pitch diam. to outside diam.
		Standard	Narrow			
A-AA	0.504	$\frac{5}{8}$	$\frac{9}{16}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{3}{8}$
B-BB	0.656	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{11}{16}$	$\frac{1}{2}$
C-CC	0.895	1	1	$\frac{11}{16}$	$\frac{7}{8}$	$\frac{3}{4}$
D-DD	1.282	$1\frac{7}{16}$	$1\frac{3}{8}$	$\frac{7}{8}$	$1\frac{1}{8}$	$\frac{7}{8}$
E-EE	1.563	$1\frac{3}{4}$	$1\frac{5}{8}$	$1\frac{1}{8}$	$1\frac{5}{16}$	$1\frac{1}{8}$

Deep groove dimensions

Belt section	W			S	R	D	Nominal pitch diam. to outside diam.
	Groove angle						
	30 Deg.	34 Deg.	38 Deg.				
A	0.530	0.560	0.580	$\frac{5}{8}$	$\frac{3}{8}$	$\frac{5}{8}$	0.60
B	0.740	0.770	0.810	$\frac{7}{8}$	$\frac{1}{2}$	$\frac{15}{16}$	0.95
C	1.000	1.050	1.100	$1\frac{3}{16}$	$\frac{11}{16}$	$1\frac{3}{16}$	1.35
D	1.420	1.480	1.540	$1\frac{5}{8}$	$\frac{15}{16}$	$1\frac{1}{2}$	1.62
E	1.700	1.780	1.850	2	$1\frac{1}{16}$	$1\frac{3}{4}$	1.96

Tolerance of dimension $W \pm 0.002$ in.

Commercial groove angles

Belt section	Pitch diam. of sheave, in.	Groove angle, deg.
A	Under 3 3–5 Over 5	30 34 38
B	Under 5.4 5.4–8 Over 8	30 34 38
C	Under 9 9–12 Over 12	30 34 38
D	Under 13 13–16 Over 16	30 34 38
E	Under 21.6 21.6–24 Over 24	30 34 38

Courtesy Gates Rubber Company.

TABLE 68.—SERVICE FACTORS FOR V-BELTS

Type of machinery	Nature of load	Factor K_s
Light fans and blowers Small centrifugal pumps	Starting load light, operating load normal	1 0
Machine tools Small pumps and compressors Line shafts with light loads	Light pulsations, starting or peak loads up to 125% of normal	1.10
Heavy-duty machine tools Reversing drives Large compressors and pumps Sawmill machinery (band saws, resaws, and hogs) Cotton-ginning machinery Clay-working machinery (dry pans, wet pans, auger machines, and pug mills) Paper-mill machinery (Jordans, beaters, calenders, grinders) Large mine fans and blowers Large flour and feed mills and pulverizers	Moderate shocks or pulsations, starting or peak loads up to 150% of normal	1.25
Mining machinery (ball, rod, and tube mills) Ore crushers Oil-well machinery Continuous-operation drives Small hand-feed mills	Severe shocks or pulsations, starting or peak loads up to 200% of normal	1.4
Textile machinery (spinning frames, twisters)	Starting or peak loads up to 250% of normal	1.5
Crushers, drag lines, and other heavy-duty machines	Possibility of stalling	2

Drives using V-belts are suitable for speed ratios up to 7:1, and for belt speeds up to 5,000 fpm. For short-center drives they have several advantages: since the belts are made endless there are no joint troubles; high speed ratios are possible, the wedging action offsetting the decrease in the arc of contact; the close centers permissible give compactness as required in many machine layouts; and there is more positive drive and less

slippage than with flat belts. Like other belt drives, they cushion the fluctuating loads on the motor and bearings as compared to chain drives. When a large number of individual belts is required by the power to be transmitted, the center of pull may be far from the supporting bearing and excessive bending of the shaft may be encountered.

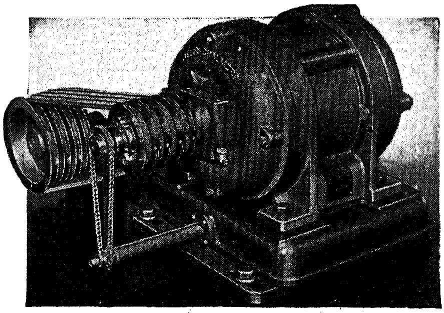


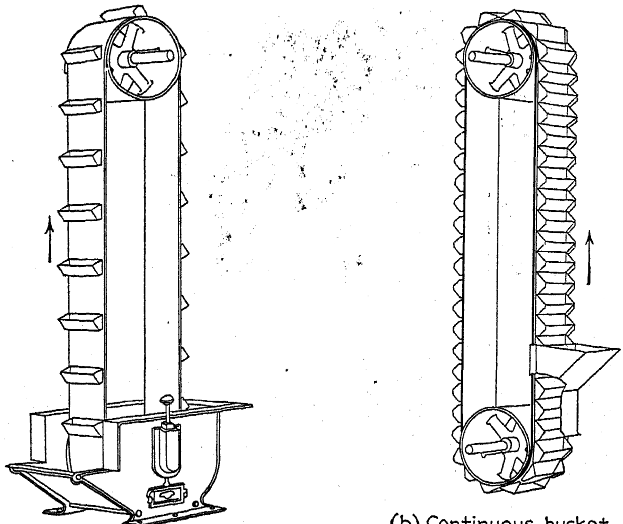
FIG. 228.—The Vari-Pitch drive consists of a Texrope V-belt drive having one or both sheaves arranged to permit adjustment of the grooves, thus providing a larger or smaller diameter, resulting in an increase or decrease in speed of the driven shaft. It has a range of speed variation from 15 to 25 per cent per sheave. The drive is made in two types, stationary and motion control. The stationary-control type is used when speed changes are infrequent and when adjustment is to be made while the sheave is stationary. The motion-control-type Vari-Pitch sheave, shown in the figure, is used when speed changes are frequent and when adjustment is to be made while the sheave is in motion. A handwheel, conveniently located on the automatic motor base, increases or decreases the pitch diameter of the Vari-Pitch sheave, thereby varying the speed and simultaneously maintaining proper belt tension and compensating for the slight change in center distance occurring during the adjustment. (*Courtesy Allis-Chalmers Manufacturing Company.*)

238. Belt Elevators.* When the lift is not too great, belt elevators may be used to lift a continuous supply of material from a low to a high level. An elevator consist of buckets or other attachments for holding the material, belts or chains for supporting the buckets, machinery for driving the belt, and auxiliaries for loading and discharging, maintaining belt tension, and casings for enclosing the elevator.

Belts are used on centrifugal-discharge and continuous-discharge elevators, either of which may be inclined as much as 45 deg with the vertical. The centrifugal-discharge elevator has malleable-iron or sheet-steel buckets arranged on a belt at intervals slightly greater than the depth of the buckets. With

* For a complete treatise on elevators, see F. V. Hetzel, "Belt Conveyors and Belt Elevators," John Wiley & Sons, Inc., 1926.

belt speeds of 200 to 250 fpm and the proper size head wheel, the material will be discharged from the buckets by centrifugal force. The continuous-discharge elevator travels about 100 fpm and has the buckets set close together on the belt with the flanged front of each bucket acting as a discharge chute for the succeeding bucket, allowing a clean discharge at low speeds.



(a) Centrifugal discharge elevator

(b) Continuous bucket elevator-buckets on belt. Note loading position

FIG. 229.—(*Courtesy Link-Belt Company.*)

Belt elevators are preferable to chain elevators when handling gritty or abrasive materials since these materials cause more rapid wear on the chain links than on the rubber-covered belts.

239. Elevator Belts. Stitched canvas, woven cotton, rubber, and balata belts are commonly used. Leather belts have been used when no moisture was present but are considered to be too expensive. The construction of the belts is similar to that of power-transmission belts, except that elevator belts are much thicker in order to furnish adequate support for the bolts and rivets used in attaching the buckets.

Rubber belts made with 32- or 36-oz duck are the most common type. On account of the severe wear, a rubber cover from $\frac{1}{32}$ to $\frac{1}{16}$ in. thick is used, and when handling wet materials, covers up

to $\frac{3}{16}$ in. on the pulley side and $\frac{3}{32}$ in. on the bucket side are used. Belts for continuous elevators are usually heavier and are made of 36- and 42-oz. duck.

240. Belt Conveyors. A belt conveyor consists of an endless belt supporting the material, a belt-driving pulley, roller supports

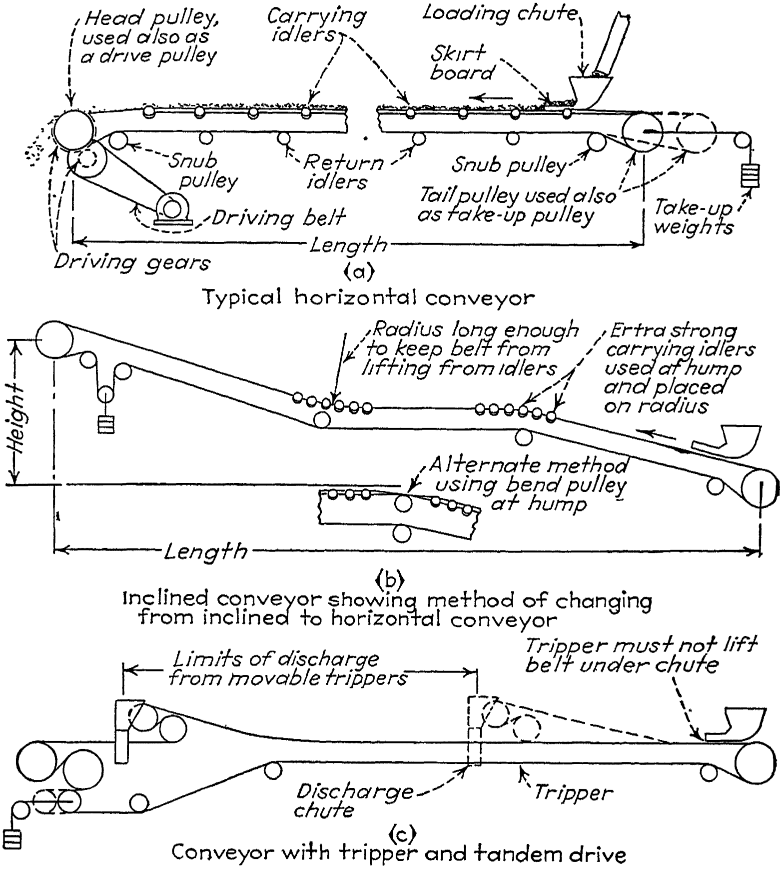


FIG. 230.—(*Courtesy Goodyear Tire and Rubber Manufacturing Company.*)

for both runs of belt, loading and discharge chutes, and driving machinery. Belt conveyors are simple in construction, require less power, and have greater adaptability than any other type of conveyor. They operate on the horizontal or on inclines up to 30 deg, the maximum incline depending on the material carried. Typical belt conveyor layouts are shown in Fig. 230.

The belt used depends on the material to be handled, stitched canvas and woven cotton being used for wrapped package, laundry, bakery, and food-product conveyors. When used in canneries for conveying fruits and vegetables, a tasteless waterproofing (consisting chiefly of wax), not affected by the hot water and steam used for cleaning, is used.

Rubber belting is the most commonly used, the construction being similar to that used for transmission belts, except that the material-carrying side has a rubber covering from $\frac{1}{16}$ to $\frac{1}{4}$ (usually $\frac{1}{8}$) in. thick, to provide a wearing surface and to protect the duck from being cut and abraded during the loading period. When guide and troughing pulleys are used, the belt is reinforced at the edges. These belts are made with one ply for each 4 to 5 in. of width, with 12-in. three-ply belts as a minimum, and 48-in. eight-ply belts as a maximum.

241. Conveyor Capacity. Capacity is determined by the belt width, the velocity, and the depth of the load. The maximum amount of material that can be held on a belt is computed from the volume of a pile extending to one-twelfth the belt width from the edge and having a slope of 4 in 10. In practice, the capacity is taken as approximately 50 per cent of this value. For average conditions the capacity is given by the relation

$$\begin{aligned} C &= 1.6\ b^2 \text{ for flat belts,} \qquad (284) \\ &= 2.0\ b^2 \text{ for belts on flared idlers,} \\ &= 3 \text{ to } 3.5\ b^2 \text{ for belts on three- to five-step idlers,} \end{aligned}$$

where C = capacity, cu ft per hr at 100 fpm.
b = belt width, in.

The belt speed should be limited to 250 fpm with a 10-in. belt, with an additional 10 fpm for each additional inch of width. Package conveyors should travel from 75 to 125 fpm, and picking belts from 40 to 50 fpm. When the belt is inclined, the speed should be reduced about 2 per cent for each degree above the horizontal.

242. Loading and Discharge. The loading chute should be designed to deliver the material in the direction of belt travel at as near the belt speed as is possible. For bulk material, the loading chute should be not over two-thirds the belt width, at least three times the size of uniform lumps, or two times the size of the largest lumps. A common rule is that the belt width

should be four times the uniform-lump size plus 6 in. but not less than twice the size of the largest lumps.

Discharge is accomplished by spilling over the end pulley, by scrapers, by tilting the belt, or by trippers. The tripper is an arrangement of pulleys to bend the belt backwards so that the load will be discharged into suitable chutes.

243. Supporting Idlers. Package conveyors are supported on plain flat idlers of wood or iron. With bulk materials, troughing idlers are used to turn up the outer portion of the belt and increase the capacity. On the return side, the belt should be supported

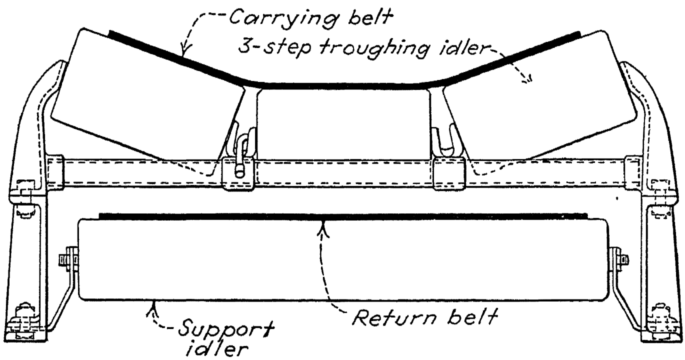


FIG. 231.—Troughed belt conveyor. (*Courtesy Link-Belt Company.*)

at intervals of 9 to 10 ft, and on the carrying side at intervals of 3 ft 6 in. for 48-in. belts to 5 ft for 12- and 16-in. belts. When the materials weigh more than 100 lb per cu ft, these distances should be decreased by 6 in. Supporting rollers should be located just ahead of and just beyond the loading chute, but never directly under it.

244. Power Required by a Horizontal Belt Conveyor. Power must be supplied to overcome the friction of the idlers due to their own weight, the belt weight, and the load, the friction of the head and foot pulleys, and the friction losses in the tripper. The power required may be expressed by the equation

$$\text{hp} = \left[(W_I + 2W_B + W_L)f\frac{d}{D}\right]\frac{LV}{33{,}000} + H_T \qquad (285)$$

where W_I = weight of revolving idlers, lb per ft of belt.
W_B = belt weight, lb per ft.
W_L = load, lb per ft.

f = coefficient of friction of the idler bearings.
d = diameter of idler bearings, in.
D = diameter of idlers, in.
L = conveyor length, ft.
V = belt velocity, fpm.
H_T = power required by tripper, hp.

The coefficient of friction for grease-lubricated idlers is 0.35, and d/D averages about 0.2, so that the term fd/D is about 0.07. With roller bearings, f may be taken as 0.15.

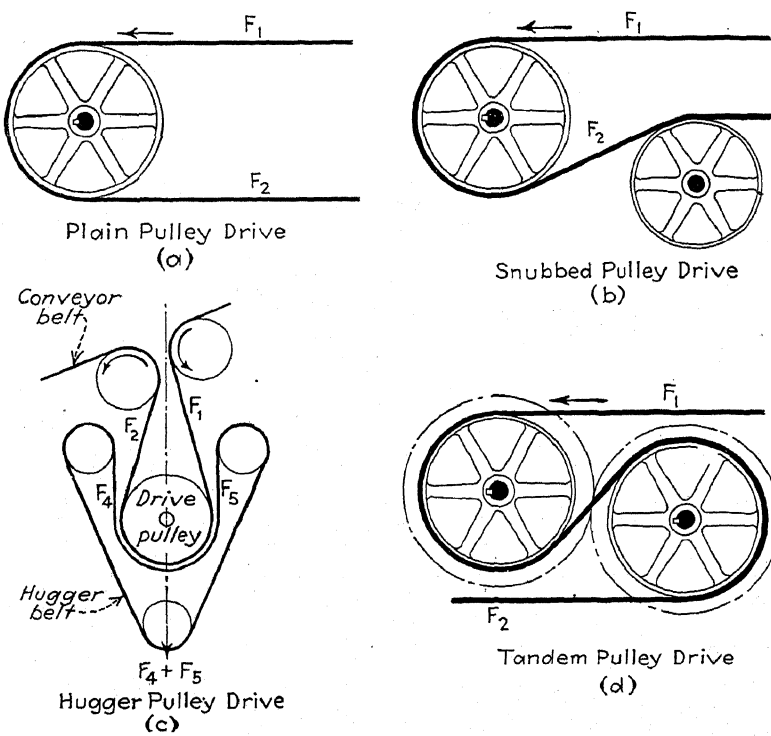


FIG. 232.

The belt must be of sufficient size to transmit the required power without excessive stress, and since it is driven by means of a pulley, just as any power-transmission belt is driven, Eqs. (277) and (278) apply. The driving pulleys are usually cast iron or rubber-covered cast iron and should be at the head or discharge end so that the return belt will not be under high

tension. For light work the drive may be at the foot-pulley if more convenient. Only 180 deg of contact are obtained with the regular head-pulley drive, and when greater contact is required a snubber pulley is used. For still greater power requirements the tandem or the hugger drive is used.

245. Selection of Conveyor-belt Size. The following procedure is suggested for use in determining a suitable belt for a conveyor:

a. Choose the minimum belt width permitted by the material size.

b. Choose the angle of incline.

c. Choose the belt speed.

d. Considering the required capacity and the belt speed, determine the belt width.

e. Select the wider of the belts found in the first and fourth items.

f. Select the type of idlers.

g. Compute the horsepower required.

h. Compute the belt pull and the belt tensions.

i. Assume a unit stress and determine the number of plies to be used in the belt.

j. If the belt is too thick, choose a higher stress, a shallower trough, or a wider belt.

TABLE 69.—MAXIMUM INCLINE OF BELT CONVEYORS

Material Conveyed	Maximum Incline, deg
Briquets and egg-shaped material	12
Wet-mixed concrete	15
Sized coal	18
Washed and screened gravel	18
Loose cement	20
Crushed and screened coke	20
Sand	20
Glass batch	20
Run-of-mine coal	22
Run-of-bank gravel	22
Crushed ore	25
Crushed stone	26
Tempered foundry sand	25
Wood chips	28

It is advisable to use values 4 to 5 deg lower than the maximums.

CHAPTER XVI

ROPE DRIVES

Ropes are round fibrous belts, and as such are especially adapted to the transmission of large amounts of power over distances of 100 to 500 ft. At the present time, the rope drive is not very popular, its field being largely absorbed by the electric transmission of power.

246. Rope Transmission. The English, or multiple-rope, system uses a number of individual ropes running in grooved sheaves and found its chief use in textile mills, flour mills, and rolling mills, where heavy fluctuating loads were transmitted between parallel shafts and where the drive was well protected from the weather. The main objection to this drive was the large number of rope splices and the difficulty of dividing the load evenly among all the ropes. This system had the advantage that the breaking of one or more of the ropes did not require shutting down the plant.

The American, or continuous-rope, system uses a continuous rope wound from groove to groove along the sheaves, and is adapted to drives where the ropes must be guided around corners or between shafts that are not parallel. Tension in the rope is automatically maintained by tension carriages and weights, and guide pulleys are provided to guide the rope leaving the last groove of the driving sheave to the proper position for entering the first groove of the driven sheave. This system gives better service than does the multiple drive when exposed to changeable weather conditions. The main objection to this drive is the complete shutdown required if the rope breaks.

247. Power-transmission Ropes. Manila, hemp, and cotton ropes are made by twisting the fibers into yarns, twisting the yarns together to form strands, and twisting the strands together to form the rope. Small ropes have three strands, but nearly all ropes over $\frac{7}{8}$ in. diameter have four strands. Sizes usually carried in stock are $\frac{3}{4}$, $\frac{7}{8}$, 1, $1\frac{1}{8}$, $1\frac{1}{4}$, $1\frac{3}{8}$, $1\frac{1}{2}$, $1\frac{3}{4}$, and 2 in.

Manila rope is hard and stiff, and, when wedged into the grooves, the fibers cut each other, reducing the strength of the rope. The ultimate strength of Manila rope is generally taken as $7{,}000d^2$ lb when new, where d is the rope diameter in inches. This figure is conservative, the best ropes having strengths of $9{,}000d^2$ lb. To allow for wear, fiber cutting, etc., the permissible working load is taken as $200d^2$ lb, equivalent to an apparent factor of safety of 35 when the rope is new. The weight is approxi-

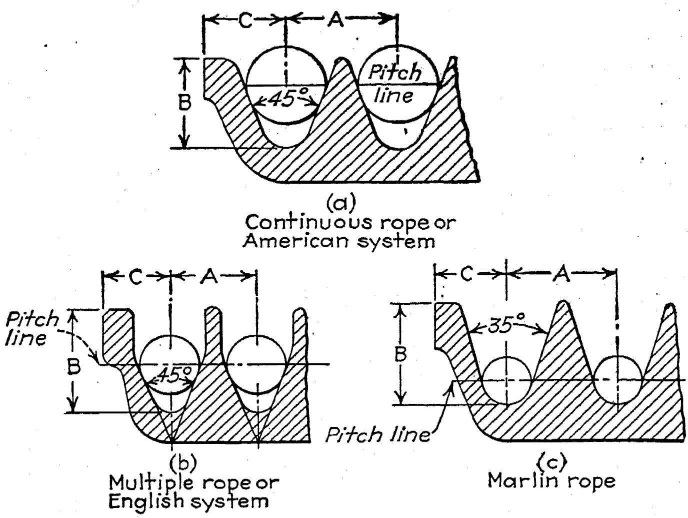


FIG. 233.—Grooves for rope sheaves.

mately $0.33d^2$ lb per ft. The most economical operating speeds are from 4,000 to 5,000 fpm.

Hemp rope is similar to Manila rope but is not so strong or so stiff. The strength is about $6{,}600d^2$ lb, the working load $200d^2$ lb, and the weight $0.33d^2$ lb per ft.

Cotton rope is lighter and more flexible than Manila or hemp rope and can be operated at higher speeds. The strength is about $4{,}600d^2$ lb, the working load $200d^2$ lb, and the weight $0.29d^2$ lb per ft. Operating speeds up to 7,000 fpm are recommended.

248. Power Transmitted by Ropes. The equations developed in Arts. 223 and 224 apply to ropes as well as to belts. Hence

$$\text{hp} = \frac{n(F_1 - F_c)v}{550}\left(\frac{e^{f\theta} - 1}{e^{f\theta}}\right) = \frac{n(F_1 - F_2)v}{550} \tag{286}$$

where F_1 = tension on the tight side, lb.
F_2 = tension on the slack side, lb.
F_c = centrifugal force, wv^2/g, lb.
n = number of ropes.
v = rope velocity, fps.
w = weight of rope, lb per ft.

In the multiple-rope system, the ropes are usually run without initial tension so that F_2 is practically zero.

Equation (286) is not strictly true for the continuous system, since the rope tension gradually decreases as the rope progresses along the sheave.

249. Coefficient of Friction for Ropes. The coefficient of friction of a round rope on a flat cast-iron pulley face varies from 0.15 to 0.30 depending upon the speed, surface finish, and rope condition. For design purposes, 0.12 is a fair value. The rope wedges into the sheave grooves, hence from Eq. (281)

$$f = \frac{f'}{\sin\theta} = \frac{0.12}{\sin 22\frac{1}{2}} = 0.314 \qquad \text{for 45-deg grooves}$$

and

$$f = \frac{0.12}{\sin 30} = 0.240 \qquad \text{for 60-deg grooves}$$

Because of uncertainties in the coefficient of friction and in the arc of contact, it is common practice to assume the term

$$\frac{(e^{f\theta} - 1)}{e^{f\theta}}$$

in Eq. (286) to be $\frac{2}{3}$.

250. Rope Drums. Power drums and sheaves have tapered grooves to provide wedging action and increased traction, but idler and guide sheaves are round. The angle included between the sides of driving grooves is usually 45 deg for both multiple- and continuous-rope drives, although a few manufacturers use 60-deg grooves for the continuous system. The pitch diameter of the sheave is measured to the rope center, about three-fourths of the way up on the tapered groove. For good rope life, the pitch diameter should be not less than $40d$ for Manila rope, and $30d$ for cotton rope.

251. Rope Sag. When the center distance is great, the amount of rope sag between sheaves may cause trouble, and it is

advisable to check the amount of sag. The rope assumes the shape of the catenary curve, but sufficiently accurate results are obtained if the curve is assumed to be parabolic. The sag in ft at the center of a horizontal rope is approximately

$$\text{sag} = \frac{wL^2}{8F} \qquad (287)$$

where F = tension in rope, lb.
L = center distance, ft.
w = weight of rope, lb per ft.

252. Hoisting Tackle. A typical hoisting-block arrangement is shown in Fig. 234. As the rope passes around each sheave, there is a change in rope tension, since some work is absorbed by friction between the sheave and rope and in the sheave bearings. Then

$$\frac{F_1}{F_2} = C, \qquad \frac{F_2}{F_3} = C, \text{ etc.}$$

from which

$$F_2 = \frac{F_1}{C}, \qquad F_3 = \frac{F_1}{C^2}, \text{ etc.}$$

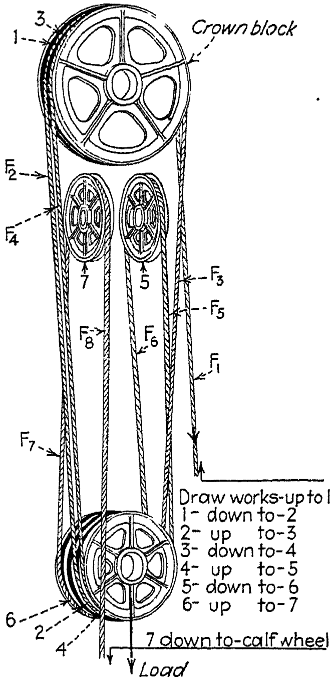


FIG 234.—Rope blocks as used in oil-derrick rigging.

The efficiency of the hoist is the ratio of the work output to the work input, and in this case

$$\begin{aligned} \text{eff} &= \frac{W}{nF_1} = \left(\frac{F_1}{C} + \frac{F_1}{C^2} + \frac{F_1}{C^3} \cdots + \frac{F_1}{C^n}\right)\frac{1}{nF_1} \qquad (288) \\ &= \frac{C^{n-1} + C^{n-2} + C^{n-3} \cdots + C + 1}{nC^n} \\ &= \frac{C^n - 1}{nC^n(C-1)} \end{aligned}$$

where n is the number of times the rope passes over a sheave, and C is a constant depending upon the rope diameter, the sheave diameter, the bearing diameter, and the coefficient of friction. Very few experimental data are available along this line, but

for average conditions with Manila rope, the constant can be assumed to have the following value: 1.15 for poorly lubricated bronze bearings; 1.08 for well lubricated bronze bearings; 1.05 for ball and roller bearings.

The maximum permissible working load for Manila rope used for hoisting purposes is $200d^2$ lb with speeds from 400 to 800 fpm, $400d^2$ lb with speeds from 150 to 300 fpm, and $1{,}000d^2$ lb for speeds below 100 fpm.

253. Wire Ropes. Wire ropes formerly found favor for long-distance transmission of power. Electric transmission has made this use practically obsolete. The chief uses of wire rope at the present time are in elevators, mine hoists, cranes, oil-well drilling, aerial conveyors, tramways, haulage devices, and suspension bridges.

As the requirements for strength and service increased, there followed in order, wrought-iron, cast-steel, extra-strong cast-steel, and plow-steel ropes. For extra-high strength, alloy-steel ropes (known by various trade names) are now available. For certain purposes, ropes are made of aluminum alloys, copper, bronze, and stainless steels.

The various grades of steel wires have minimum ultimate strengths approximately as follows: iron, 85,000; cast steel, 170,000; extra-strong cast steel, 190,000; plow steel, 210,000; and alloy steel, 230,000 psi. The smaller wire sizes have strengths from 10 to 20 per cent higher. The wires are laid in curved form in the rope; hence it is impossible to develop the full strength of the metal in the finished rope, the loss in strength amounting to 5 to 20 per cent. The ultimate strengths* of plow-steel ropes may be approximated by the formulas

$$\begin{aligned} F_u &= 76{,}000d^2 \text{ lb} \qquad \text{for 6 by 7 and 6 by 19 ropes} \qquad (289) \\ &= 75{,}000d^2 \text{ lb} \qquad \text{for 6 by 37 ropes} \end{aligned}$$

where d = diameter of the rope, in.

The designation 6 by 7 indicates that the rope is made of six strands each containing seven wires. The weight of these ropes is approximately $1.58d^2$ lb per ft.

* Tables of rope strengths are given in "Machinery's Handbook," 11th ed., pp 441–445, Industrial Press, and Marks, "Mechanical Engineers' Handbook," 3d ed., pp. 1066–1068, McGraw-Hill Book Company, Inc.

254. Wire-rope Construction. The individual wires are first twisted into strands, and then the strands are twisted around a hemp or steel center to form the rope. The ropes are right- or left-lay, depending on whether the strands form right- or left-hand helixes. Most rope is right-lay. Regular-lay rope has the wires twisted opposite to the strands and is standard construction in this country. Lang-lay rope has the wires and strands

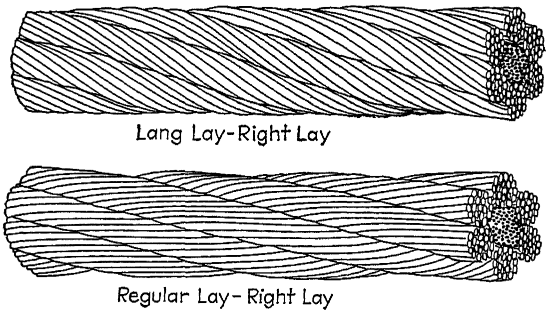


Fig. 235.

twisted in the same direction, giving a rope with a better wearing surface, but it is harder to splice and twists more easily when loaded.

The most common rope constructions are illustrated in Fig. 236, each construction being designed for particular properties. By varying the construction, the metal may be distributed to give maximum wear, maximum flexibility, or any desired intermediate quality. The standard constructions are 6 by 7 or coarse lay; 6 by 19 or flexible; and 6 by 37, together with 8 by 19, or extra flexible.

In making a selection, one must consider flexibility, wear resistance, strength, reserve strength, core strength, and corrosion resistance. For equal diameters, the use of a large number of small wires gives a rope of high flexibility. Increasing the wire diameter and decreasing the number of wires reduces the flexibility. When extreme flexibility without extreme strength is required, tiller rope (6 by 6 by 7) is used.

Large wires give better wear resistance. Two sizes of wire laid alternately in the outer layer increase the wearing qualities and retain the flexibility. For severe service, steel-clad rope is

used. This consists of a regular rope with each strand wrapped with a thin flat strip of steel to protect the wires. The strength of the rope is not increased, but its life is lengthened, since after the covering is worn through, the regular rope is still intact, and the metal cover, being forced down between the strands, presents more wearing surface.

The strength does not depend on the rope construction but is due entirely to the material from which the wires are made. Of course, the flexibility that is obtained with small wire sizes

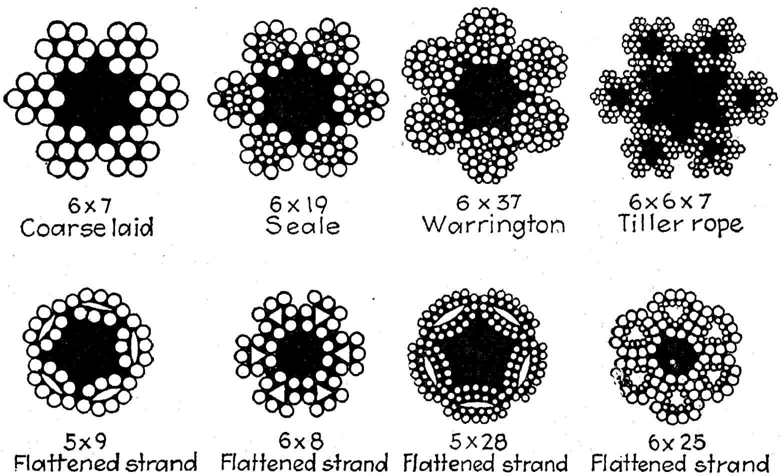


FIG. 236.—Types of wire-rope construction.

increases the fatigue strength. By reserve strength is meant the ratio of the strength of all the inside wires to the strength of all the wires in the rope. The outside wires are subject to service wear and the inside wires are protected, forming the reserve capacity that can be relied upon throughout the life of the rope. This consideration is important in choosing ropes for service in which human life is involved. The coarse-lay ropes, being made of large wires with few inner or protected wires, have the least reserve strength, and ropes like the modified Seale and the Warrington have the highest reserve strength.

255. Corrosion Resistance. Corrosion is chiefly due to moisture and stray electric currents and may be effectively overcome by using suitable rope lubricants. Galvanized ropes are suitable for some purposes, but since the zinc flakes off easily,

it is not suitable for ropes running over sheaves or those which are frequently bent. Copper, bronze, and stainless steels are sometimes used where corrosion resistance is a major consideration.

256. Bending Stresses.* As the rope passes around the drums and sheaves, the wires on the outside increase in length, and those on the inside decrease, this action producing additional tension in the outer wires. The rope does not bend as a solid bar, but there is a movement and rearrangement of the wires, this movement varying with the rope construction, the wire size, the type of center, and the amount of pinching or restraint in the grooves. It is evident that these items vary with the rope and installation, and that no mathematical formula based on a series of assumptions at static conditions can give the true bending stress under all operating conditions.

There are eight or more bending-stress formulas in use, giving a wide variety of results. Only the simplest of these is given here, as it seems to agree fairly well with experimental data available. An experienced designer of rope installations will modify the results according to his experience with similar installations.

The bending stress in the outer wire is expressed by the formula

$$s_b = \frac{E_r d_w}{D} \tag{290}$$

where s_b = bending stress, psi.
E_r = modulus of elasticity of the rope, psi of wire area.
d_w = wire diameter, in.
D = drum or sheave diameter, in.

The value E_r is not the modulus of elasticity of the wire material, but of the entire rope. Tests and theoretical investigations by J. F. Howe indicate that for steel ropes of the ordinary constructions the value of E_r may be taken as 12,000,000.

The value of the wire diameter d_w depends on the rope construction. For preliminary computations, the wire diameter and the total cross-sectional area of the metal in a rope may be taken as follows:

* For complete discussions on bending stresses the reader is referred to James F. Howe, Determination of Stresses in Wire Rope as Applied to Modern Engineering Problems, *Trans. A.S.M.E.*, Vol. 40, p. 1043, 1918, and to "Wire Engineering," John A. Roebling's Sons, October, 1932.

Rope	d_w	A
6 by 7	$0.106d$	$0.38d^2$
6 by 19	$0.063d$	$0.38d^2$
6 by 37	$0.045d$	$0.38d^2$
8 by 19	$0.050d$	$0.35d^2$

In most cases, it is more convenient to convert the bending stress into the equivalent bending load, *i.e.*, the direct tension load that would produce the same wire stress. This equivalent bending load, in pounds, is

$$F_b = A\,\frac{E_r d_w}{D} \tag{291}$$

257. Starting Stresses. When starting and stopping, the rope and the supported load must be accelerated by a force transmitted through the rope. In hoisting and elevator service where the acceleration may be as high as 10 ft per sec² the additional load becomes an important item. If, when the rope drum begins to rotate, there is any slack rope to be taken up before the load is moved, there will be considerable impact load on the rope. This impact may be determined by the usual impact equations if the acceleration of the rope is known, so that the velocity at the instant of impact can be determined. However, a rope will stretch much more than a solid bar, because of the twisted lay of the wires, and this condition relieves the impact effect to some extent. Computations for the impact load are of no practical value because of the unknown factors and the designer must use his judgment based on experience in cases of this kind.

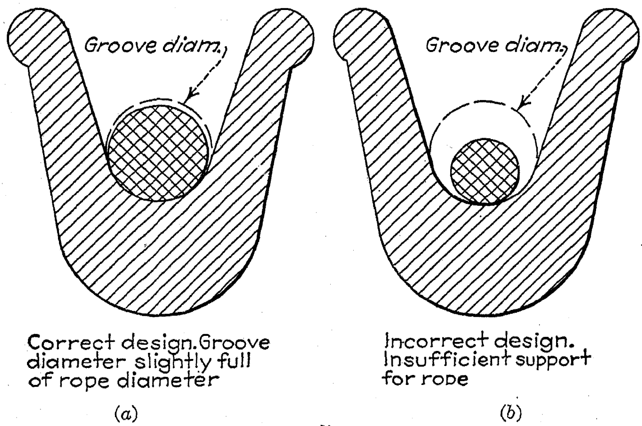
258. Sheave and Drum Diameters. On account of the bending stresses where the rope wraps around the drum, it is important that the drum diameters be kept fairly large. This is especially true with high-speed ropes in continuous service, where the fatigue action will materially affect the life of the rope. Practice varies in regard to the proper diameter, but the values recommended as standard practice are given in Table 70. Where larger diameters are possible, their use will give better and more economical service. Space requirements may lead to the use of sheaves smaller than those given in the table.

TABLE 70.—COMMON WIRE-ROPE APPLICATIONS

Type of service	Type of rope	Sheave diameter	
		Recommended	Minimum
Haulage rope: Mine haulage Factory-yard haulage Inclined planes Tramways Power transmission Guy wires	6 by 7	72*d*	42*d*
Standard hoisting rope: (most commonly used rope) Mine hoists Quarries Ore docks Cargo hoists Car pullers Cranes Derricks Dredges Tramways Well drilling Elevators	6 by 19	45*d* 60–100*d* 20–30*d*	30*d*
Extra-flexible hoisting rope	8 by 19	31*d*	21*d*
Special flexible hoisting rope: Steel-mill ladles Cranes High-speed elevators Service where sheave diameters are limited	6 by 37	27*d*	18*d*

259. Sheave Grooves. The contour of the sheave groove has a great influence on the life and service of the rope. If the groove has a bottom radius much larger than the rope, there will be insufficient support for the rope, which will flatten out from its normal circular section. This tends to increase the fatigue effects. With too small a bottom radius, the rope will be wedged into the groove and the normal rotation of the rope will be prevented. This concentrates the wear along two lines parallel to the axis instead of distributing it around the entire circum-

ference. The rope center will also be distorted and premature breaking of the wires will occur in the valleys between the strands. The correct design of the groove is shown in Fig. 237. This groove gives support to the rope on nearly half its circumference.



$$\text{Correct groove radius} = \frac{\text{rope diam.}}{2} \begin{cases} + \frac{1}{64}'' \text{ for } \frac{1}{4} \text{ and } \frac{5}{16} \text{ ropes} \\ + \frac{1}{32}'' \text{ " } \frac{3}{8} \text{ to } 1\frac{1}{4} \text{ ropes} \\ + \frac{1}{16}'' \text{ " } 1\frac{3}{8} \text{ to } 2\frac{1}{4} \text{ ropes} \end{cases}$$

FIG. 237.

260. Winding the Rope. There are four common methods of imparting motion to the rope: winding drums, winding machines, friction spools, and grip wheels.

When winding drums are used, the end of the rope is attached to the drum. The rope should always wrap once or more around the drum so that the rope is held by friction instead of by direct pull on the fastener where the rope will not sustain its full breaking strength. Cylindrical drums, either plain or with guide grooves, are the most commonly used in all classes of service, and have the advantage that several layers of rope may be wound on them, thus reducing the size required. Conical drums and combined conical and cylindrical drums are used with deep mine hoists to balance the changing torque due to the decreasing weight of the rope as the hoist rises.

Since the rope is wound under tension, the drum is subjected to external crushing loads as well as bending loads, and the body should be designed as a thick cylinder subjected to external

pressure. Hoists fitted with conical and plate clutches may also have axial loads imp essed on the drums. Drum flanges are usually subjected to tangential loads due to the brake, axial or tangential loads due to the clutch, and axial loads due to the side pressure of the ropes when wound in several layers. The complete design of drums can not be discussed here, but those interested are referred to the work of Everett O. Waters.*

261. Idlers and Guide Sheaves. Rollers and sheaves used to support and guide the rope should be of generous proportions to reduce bending and fatigue effects. Since the forces involved are usually less than on the driving sheaves, the diameters may be reduced. They should be grooved at the center and wide enough to prevent the rope moving off sideways.

262. Rope Fasteners. Spliced wire rope should be avoided, and, where it is used, the permissible working load should be reduced to about 75 per cent of the working load on unspliced ropes.

Several types of rope fasteners are shown in the accompanying illustrations. Of these, the wire-rope socket, Fig. 238*e*, is the only one that will develop 100 per cent of the rope strength. The wires at the end are separated, the hemp center is removed, and the wires are cleaned, dipped in muriatic acid, inserted in the socket, evenly distributed, and anchored in place by filling the socket with high-grade molten zinc. If properly made, the joint will be as strong as the rope. Babbitt, lead, and other antifriction metals are sometimes substituted for the zinc, but these do not bond perfectly with the wires so that such a joint will not develop full rope strength.

When thimbles or eye splices are used, the full rope strength can not be developed, such joints having the efficiencies indicated in Table 71.

TABLE 71.—EFFICIENCIES OF WIRE-ROPE FASTENERS

Method of Fastening	% of Total Rope Strength
Wire-rope socket with zinc	100
Thimble with four or five wire tucks	90
Special offset thimble with clips	90
Regular thimble with clips	85
Three-bolt wire clamps	75

* WATERS, E. O., Rational Design of Hoisting Drums, *Trans. A.S.M.E.*, Vol. 42, p. 463, 1920.

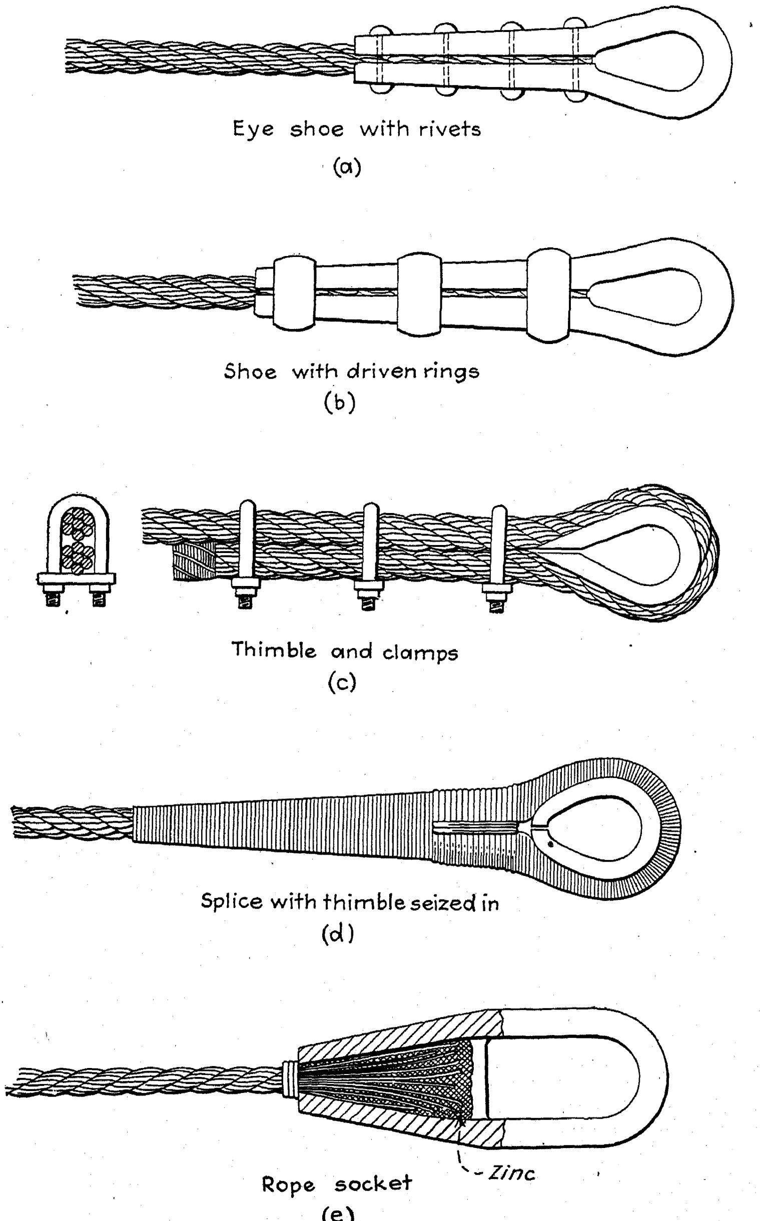

Eye shoe with rivets
(a)
Shoe with driven rings
(b)
Thimble and clamps
(c)
Splice with thimble seized in
(d)
Zinc
Rope socket
(e)

FIG. 238.

TABLE 72.—FACTORS OF SAFETY FOR WIRE ROPES
(Based on Ultimate Strength)

Service	Factor of Safety
Elevators	8 –12
Mine hoists	2 5– 5
Cranes, motor driven	4 – 6
hand power	3 – 5
Derricks	3 – 5

263. Examples of Wire-rope Selection. To bring out the principles of rope applications, several examples with solutions are presented.

Example 1. Vertical Shaft Hoist. Select a wire rope for a vertical mine hoist to lift 1,200 tons of ore in each 8-hr shift from a depth of 2,400 ft. Assume a two-compartment shaft with the hoisting skips in balance.

Solution. Several combinations of rope velocity, loading, etc., could be used, and, in practice, several combinations would be worked out and the proper combination selected after due consideration of the effect of each on the hoist design and power requirements. Calculations for only one combination are given here.

A maximum rope velocity of 2,500 fpm with acceleration and deceleration periods of 12 sec each, and a rest period of 10 sec for discharging and loading are assumed.

During the acceleration period, the skip travels

$$S_a = \frac{vt}{2} = \frac{2{,}500 \times 12}{60 \times 2} = 250 \text{ ft}$$

with an acceleration of

$$a = \frac{2{,}500}{60 \times 12} = 3.47 \text{ ft per sec}^2$$

The total distance traveled at full speed is 2,400 ft less $2S_a$, or 1,900 ft, and the time required is

$$t = \frac{1{,}900 \times 60}{2{,}500} = 45.6 \text{ sec}$$

The time required for one trip is

	Second
Acceleration	12
Full speed	45.6
Deceleration	12
Discharging and loading	10
Total	79.6

This allows 45 trips per hour and requires a skip of 3.3 tons, say 3.5 tons capacity. A hoisting skip weighs approximately 0.6 of its load capacity, or, in this case, about 2 tons, making a total load of 5.5 tons to be hoisted.

The rope selected must have strength sufficient to support the load, the rope weight, the accelerating load, and the bending load with a factor of safety of approximately 5. As there are several unknown quantities involved, trial diameters must be assumed, and later modified if not satisfactory. Try a $1\frac{1}{4}$ in. 6 by 19 plow-steel rope, weighing 2.47 lb per ft and having an ultimate strength of 59.4 tons. Average mine-hoist practice is to use drums 60 to 100 times the rope diameter. Assume the rope drum to be 9 ft. 6 in. in diameter.

Then the loads are

	Tons
Useful load	3.5
Weight of skip	2.0
Weight of rope, $\dfrac{2{,}400 \times 2.47}{2{,}000}$	2.96
Acceleration of load, $\dfrac{(3.5 + 2)3.47}{32.2}$	0.59
Acceleration of rope, $\dfrac{2.96 \times 3.47}{32.2}$	0.32
Equivalent bending load	2.47
Total	11.84

The factor of safety for this rope is

$$FS = \frac{59.4}{11.84} = 5.02$$

which is slightly greater than the desired value; thus the rope is satisfactory.

Example 2. Inclined Shaft Hoist. Select a wire rope for an inclined shaft whose length is 2,400 ft, at a 60 per cent slope, with a loaded skip weighing 22,000 lb, a rope velocity of 2,000 fpm, an acceleration period of 10 sec, and a factor of safety of 5.

Solution. Assume a $1\frac{1}{8}$ in. 6 by 19 plow-steel rope, weighing 2.00 lb per ft and having an ultimate strength of 48 tons. Assume the rope drum to be 8 ft in diameter.

Since the hoist operates on an incline, the friction of the skip and of the wire supported on track rollers must be overcome by the rope pull. The car friction will amount to about 50 lb per ton of weight normal to the incline, using bronze bushings in the wheels. The friction of the rope on proper track rollers is about 100 lb per ton of weight normal to the incline.

From the data

$$a = \frac{2{,}000}{60 \times 10} = 3.33 \text{ ft per sec}^2$$

$$\text{Rope weight} = 2{,}400 \times 2 = 4{,}800 \text{ lb}$$

and the angle of incline is 30 deg 58 min.

The loads on the rope are

	Tons
Weight of skip and load, $\frac{22{,}000 \sin \theta}{2{,}000}$	5 66
Weight of rope, $\frac{4{,}800 \sin \theta}{2{,}000}$	1 23
Skip friction, $\frac{50 \times 22{,}000 \cos \theta}{2{,}000 \times 2{,}000}$	0.24
Rope friction, $\frac{100 \times 4{,}800 \cos \theta}{2{,}000 \times 2{,}000}$	0 10
Acceleration, $\frac{(22{,}000 + 4{,}800)3.33}{2{,}000 \times 32.2}$	1.39
Equivalent bending load	2.13
Total	10.75

The factor of safety with this rope is

$$FS = \frac{48}{10.75} = 4.46$$

which is less than the required factor of 5. A $1\frac{1}{4}$-in. rope should now be assumed, and the calculations repeated. This calculation is left to the student.

Example 3. High-speed Passenger Elevator. Select a wire rope for the elevator in a building where the total lift is 600 ft, the rope velocity 900 fpm, and the full speed is to be reached in 40 ft. The lifting sheaves are to be of the traction type. The cage will weigh 2,500 lb and the passengers 2,000 lb.

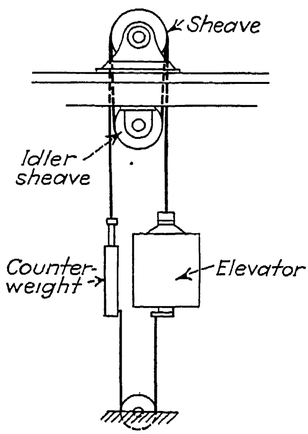


FIG. 239.

Solution. State laws generally require at least 4 ropes and a factor of safety of 8 on passenger elevators. The total weight to be lifted is 4,500 lb, and allowing the same amount for the weight of the rope, acceleration, and bending loads, the total required strength is $2 \times 4{,}500 \times 8$ or 72,000 lb, which is 18,000 lb per rope.

Elevator ropes are usually 6 by 19 construction, although constructions like the Special Seale are being used on the longest high-speed elevators. For this elevator, assume $\frac{1}{2}$-in. 6 by 19 plow-steel ropes, weighing 0.39 lb per ft and having an ultimate strength of 10 tons. The head-shaft sheaves for a $\frac{1}{2}$-in rope should be about 36 in. in diameter.

The rate of acceleration is

$$a = \frac{v^2}{2S} = \frac{900^2}{2 \times 40 \times 3{,}600} = 2.81 \text{ ft per sec}^2$$

Then the rope loads are

	Tons
Elevator and passengers, $\dfrac{4{,}500}{4 \times 2{,}000}$	0 563
Weight of rope, $\dfrac{0.39 \times 600}{2{,}000}$	0 117
Acceleration of load, $\dfrac{4{,}500 \times 2\ 81}{4 \times 2{,}000 \times 32.2}$	0 049
Acceleration of rope, $\dfrac{0.117 \times 2.81}{32.2}$	0 010
Equivalent bending load	0 497
Total	1 236

The factor of safety with this rope is

$$FS = \frac{10}{1.236} = 8.10$$

which is slightly greater than the factor desired.

Since this is a traction-type elevator, the number of wraps of rope around the driving sheaves must be determined. The driving-sheave layout will be similar to Fig. 239. The counterweight is usually made sufficient to balance the elevator plus one-third the live weight, making the counterweight, in this case, 3,167 lb. When the elevator is at the bottom, and the counterweight at the top, the rope tensions are

$$F_1 = \frac{2{,}500 + 2{,}000}{4} + 234 + 98 + 20 = 1{,}477 \text{ lb per rope}$$

$$F_2 = \frac{3{,}167}{4}\left(1 - \frac{2.81}{32.2}\right) = 723 \text{ lb}$$

and

$$F_1 - F_2 = 754 \text{ lb}$$

which must be supplied by sheave friction. The arc of contact is 180 deg and the coefficient of friction for greasy wire rope on round-bottom cast-iron sheaves is about 0.18 to 0.23 with hard-rubber groove linings. Hence

$$\frac{F_1}{F_2} = e^{f\theta} = 2.718^{0\ 18 \times 3\ 14} = 1.76$$

and

$$F_2 = \frac{F_1}{e^{f\theta}} = \frac{1{,}477}{1.76} = 843 \text{ lb}$$

from which

$$F_1 - F_2 = 634 \text{ lb} \qquad \text{for one wrap around the sheave}$$

Then the required number of wraps is

$$n = \frac{754}{634} = 1.19, \text{ say 2 wraps}$$

The number of wraps required should also be checked with the elevator in its top position.

CHAPTER XVII

HOISTING AND POWER CHAINS

In the construction of machines there are three general services for which chains are suitable: hoisting, conveying, and power transmission. For each class of service, several types of chain are available.

264. Hoisting Chains. Coil chains, Fig. 240, are made of wrought iron or of soft steel with the links welded together,

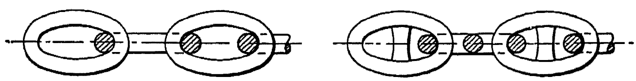


FIG. 240.

or rolled weldless from a cross-shaped rolled bar. The stud-link chain is used to prevent kinking of the chain. The stud increases the strength of the chain at the elastic limit about 20 per cent, but decreases the breaking strength. The strengths* of hoisting chains, in terms of the bars from which they are made, can be estimated as follows:

Standard close link	138% of the bar
Coil chain	120% of the bar
BB crane chain	145% of the bar
Stud chain	165% of the bar

For the ordinary steel coil chains, the working load is as follows:

Common coil chain	$12,300d^2$ lb
BB crane chain	$13,600d^2$ lb

where d is the chain size, *i.e.*, the diameter of the rod from which

* Tables of chain strengths are given in "Machinery's Handbook," 11th ed., pp. 882–884, Industrial Press, New York, and Marks, "Mechanical Engineers' Handbook," 4th ed., p. 996, McGraw-Hill Book Company, Inc

it is made, in inches. The proof test load is twice this value, and the breaking load approximately four times this value.

The drums or sheaves on which coil chains operate are usually grooved or pocketed as shown in Figs. 241 and 242.

265. Power Chains. For low speeds where the loads are not great, detachable chain is used. The links are usually made of malleable cast iron or manganese steel, cast in one piece with no separate bushings or pins at the joints. Because of the open construction at the joint, links are readily removed from, or added to, the chain. For conveyor and elevator service, this type of chain is furnished with special flanges and other attachments to provide connections to the other parts of the conveyors. Detachable chain is suitable for power transmission up to speeds of 400 fpm.

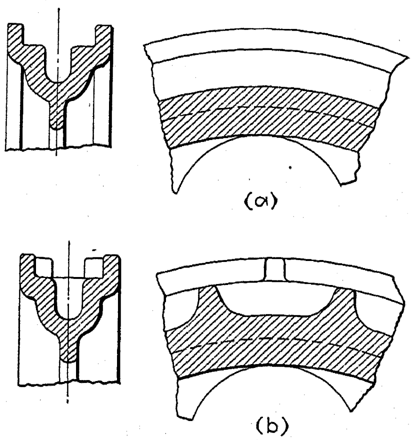


FIG. 241.

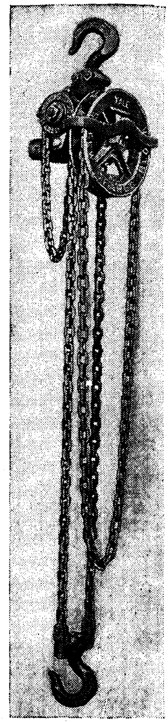


FIG. 242.

When the chains are exposed to grit, closed-end pintle chains are preferable to the detachable chains. The two types are interchangeable. The pins at the joints are either riveted over or made removable so that chain links may be removed. Closed-end pintle chains are slightly stronger than detachable chains and are suitable for speeds up to 400 fpm.

Bushed closed-end pintle chains are made of malleable iron with casehardened steel pins and bushings, the pins being either

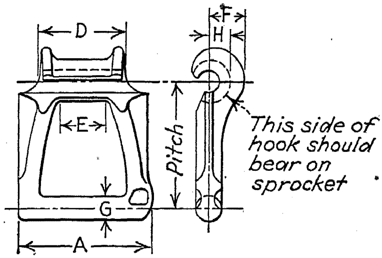


FIG. 243.—Ewart detachable chain.

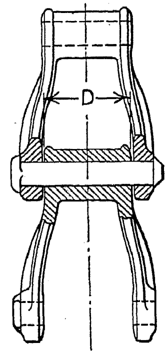


FIG. 244.—Closed-end pintle chain.

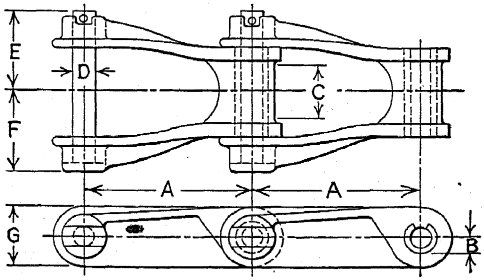


FIG. 245.—Bushed pintle chain.

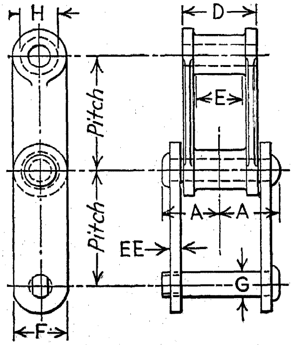


FIG. 246.—Combination chain.

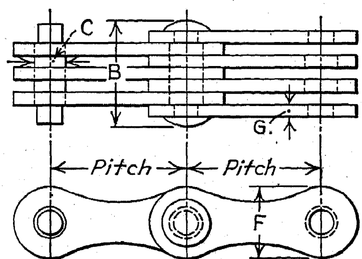


FIG. 247.—Leaf or balance chain.

riveted over or made removable. The bushed chain is suitable for power transmission under severe service conditions at speeds up to 400 fpm.

All of the chains mentioned are cast and operate on cast-tooth sprockets. Their pitch is only approximate. Where more accurate chains are required, block, roller, and silent chains are used. Roller chains are made with steel side bars, with hardened steel bushings and pins. In the cheaper grades, the rollers are malleable iron or cast steel. This type (Fig. 248) is suitable for power transmission at speeds up to 600 fpm. The chains shown in Figs. 249 and 250 have alloy-steel side plates

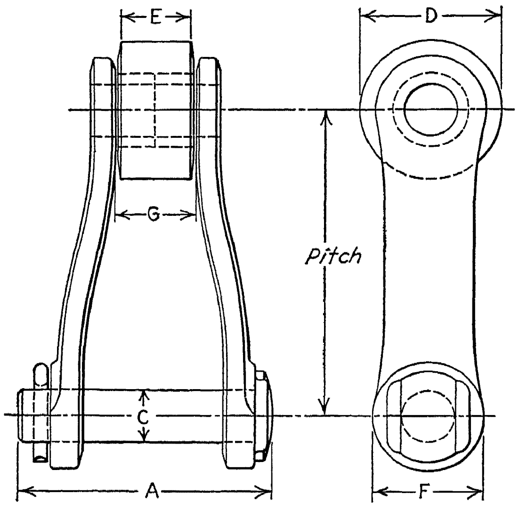


FIG. 248.—Malleable-iron roller chain.

and heat-treated steel rollers and are suitable for heavy duty and shock loading at speeds up to 700 fpm.

The highest grade roller chains, usually referred to as finished-steel roller chains, are used for accurate timing, for the transmission of power, and as the basic members for many types of conveyors. Dimensions of these chains are shown in Table 73. The individual parts of these chains—pins, bushings, rollers, and link plates—are made of special alloy steels, selected, processed and heat-treated so that each part may function with maximum efficiency in its own field as a tension member or as a bearing member. It is characteristic of roller chain that no single part is required to resist both tension and wear.

266. Speed and Sprocket Limits.* Linear speeds of these chains are not limited except by the number of teeth and the rotative speed of the smallest sprocket in the drive. The shorter

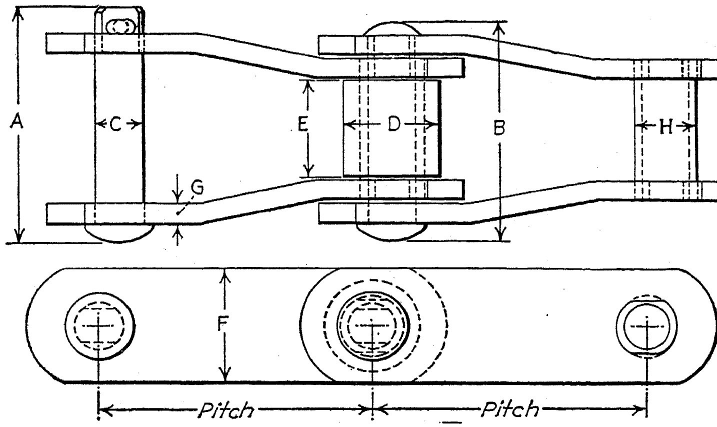


Fig. 249.—Steel roller chain, offset links.

the pitch, the higher the permissible rotative speed, with speeds as high as 8,800 rpm approved for $\frac{1}{4}$ in. pitch chains.

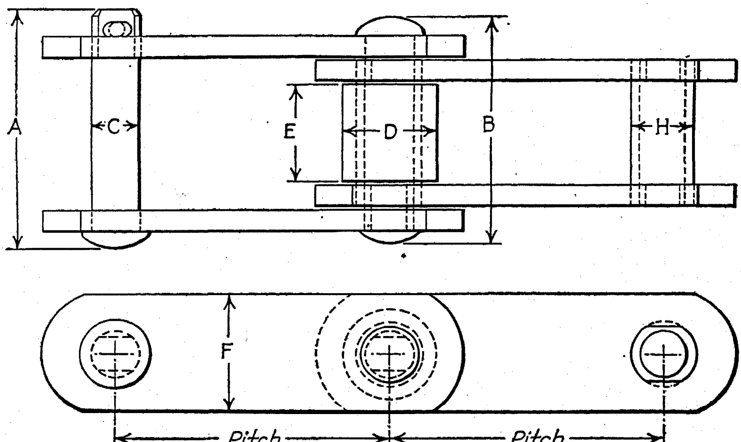
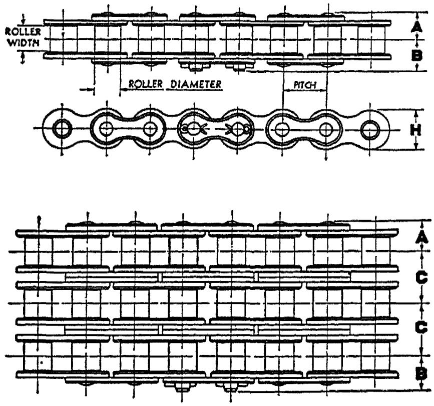


Fig. 250.—Steel roller chain, straight links.

Sprockets with fewer than 16 teeth may be used for relatively slow speeds, but 18 to 24 teeth are recommended as minima for

* Material on finished-steel roller chains, chain sprockets and chain selection furnished by H. G. Taylor, Diamond Chain and Manufacturing Company, Indianapolis, Ind.

TABLE 73.—FINISHED STEEL ROLLER CHAINS



Chain No.		Pitch p	Roller		Pin diameter	Roller link plate		Transverse dimensions			Tensile strength, lb per strand	Weight, lb per foot	Recommended maximum rpm		
													Teeth		
A S A.	Diamond		Width w	Diameter d		Thickness t	Height H	A	B	C			12	18	24
25	89	$\frac{1}{4}$	$\frac{1}{8}$	0 130	0 0905	0 030	0 226	0 149	0 188	0 254	875	0 084	5,800	7,800	8,880
35	82	$\frac{3}{8}$	$\frac{3}{16}$	0 200	0 141	0 050	0 344	0 224	0 290	0 400	2,100	0 21	2,380	3,780	4,200
41	65*	$\frac{1}{2}$	$\frac{1}{4}$	0 306	0 141	0 050	0 383	0 256	0 315		2,000	0 26	1,750	2,725	2,850
40	66	$\frac{1}{2}$	$\frac{5}{16}$	0 312	0 156	0 060	0 452	0 313	0.358	0 563	3,700	0 41	1,800	2,830	3,000
50	449*	$\frac{5}{8}$	$\frac{3}{8}$	0 400	0 200	0 080	0 594	0 384	0 462		6,100	0 68	1,300	2,030	2,200
50	148	$\frac{5}{8}$	$\frac{3}{8}$	0 400	0 200	0 080	0 545	0 384	0 462	0 707	6,600	0 65	1,300	2,030	2,200
60	433	$\frac{3}{4}$	$\frac{1}{2}$	0 469	0 234	0 094	0 679	0 493	0 567	0 892	8,500	0 99	1,025	1,615	1,700
80	434	1	$\frac{5}{8}$	0 625	0 312	0 125	0 903	0 643	0 762	1 160	14,500	1 73	650	1,015	1,100
100	470	$1\frac{1}{4}$	$\frac{3}{4}$	0 750	0 375	0 156	1 128	0 780	0 910	1 411	24,000	2 51	450	730	850
120	472	$1\frac{1}{2}$	1	0 875	0 437	0 187	1 354	0 977	1 123	1 796	34,000	3 69	350	565	650
140	474	$1\frac{3}{4}$	1	1 000	0 500	0 220	1 647	1 054	1 219	1 929	46,000	5 00	260	415	500
160	478	2	$1\frac{1}{4}$	1 125	0 562	0 250	1 900	1 250	1 433	2 301	58,000	6 53	225	360	420
200	480	$2\frac{1}{2}$	$1\frac{1}{2}$	1 562	0 781	0 312	2 275	1 533	1 850	2.800	95,000	10.65	170	260	300

Courtesy of Diamond Chain and Mfg Co

* Not made in multiple strands. All dimensions are in inches.

the higher speeds. Ordinarily, sprockets with fewer than 25 teeth, running at speeds above 500 rpm, should be heat-treated to have a tough, wear-resistant surface of 35 to 45 Rockwell C hardness.

If the speed ratio requires a large sprocket with more than 120 teeth, or with more than eight times the number of teeth on the small sprocket, it is advisable to make the desired speed change

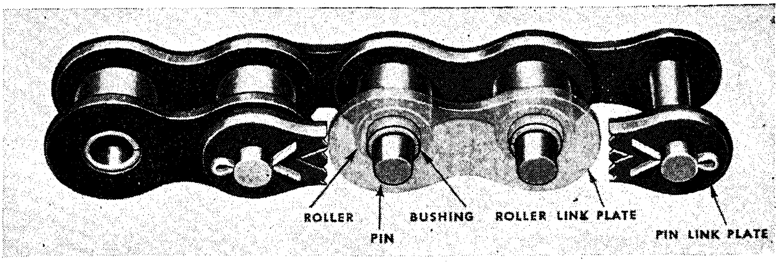


FIG. 251.—Construction of roller chain. (*Courtesy Diamond Chain and Manufacturing Company.*)

in two or more steps. Comparatively small numbers of teeth account for a faster rate of wear in the articulating joints of the chain, and comparatively large numbers of teeth allow the chain to top the teeth before the chain is elongated enough to be unsuitable for further service on sprockets with fewer teeth. The rate of wear and resultant elongation of chains vary directly with the angle of articulation in the pin-bushing joint. This angle is inversely proportional to the number of teeth and is equal to 360 deg divided by the number of teeth.

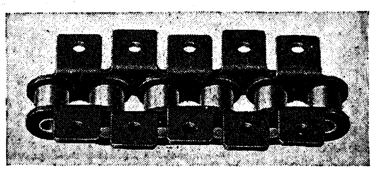


FIG. 252.—Transmission roller chain. (*Courtesy Diamond Chain and Manufacturing Company.*)

The American Standards Association tooth form (Fig. 255) allows the chain to adjust itself to a larger effective pitch polygon as the chain elongates. To determine the allowable elongation, it is assumed that the diameter of the maximum effective pitch circle followed by the elongated chain is equal to the nominal pitch diameter plus the diameter of the chain roller. The A.S.A. roller diameter is five-eighths of the pitch. Based upon these figures, the theoretical allowable elongation, in percentage, will be approximately 200 divided by the number of teeth. For 200 teeth, therefore, the percentage of allowable elongation is 1 per cent; for 133 teeth, $1\frac{1}{2}$ per cent; for 100 teeth, 2 per cent; and for 57 teeth, $3\frac{1}{2}$ per cent. Chains which have elongated $3\frac{1}{2}$ per cent should be replaced.

In combining the effects of small and large numbers of teeth, it may be concluded that the service life of a chain on 12- and 60-tooth sprockets, for example, will be about the same as for 24 and 120 teeth if the rotative speeds and chain pulls were the same in each case, the greater wear and elongation due to the 12-tooth sprocket being counterbalanced by the greater allowable elongation on the 60-tooth sprocket. A multiple chain, four strands wide on the smaller sprockets, would have practically the same power capacity as a double-strand chain of the same pitch

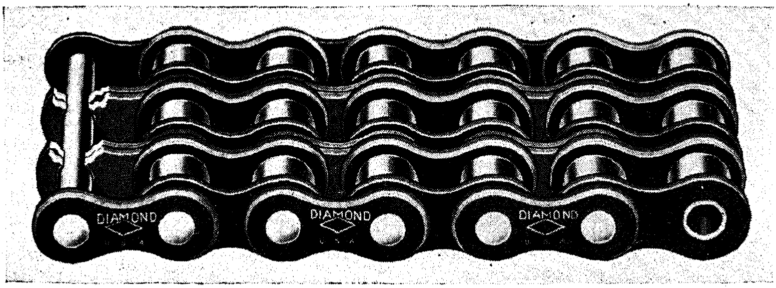


FIG. 253.—Multiple-strand roller chain. (*Courtesy Diamond Chain and Manufacturing Company.*)

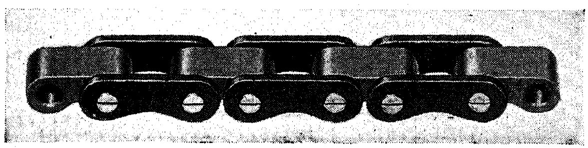


FIG. 254.—Block chain. (*Courtesy Diamond Chain and Manufacturing Company.*)

on the larger sprockets. The four-strand drive would require less space radially, but the double-strand drive would have all other advantages—less axial width, smoother action, quieter operation, less load on shaft bearings due to only half the chain pull, higher permissible rotative speed, equivalent or slightly lower cost, and the lower rate of wear on the larger number of teeth will offset the lower allowable percentage of elongation.

Idler sprockets may be meshed with either face of standard roller chain, to take up slack, to guide the chain clear of obstructions, to obtain direction of rotation opposite to that of the driver sprocket, or to provide a longer arc of chain wrap on another sprocket. Idlers should not turn faster than the rotative

speeds recommended as maxima for other sprockets with the same number of teeth. It is desirable that the idlers have at least three teeth in mesh with the chain, preferably with the idle span of the chain. Idler sprockets should have accurately machined teeth, the same as other sprockets. Plain disks, plain or grooved cylinders, and flanged pulleys should not be used, especially those which would allow contact with the edges of the link plates.

267. Chain Selection. Horsepower ratings for two-sprocket drives are based upon the number of teeth and the rotative speed of the smaller sprocket, either driver or follower. The pin-bushing bearing area, as it affects the allowable working load, is the important factor for medium and high speed drives. Due to the relationship between lightness and tensile strength of roller chains, the effect of centrifugal force does not need to be considered. Even at the unusual speed of 6,000 fpm, the added tension in the chain due to centrifugal force is only 3 per cent of the ultimate tensile strength. The permissible working load, as suggested by the *A S.A. Publications* B29a, 1936 ed., may be calculated by using the empirical formula

$$F_w = \frac{2{,}600{,}000A}{V + 600} \tag{292}$$

where F_w = permissible load, lb.
A = projected bearing area of the pin-bushing joint, sq in.
V = chain velocity, fpm.

For standard roller chain, the pin diameter is five-sixteenths of the pitch, or one-half of the roller diameter, and the bushing length is seven-eighths of the pitch, or roller length (five-eighths of pitch) plus two times the link plate thickness (one-eighth of pitch). Therefore

$$A = \frac{5p}{16} \times \frac{7p}{8} = 0.273p^2. \tag{293}$$

Using accurately machined sprockets, a higher permissible working load may be used, or

$$F_w = \frac{3{,}240{,}000A^*}{V + 617} \tag{294}$$

* Practice of the Diamond Chain and Manufacturing Company, Indianapolis, Ind.

TABLE 74.—HORSEPOWER RATINGS FOR SINGLE-STRAND ROLLER-CHAIN DRIVES

A.S.A. No. 25, ¼ in. pitch, Diamond No. 89

(Rpm of sprocket)

Teeth	200	400	600	800	1,200	1,800	2,400	3,000	4,000	5,000	6,000	8,000
12	0.10	0.19	0.27	0.33	0.45	0.58	0.67	0.73	0.78	0.73		
15	0.12	0.24	0.35	0.43	0.60	0.79	0.94	1.04	1.14	1.20	1.14	
18	0.15	0.29	0.42	0.53	0.74	0.98	1.17	1.32	1.49	1.56	1.56	
21	0.18	0.36	0.50	0.62	0.86	1.16	1.38	1.57	1.77	1.86	1.88	1.66
24	0.21	0.40	0.57	0.71	0.98	1.31	1.58	1.78	2.02	2.12	2.14	1.89

A.S.A. No. 35, ⅜ in. pitch, Diamond No. 82

(Rpm of sprocket)

Teeth	200	400	800	1,200	1,600	2,000	2,400	2,800	3,200	3,600	4,000	4,500
12	0.34	0.60	1.01	1.31	1.53	1.66	1.72	1.73				
15	0.43	0.78	1.35	1.78	2.12	2.37	2.54	2.65	2.70	2.69		
18	0.52	0.96	1.65	2.21	2.65	2.98	3.24	3.43	3.52	3.57	3.55	
21	0.61	1.12	1.95	2.61	3.14	3.53	3.86	4.08	4.22	4.28	4.28	
24	0.70	1.28	2.22	2.98	3.57	4.04	4.38	4.65	4.81	4.86	4.87	4.75

A.S.A. No. 40, ½ in. pitch, Diamond No. 66

(Rpm of sprocket)

Teeth	200	400	600	800	1,000	1,200	1,600	1,800	2,000	2,400	2,800	3,200
12	0.77	1.34	1.81	2.16	2.46	2.71	2.99	3.07	3.10			
15	0.99	1.76	2.40	2.93	3.38	3.77	4.32	4.52	4.67	4.81		
18	1.20	2.15	2.94	3.63	4.21	4.71	5.48	5.76	5.97	6.27	6.35	
21	1.41	2.52	3.47	4.27	4.97	5.57	6.50	6.86	7.13	7.50	7.63	
24	1.60	2.88	3.96	4.87	5.67	6.35	7.40	7.80	8.12	8.51	8.68	8.57

A.S.A. No. 50, ⅝ in. pitch, Diamond No. 449 (single), No. 148 (multiple)

(Rpm of sprocket)

Teeth	100	200	300	400	600	800	1,000	1,200	1,400	1,600	1,800	2,200
12	0.80	1.44	1.99	2.48	3.26	3.86	4.3	4.6	4.8			
15	1.02	1.87	2.61	3.27	4.39	5.31	6.0	6.8	7.0	7.3	7.5	
18	1.23	2.27	3.19	4.01	5.41	6.58	7.5	8.3	8.9	9.4	9.7	
21	1.45	2.66	3.75	4.70	6.38	7.77	8.9	9.8	10.6	11.1	11.6	11.9
24	1.65	3.05	4.27	5.37	7.28	8.85	10.2	11.2	12.1	12.6	12.1	13.6

A.S.A. No. 60, ¾ in. pitch, Diamond No. 433

(Rpm of sprocket)

Teeth	50	100	200	300	400	600	800	1,000	1,200	1,400	1,600	1,800
12	0.73	1.34	2.41	3.30	4.05	5.2	6.1	6.6	6.9			
15	0.92	1.72	3.14	4.34	5.39	7.1	8.5	9.5	10.2	10.6		
18	1.12	2.10	3.82	5.31	6.63	8.9	10.6	12.0	13.0	13.7	14.1	
21	1.31	2.46	4.49	6.24	7.80	10.4	12.5	14.1	15.4	16.3	16.9	
24	1.50	2.80	5.11	7.12	8.90	11.9	14.3	16.1	17.6	18.6	19.2	19.5

A.S.A. No. 80, 1 in. pitch, Diamond No. 434

(Rpm of sprocket)

Teeth	50	100	150	200	300	400	500	600	700	800	1,000	1,160
12	1.68	3.07	4.28	5.3	7.2	8.7	9.8	10.7	11.4			
15	2.14	3.95	5.57	7.0	9.6	11.8	13.6	15.1	16.3	17.3		
18	2.59	4.81	6.79	8.6	11.8	14.5	16.9	18.9	20.5	21.9	24.0	
21	3.03	5.62	7.96	10.1	13.9	17.1	19.9	22.3	24.3	26.0	28.5	
24	3.46	6.43	9.10	11.5	15.8	19.5	22.7	25.4	27.7	29.6	32.5	33.9

TABLE 74.—HORSEPOWER RATINGS FOR SINGLE-STRAND ROLLER-CHAIN DRIVES.—(*Continued*)

A.S.A No. 100, 1¼ in pitch, Diamond No. 470

(Rpm of sprocket)

Teeth	25	50	100	200	300	400	500	650	700	750	800	870
12	1 72	3 19	5 8	9 9	13 0	15 6	17 2					
15	2 19	4 10	7 5	13 1	17 5	21 3	24 0	27 2	28 1			
18	2 55	4 97	9 1	16 0	21 6	26 6	30 2	34 5	35 7	36 8		
21	3 08	5 80	10 7	18 9	25 5	31 4	35 7	40 9	42 3	43 5	44 6	
24	3 52	6 62	12 2	21 5	29 2	35 4	40 5	46 5	48 1	49 5	50 6	52 0

A S A No 120, 1½ in. pitch, Diamond No. 472

(Rpm of sprocket)

Teeth	25	50	75	100	150	200	250	300	350	400	500	600
12	2 90	5 4	7 6	9 6	13 2	16 2	18 7	21 0	22 8	24 3		
15	3 71	6 9	9 8	12 5	17 3	21 6	25 3	28 6	31 4	33 9	38 0	
18	4 74	8 4	12 0	15 3	21 3	26 6	31 3	35 4	39 2	42 4	47 9	
21	5 24	9 9	14 0	17 9	24 9	31 2	36 8	41 7	46 2	50 0	56 7	61 7
24	5 99	11 3	16 0	20 4	28 5	35 7	41 9	47 6	52 6	57 1	64 6	70 3

A S A. No 140, 1¾ in. pitch, Diamond No. 474

(Rpm of sprocket)

Teeth	20	30	50	100	150	200	250	300	350	400	450	475
12	3 72	5 4	8 4	14 8	20 1	24 5	28 1	31 0				
15	4 73	6 9	10 8	19 3	26 6	32 8	38 2	42 8	46 7			
18	5 73	8 3	13 1	23 7	32 7	40 5	47 3	53 2	58 4	62 9		
21	6 70	9 7	15 3	27 7	38 4	47 6	55 7	62 8	69 0	74 5	79 0	
24	7 65	11 1	17 5	31 7	43 7	54 3	63 6	71 6	78 7	84 8	89 9	92 4

A.S.A. No. 160, 2 in. pitch, Diamond No. 478

(Rpm of sprocket)

Teeth	10	20	40	80	120	160	200	240	280	320	360	400
12	2 9	5 5	10 1	18 0	24.6	30 1	34 8	38.6				
15	3 7	7 0	13 0	23 5	32 4	40 2	47 0	52 9	58 0	62 4		
18	4 4	8 5	15 8	28 7	39 7	49 5	58 1	65 7	72 4	78.3		
21	5 2	9 9	18 5	33 6	46 7	58 1	68 3	77 5	85 5	92 5	99	
24	5 9	11 3	21 1	38 4	53 5	66 5	78 0	88 3	97 4	105.4	112	118

A S A. No. 200, 2½ in. pitch, Diamond No. 480

(Rpm of sprocket)

Teeth	10	20	40	60	80	100	120	160	200	240	260	280
12	5 6	10 5	19 1	26 8	33 6	39.6	45 1	54 4				
15	7 1	13 4	24 7	34.8	43 9	52.2	59 8	73 4	85			
18	8 6	16 2	30 0	42 4	53.7	64.1	73 7	90.7	105	118		
21	10.0	18 9	35 1	49 7	63.1	75 3	86.6	106 9	124	139	146	
24	11 4	21 6	40.2	56 8	71.9	86.0	98.8	121.8	142	159	166	173

These tables are abbreviated. Intermediate values may be interpolated and some values for greater numbers of teeth may be extrapolated. Blank spaces indicate that these numbers of teeth are not approved for these speeds. Ratings for multiple-strand chains are proportional to the number of strands. The recommended numbers of strands for multiple-strand chains are 2, 3, 4, 6, 8, 10, 12, 16, 20, and 24, with a maximum over-all width of 24 in. The horsepower ratings are conservatively based upon a satisfactory service life of 15000 hours and a chain length of 100 pitches. These ratings also take for granted that reasonable care will be given to installation and maintenance, including adequate lubrication at all times Theoretically, a drive operating 24 hr a day should have double the horsepower capacity of a drive required to operate only 10 to 12 hr per day, if satisfactory service is specified for the same number of years.

The horsepower at normal speeds and medium numbers of teeth is

$$\mathrm{Hp} = F_w \times V \tag{295}$$

The horsepower ratings given in Table 74 are based upon a more comprehensive formula which includes correction factors for chordal action and for velocity.

For very slow speeds and favorable operating conditions, including intermittent service, the chain selection may be based upon the ultimate tensile strength of the chain rather than durability. For chain speeds of 25 fpm or less, the chain pull may be as much as one-fifth of the ultimate strength; for 50 fpm, one-sixth; for 100 fpm, one-seventh; for 150 fpm, one-eighth; for 200 fpm, one-ninth; and for 250 fpm, one-tenth of the ultimate tensile strength.

268. Center Distance and Chain Length. If a center distance is to be nonadjustable after installation, it should be selected for an initially snug fit for an even number of pitches of chain. For the average application, a center distance equivalent to 40 ± 10 pitches of chain represents good practice, and it must be greater than half the sum of the outside diameters of the sprockets. Extremely short center distances should be avoided, if possible, especially for ratios greater than 3:1. It is desirable to have at least 120-deg wrap in the arc of contact on a power sprocket. For ratios of 3:1 or less, the wrap will be 120 deg or more for any center distance or numbers of teeth. To have a wrap of 120 deg or more, for ratios greater than $3\frac{1}{2}$:1, the center distance must not be less than the difference between the outside diameters of the two sprockets.

The chain length is

$$L = 2C\cos\alpha + \frac{T_1p(180 + 2\alpha)}{360} + \frac{T_2p(180 - 2\alpha)}{360} \tag{296}$$

$$L_p = 2C_p\cos\alpha + \frac{T_1}{2} + \frac{T_2}{2} + \frac{\alpha(T_1 - T_2)}{180} \tag{297}$$

$$L_p = 2C_p + \frac{T_1}{2} + \frac{T_2}{2} + \frac{K(T_1 - T_2)^2}{C_p} \tag{298}$$

where L = chain length, in.

L_p = chain length, pitches.

C = center distance, in.

C_p = center distance, pitches.

p = pitch of chain, in.
R = pitch radius of large sprocket, in.
r = pitch radius of small sprocket, in.
α = angle between tangent and center line, deg.
T_1 = number of teeth on large sprocket.
T_2 = number of teeth on small sprocket.
$180 + 2\alpha$ = angle of wrap on large sprocket, deg.
$180 - 2\alpha$ = angle of wrap on small sprocket, deg.
K = a variable, its value depending upon the value of $\dfrac{T_1 - T_2}{C_p}$.

$$\alpha = \frac{\sin^{-1}(R - r)}{C}.$$

TABLE 75.—CONDENSED VALUES OF K IN EQ. (298)

$\dfrac{T_1 - T_2}{C_p}$	0 1	1 0	2 0	3.0	4 0	5 0	6 0
K	0 02533	0 02538	0 02555	0 02584	0 02631	0 02704	0 02828

Formulas for chain length on multisprocket drives are too cumbersome to be useful. Multisprocket drives should be laid out accurately to scale, preferably full size, or larger.

Since standard roller chains are made up of alternate roller links and pin links, it is preferable to use chain lengths in even numbers of pitches. An odd number of pitches requires an offset link one pitch long. The link plates are offset to affect half a roller link at one end and half a pin link at the other end. Offset links should be avoided if possible.

Since roller chain cannot slip on a sprocket, it is advisable to avoid nonproductive pull on the chain, which is due to the unnecessary tension in the slack span. The permissible amount of slack depends upon several factors—length of span, weight of chain, character of load, whether impulsive or jerky, and slope of center line. Extremely long horizontal center distances for comparatively heavy chains should be avoided. The relationship between depth of sag and tension due to weight of chain in the catenary are approximately

$$h = 0.433\sqrt{S^2 - L^2} \qquad \text{and} \qquad T = w\left[\frac{S^2}{8h} + \frac{h}{2}\right] \tag{299}$$

where h = depth of sag, in.
L = distance between points of support, in.
S = catenary length of chain, in. (approximately equal to the length tangent to the sprockets).
T = tension or chain pull, lb.
w = weight of chain, lb per in.

269. Chain Sprockets. The tooth form adopted by A.S.A. for roller chain is shown in Fig. 256. One of the most important requirements of a sprocket cutter is that it cut the space (roller seat) between teeth slightly oversize, to allow rollers to seat without being pinched.

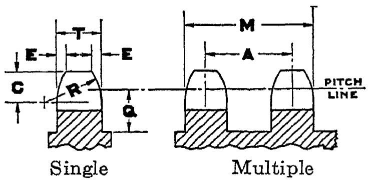


FIG. 255.—Standard tooth section profile. (*Courtesy Diamond Chain and Manufacturing Company.*)

n = number of strands in multiple strand chain.
p = pitch of chain, in.
t = nominal thickness of link plate.
T = thickness of tooth.
$T = 0.93W - 0.006$, for single-strand chain.
$T = 0.90W - 0.006$, for double- and triple-strand chain.
$T = 0.88W - 0.006$, for four- and five-strand chain.
$T = 0.86W - 0.006$, for six-strand chain or wider.
M = over-all width of tooth-profile section.
W = inside width of single-strand roller chain or length of roller.
$C = 0.5p$.
$E = 0.125p$.
$R = 1.063p$.
$Q = 0.5p$ for link plates with figure-of-eight contour
$A = W + 4.15t + 0.003$.
$M = A(n - 1) + T$.
Tolerance on T or
$M = \pm(0.02W + 0.002)$.

The A.S.A. tooth-section profile for standard-width chains is shown in Fig. 255 with all dimensions in inches.

Under normal conditions the shaft diameter and keyway dimensions determine the minimum hub diameter of a sprocket, usually equal to the shaft diameter plus four times the keyway depth. If there is to be a setscrew over the key, the minimum hub diameter is the shaft diameter plus six times the keyway depth, assuming a square key.

To provide clearance between ends of adjacent link plates, the end radius must be less than half of the pitch, or the height of the link plate must be less than the pitch. Roughly, therefore, the maximum hub diameter to allow clearance under the edges of the link plates with figure-eight contour would be the pitch diam-

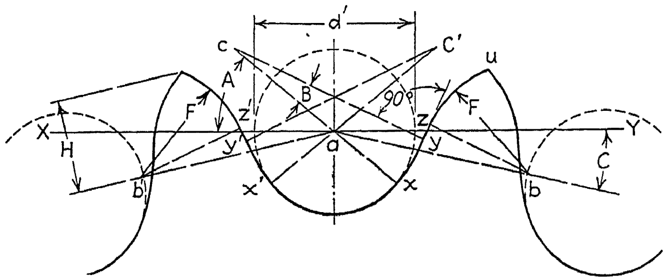


FIG. 256.—Design of standard sprocket tooth for roller chains. (*Courtesy Diamond Chain and Manufacturing Company.*)

p = pitch; d = nominal roller diameter; T = number of teeth; D = pitch diameter $= \dfrac{p}{\sin \dfrac{180}{T}}$; H = height of tooth above center line between two adjacent seating-curve centers.

$d' = 1.005d + 0.003''$; $A = 35 \text{ deg} + \dfrac{60 \text{ deg}}{T}$; $B = 18 \text{ deg} - \dfrac{56 \text{ deg}}{T}$; $ac = 0.8d$; $C = \dfrac{180 \text{ deg}}{T}$.

Draw line XY. Locate point a, and with that as center and radius ax equal to $\frac{1}{2}d'$ draw circular arc for the "seating curve" xx'.

Draw line xac making angle A with line XY and locate point c so that $ac = 0.8d$. Draw line cy making angle B with line cx. With center at c and radius cx, draw arc xy for the "working curve."

Draw line yz perpendicular to line cy. Draw line ab making angle C with line XY, and locate point b so that $ab = 1.24d$. Draw line bz parallel to line yc. With b as center and radius bz, draw the "topping curve," arc zu tangent to line zy.

A similar construction for the other half will complete the tooth outline.

Outside diameter of sprocket when tooth is pointed $= p \cot \dfrac{180 \text{ deg}}{T} + 2H$.

The recommended value for H is $0.3p$; and when this value is chosen, the outside diameter of the sprocket will be $p\left(0.6 + \cot \dfrac{180 \text{ deg}}{T}\right)$.

The pressure angle, when the chain is new, is $xab = 35 \text{ deg.} - \dfrac{120 \text{ deg}}{T}$.

The minimum pressure angle is abz or $17 \text{ deg.} - \dfrac{64 \text{ deg}}{T}$,

The average pressure angle is $26 \text{ deg} - \dfrac{92 \text{ deg}}{T}$.

The standard tooth form is designed to give maximum efficiency throughout the life of the drive. Because of the large pressure angle and the distribution of the load over a number of teeth the tendency of the teeth to wear hook-shaped is greatly reduced. The reason for this is that the chain rides higher on the teeth as it elongates, thus accommodating itself to a larger pitch circle. This is the standard approved by the A.S.M.E., S.A.E., and A.G.M.A.

eter minus pitch (see Q under Fig. 255). The extra clearance needed under standard offset link plates and straight-edge link plates require the use of a smaller maximum hub diameter, which can be obtained from the equation

$$\text{Maximum hub diameter} = D \times \cos \frac{180}{T} - (H + 0.05) \quad (300)$$

where D = pitch diameter, in.
T = number of teeth.
H = height of link plate, in.

In some forms of cast sprockets, clearance is provided between the rollers and teeth by a slight increase of the pitch diameter of

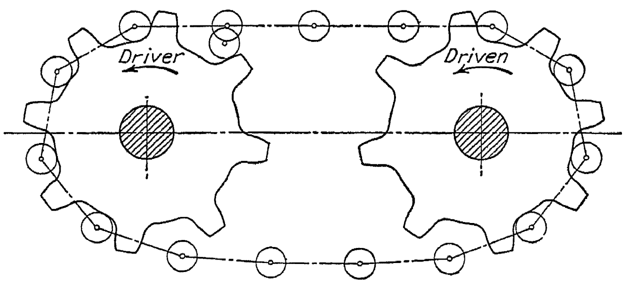


FIG. 257.—Roller-chain drive showing tooth action when clearance is provided.

the driving sprocket, and a decrease in that of the driven sprocket. This construction causes one tooth to carry all the load and is not desirable except with cast teeth and cast chain in which the pitch can not be accurately maintained.

TABLE 76.—SERVICE FACTORS, WITH REQUIRED HORSEPOWER = 1.00

Operating characteristics	Intermittent Few hours per day; few hours per year	Normal 8 to 10 hr per day, 300 days per year	Continuous 24 hr per day
Easy starting, smooth, steady load.	0.60–1.00	0.90–1.50	1 20–2 00
Light to medium shock or vibrating load	0.90–1.40	1.20–1 90	1.50–2 40
Medium to heavy shock or vibrating load.	1.20–1.80	1.50–2 30	1 80–2.80

Courtesy Diamond Chain and Manufacturing Company.

270. Service Factors. The horsepower ratings in Table 74 should be modified for various service conditions by using service factors as given in Table 76.

The following conditions justify service factors in the higher range: sprocket ratios greater than 6:1; more than two sprockets in the drive; less than 120-deg wrap on sprockets other than idlers; very short center distances, especially with few teeth; vertical or near-vertical center lines, small sprocket below; frequent starting and stopping, especially under heavy load.

271. Lubrication. Chain drives suffer more from lack of proper lubrication than from many years of normal service. Light-bodied oil of good quality, fluid at the prevailing temperature, is

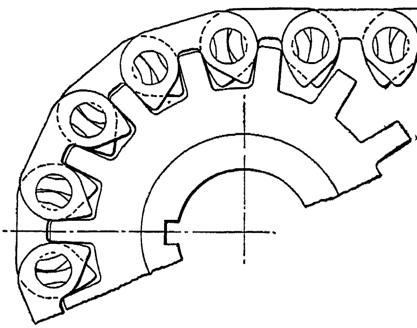


FIG. 258.—Morse rocker-joint silent chain and straight-sided sprocket teeth.

the best general purpose lubricant for both high- and low-speed drives; S.A.E. 20 or S.A.E. 30 for the shorter pitches and S.A.E. 40 for the heavier chains; winter oil when temperatures are very low. Heavy oils and greases applied at ordinary temperatures cannot pass through the narrow clearance spaces leading to the pin-bushing bearing surfaces.

For other than splash and bath systems which are generally approved only for speeds under 1,500 fpm, the oil should be delivered to the upper edges of link plates in the bottom span of chain. Oil applied to the upper span of chain or to the chain rollers is of little value in prolonging chain life and retarding wear. Enough oil should be applied to prevent overheating and drawing of the hardened bearing surfaces, the amount depending upon operating conditions.

272. Silent Chains. Silent chains (Fig. 258) are made of alternating steel links shaped so that they have straight tooth

edges bearing along the entire length of the sprocket teeth. They are suitable for speeds up to 1,600 fpm in general service, and up to 2,500 fpm if enclosed and well lubricated. The steps to be observed in the selection of silent chains are essentially the same as those given for roller chains in Arts. 267 and 268. To aid in

TABLE 77.—DATA FOR THE SELECTION OF SILENT CHAINS

Chain pitch, in.	Rpm of the small sprocket	Minimum number of sprocket teeth		Allowable chain pull,* lb. per in. width	Chain widths
		Driver	Driven		
$\frac{3}{8}$	2,000–5,000	15	17	60 @ 2,400+	$\frac{3}{8}$– $2\frac{1}{2}$
$\frac{1}{2}$	1,500–2,000	17	19	80 @ 1,800+	$\frac{1}{2}$– 3
$\frac{5}{8}$	1,200–1,500	17	19	100 @ 1,500+	$\frac{3}{4}$– 4
$\frac{3}{4}$	1,000–1,200	17	21	120 @ 1,200+	$\frac{3}{4}$– 5
1	800–1,000	17	21	170 @ 1,000+	1 –12
$1\frac{1}{4}$	650– 800	17	23	220 @ 800+	2 –14
$1\frac{1}{2}$	500– 650	17	23	280 @ 600+	2 –15
2	300– 500	19	25	450 @ 400+	2 –18
$2\frac{1}{2}$	up to 300	19	25	750 @ 250+	3 –25

* If the rpm is below the value given, the chain pull may be increased in the inverse ratio of the revolutions.

the selection, the dimensions and strengths are given in Table 77. The pitch is governed by the speed of the smaller sprocket, the short-pitch chains giving quieter operation, but space limitations may require narrow long-pitch chains. For preliminary computations, the pitch may be selected from Table 77. For best results the velocity of silent chains should be between 1,200 and 1,400 fpm, although enclosed chains, well lubricated, may run at 2,500 fpm. The permissible chain pull per inch of width can be taken from Table 77. The chain width, determined by dividing the required chain pull by the permissible pull per inch of width, should be from two to six times the pitch. If the trial chain is too narrow, try a shorter pitch chain, and if too wide, try a longer pitch.

Silent chains are used on sprockets having straight-sided teeth, as shown in Fig. 258. Since the angle included between the working faces of the chain links is 60 deg, the flanks of adjacent teeth on all sizes of sprockets must be at 60 deg. The chains are held on the sprockets by center plates riding in grooves

in the sprocket, outside links extending down over the outer ends of the teeth, or by wires held in grooves in the sprockets.

273. Examples of Chain Selection. Example 1. Select a roller chain drive to transmit 40 hp from a 1,200-rpm motor to a line shaft at 250 rpm. The motor shaft diameter is $2\frac{3}{8}$ in., and the center distance is adjustable from 24 in. Service will be 10 hr per day, 6 days per week, and good lubrication will be provided.

Solution. As shown in Table 73, the pitch of the chain is governed by the number of teeth and the speed of the small sprocket. For economy in price, the longest approved pitch is usually selected unless extreme quietness is desirable.

The longest pitch chain for 1,200 rpm is $\frac{3}{4}$ in.

The minimum hub diameter for $2\frac{3}{8}$ in. bore is $3\frac{5}{8}$ in.

The minimum trial pitch diameter is $3\frac{5}{8}$ in. $+ \frac{3}{4}$ in. $= 4\frac{3}{8}$ in. (see Fig. 255).

Since the perimeter of the pitch polygon is approximately equivalent to the circumference of the pitch circle, the minimum number of teeth will be

$$\frac{4.375\pi}{0.75} = 18.35, \qquad \text{say } 19.$$

The pitch diameter $D = \dfrac{p}{\sin \dfrac{180}{T}} = \dfrac{0.75}{\sin \dfrac{180}{19}} = 4.557$ in.

$$\text{The outside diameter } = p\left(0.6 + \cot \frac{180}{T}\right)$$

$$= 0.75\left(0.6 + \cot \frac{180}{19}\right) = 4.945 \text{ in.}$$

The maximum hub diameter for 19 teeth, $\frac{3}{4}$ in. pitch, from Eq. (300) is found to be $3\frac{49}{64}$ in. This is adequate for $2\frac{3}{8}$ in. bore, being more than the recommended $3\frac{5}{8}$ in. minimum hub diameter.

The reduction ratio is 4.8:1, and the nearest ratio which can be provided with a 19-tooth driver is 91:19, or 4.79:1.

For 91 teeth the pitch diameter is 21.729 in. and the outside diameter is 22.166 in.

One-half of the sum of the outside diameters is 13.143 in., providing approximately 9 in. clearance between the two sprockets.

The difference between the pitch diameters is 17.172 in. and will provide more than 120 deg of wrap, the desired minimum on the small sprocket.

The speed ratio is 4.8:1, and a combination of 19 and 91 teeth represents a ratio of 4.79:1. If the exact ratio is more important than the minimum number of teeth, 20 and 96 teeth should be used.

From Table 74, the rating for $\frac{3}{4}$ in. pitch chain on 19 teeth at 1,200 rpm is 13.8 hp per strand, making it necessary to use triple-strand chain, with a rating of 51.4 hp.

The chain speed is $\dfrac{19 \times 1{,}200 \times 0.75}{12} = 1{,}425$ fpm.

The chain pull for 40 hp is $\dfrac{40 \times 33{,}000}{1425} = 925$ lb.

With an ultimate tensile strength of 25,500 lb for this chain, the safety factor will be slightly more than 27 Tensile strengths are always adequate, and usually more than adequate, for chain having requisite power capacity

The center distance 24 in. divided by the pitch, $\frac{3}{4}$ in., is equivalent to 32 pitches. Using Eq. (298), the chain length in pitches

$$L_p = 2 \times 32 + \frac{91}{2} + \frac{19}{2} + \frac{0\ 02562(91 - 19)^2}{32} = 123.15 \qquad \text{pitches}$$

A center distance slightly less than 24 in. would require exactly 123 pitches of chain, but this length would include an offset link. It is advisable, therefore, to specify a length of 124 pitches The 0 85 pitches, or approximately $\frac{5}{8}$ in. of slack, can be taken up by adjusting the center distance after the chain is installed.

To give an initially snug fitting chain on a fixed center distance, C_p in Eq. (298) must be increased a little more than half of the $\frac{5}{8}$ in slack. (The rate of increase for values of K becomes less as the center distance is increased.)

Substituting $25\frac{5}{16}$ in. for C_p in Eq. (298), the exact chain length would be 123 91 pitches. A value of $24\frac{3}{8}$ in. gives 123 99 pitches, for which 124 pitches would be entirely satisfactory.

Example 2. Select a chain drive for a concrete mixer driven by a 15-hp motor. The driving shaft rotates at 200 rpm and the driven shaft at 50 rpm. Center to center of shafts is 5 ft.

Solution. This drive will operate with intermittent shock loads and under very gritty conditions This suggests the use of a chain similar to the Link-Belt SS type (Fig. 249) with cast tooth sprockets.

Reference to Link-Belt Catalog No 800, Table III, page 62, indicates that chain SS-378 having a pitch of 1.654 in will transmit 16 hp at 300 fpm on a 12-tooth sprocket. Also, chain SS-433 with a pitch of 2.609 in. will transmit 18 1 hp at 500 fpm on a 12-tooth sprocket.

Considering chain SS-378, we find that the required number of teeth on the small sprocket is

$$T = \frac{300 \times 12}{1.654 \times 200} = 10.85, \text{ say 11 teeth}$$

From Table IV on page 64 of the catalog, the correction factor for 11 teeth is 0 97, and we have for the capacity of the chain

$$hp = 16 \times 0.97 = 15.5$$

which is satisfactory.

Similarly, chain SS-433 requires 11 5, say 12 teeth at 500 fpm, giving the capacity as 18.1 hp.

Computing the sprocket diameters and tabulating the results, we have the table on page 367

Both chains are satisfactory, but it is desirable to have at least 12 teeth on the driver, and with a smaller number of joints there will be longer life when operating in grit, so that the long-pitch chain SS-433 will be used.

Chain	SS-378	SS-433
Pitch	1 654 in.	2.609 in.
Chain velocity	300 fpm	500 fpm
Horsepower	15 5	18.1
Teeth on sprocket:		
Driver	11	12
Driven	44	48
Pitch diameter:		
Driver	5 92 in.	10 08 in.
Driven	23 24 in.	39 89 in.

Recheck for capacity

$$\text{Chain velocity} = \frac{12 \times 2.609 \times 200}{12} = 521 \text{ fpm}$$

From Table V (Link-Belt Catalog), the factor of safety is 10.9 and we have

$$\text{hp} = \frac{F_u V}{33{,}000 \times FS} = \frac{13{,}000 \times 521}{33{,}000 \times 10\ 9} = 18.8$$

which is greater than the required value, 15 hp, and therefore satisfactory.

Example 3. Select a silent chain for the drive between a 45-hp 690-rpm motor and a compressor running at 135 rpm The distance between centers is to be made as short as practical, and the driven sprocket is not to exceed 36 in. in diameter.

Solution. A compressor drive should be designed for an overload capacity of 30 per cent, which in this case is 1.30 × 45 or 58.5 hp, say 60 hp.

The speed ratio is 690:135, or 5 11, which can be obtained with sprockets having 17 and 87 or 19 and 97 teeth. Table 77 gives $1\frac{1}{4}$ in. as the pitch for a 17-tooth driving sprocket. If 19 and 97 teeth were used, the limit of 36 in. for the driven sprocket would be exceeded. Try $1\frac{1}{4}$ in. pitch with 17 and 87 teeth.

The computed chain velocity is 1,222 fpm, and the required chain pull at this velocity is 1,620 lb. From Table 77 the permissible pull is 255 lb per in. width. The required chain width is 1,620/255, or 6.36 in. The pitch diameters of the sprockets are 6 803 and 34.79 in and the minimum center distance is

$$\frac{D_1 + D_2}{2} = \frac{34.79 + 6.803}{2} = 20.796 \text{ in.}$$

The preferred minimum center distance is

$$1.5D_1 + 0.5D_2 = 52.19 + 3.40 = 55.59 \text{ in.}$$

A silent chain of $1\frac{1}{4}$ in. pitch, 7 in. wide, and operating on 17- and 87-tooth sprockets at a center distance of approximately 24 in. is satisfactory for this drive. The center distance should be increased if possible. The chain width is more than five times the pitch. This ratio varies from 2 to 8.

CHAPTER XVIII

SPUR GEARS

Belts, friction pulleys, and other types of power transmission that depend upon friction are subject to slippage and hence do not transmit a definite and invariable speed ratio. Chains and gears are used when positive drives are necessary, and, where the center distances are relatively short, toothed

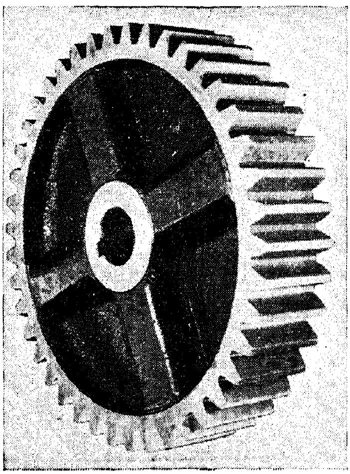


FIG. 259.—Spur gear, welded construction. (*Courtesy Lukenweld, Inc.*)

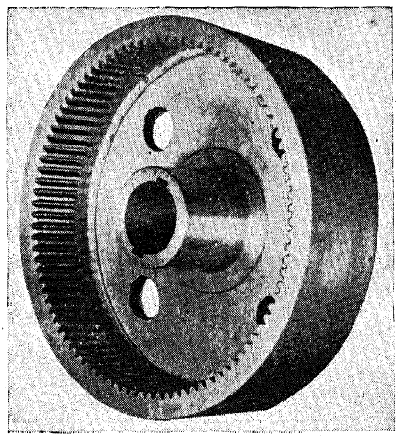


FIG. 260.—Internal gear. (*Courtesy Lukenweld, Inc.*)

gears are preferred. Some types of toothed gears are illustrated in Figs. 259 to 261.

274. Spur Gear Nomenclature. Power may be transmitted by friction between two rotating cylinders (cones, etc.) when they are pressed together, but positive driving action without slippage will only be obtained if spurs, or teeth, are built upon these surfaces to form gears. The surfaces upon which the teeth are built are called the pitch surfaces, and the intersections of these surfaces with planes perpendicular to the axes of rotation are

called the pitch lines, or pitch circles. The diameter of the pitch circle designates the size of the gear.

Spur gears (Fig. 259) are gears rotating on parallel axes and may be straight-tooth with the teeth parallel to the axis, or helical with the teeth forming helices. Annular gears consist of a small gear, or pinion, mating with an internal gear (Fig. 260) having the teeth cut on the inside of the pitch line. A rack is a spur gear of infinite diameter, *i.e.*, its pitch surface is a plane surface, and the pitch line a straight line. The common parts of spur gears are defined in Fig. 262. The circular pitch p of the gear is the distance from one face of a tooth to the corresponding face of the next adjacent tooth, measured along the pitch circle. The diametral pitch is the ratio of the number of teeth to the pitch diameter, *i.e.*, the number of teeth per inch of diameter. The normal pitch of involute gears is the distance between corresponding tooth faces measured along the line of action, or along the circumference of the base circle from which the involute-tooth outline is generated. The arc of action is the arc traversed by a point on the pitch circle while any tooth is in contact with its mating tooth, and the angle of action is the subtending angle. The addendum is the height of the tooth outside of the pitch line and the dedendum is the height of the tooth inside of the pitch line. The sum of the addendum and dedendum is the total tooth height.

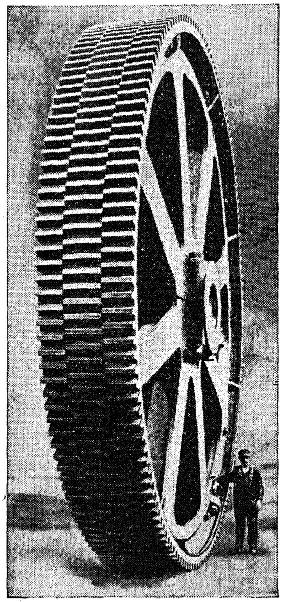


FIG. 261.—Stepped gears. (*Courtesy Mesta Machine Company.*)

275. Tooth Profiles. In order that the teeth of two mating gears will transmit uniform angular velocity, the common normal to the mating tooth surfaces at their point of contact must always pass through the same pitch point,* *i.e.*, the point where the line of centers intersects the pitch circles. Tooth profiles

* For proof and a more complete treatment of the theory of tooth profiles, see Vallance and Farris, "Principles of Mechanism," p. 229, The Macmillan Company.

fulfilling this requirement may be generated by rolling a template, with a suitable tracing point, on the outside of one pitch circle and on the inside of the mating pitch circle as in Fig. 263. Templates of many shapes may be used, but only two are commonly

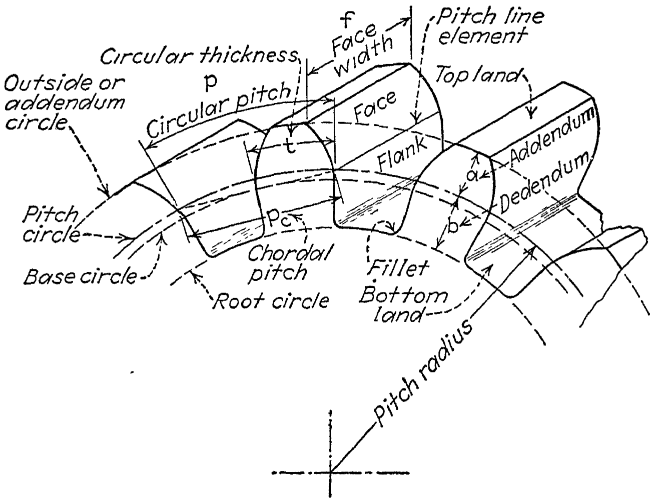


Fig. 262.—Representation of terms used with gear teeth.

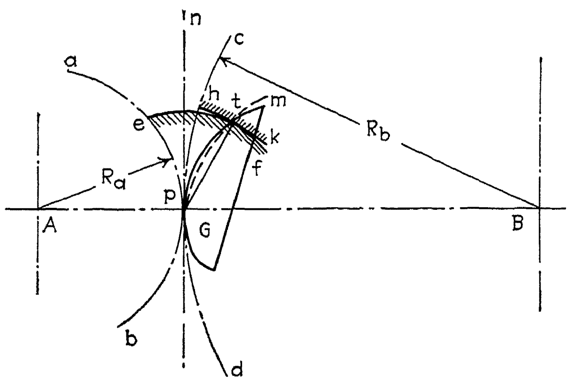


Fig 263.

employed in modern gear practice. These are the circle for cycloidal teeth, and the logarithmic spiral for involute teeth. The involute, however, is more readily generated from a base circle as explained in Art. 277.

276. Cycloidal Gears. The cycloidal form was one of the first regular profiles used for gear teeth. The difficulties encountered in producing accurate profiles have gradually forced this system into obsolescence. Cycloidal teeth are seldom used in gears

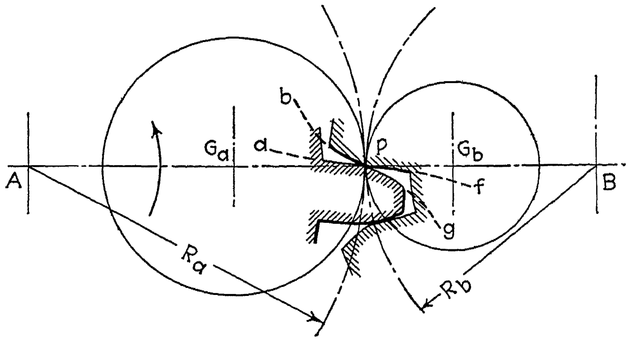


FIG. 264.

at the present time, although the cycloidal form is widely used for the impellers of rotary blowers, superchargers, and similar devices.

The cycloidal tooth curves are generated by rolling a circular template, or generating circle, on the outside and the inside of the pitch circle. In Fig. 264 the tracing point p traces the epicycloid pb, or face curve, when the circular template G_a is rolled on the pitch circle B, and the hypocyloid pa, or flank curve, when rolled on the inside of the pitch circle A. In an interchangeable set, all gears should operate properly with every other gear of the set. This condition is obtained by using the same generating circle for all faces and flanks, the generating circle usually having a diameter equal to the radius of a 15-tooth gear.

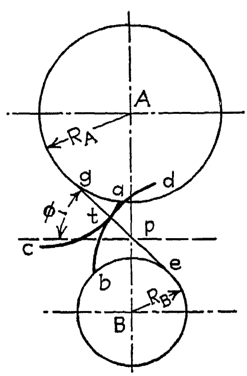


FIG. 265.

277. Involute Gears. The involute curve is the basis of nearly all tooth profiles now in general use. The tooth profile is the involute of a base circle slightly smaller than the pitch circle. In Fig. 265, a cord with a tracing point t is shown wrapped around two disks with centers at A and B. When A is rotated,

the tracing point describes the involute *cta* on the disk A and the involute *btd* on the disk B. The tracing point is always on the tangent to the circles A and B, and the tangent is always the normal to the tooth curves at their point of contact; hence p is the pitch point and Ap and Bp are the radii of the pitch circles. Pressure transmitted between the tooth surfaces will always act along the common normal; hence the pressure angle ϕ is constant. The angular-velocity ratio is given by the equation

$$\frac{\omega_A}{\omega_B} = \frac{Bp}{Ap} = \frac{R_B}{R_A} = \text{a constant} \tag{301}$$

hence changing the center distance ApB does not alter the velocity ratio or destroy the proper tooth action of the involutes. The pitch diameters and the pressure angle change when the center distance is altered. The possibility of altering the center distance without destroying the correct tooth action is an important property of the involute gear.

In practice, the base circles and center distances are chosen so that particular values of the pressure angle are obtained, generally $14\frac{1}{2}$ or 20 deg and occasionally $17\frac{1}{2}$ and $22\frac{1}{2}$ deg.

278. Interference in Involute Gears. It is evident that the involute can not extend inward beyond the base circle from which it is generated; hence, if the pinion revolves counter-

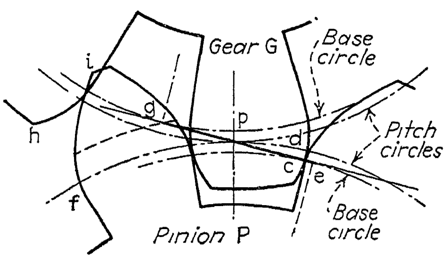


FIG. 266.

clockwise, the first point of contact between the tooth curves will be at e (Fig. 266), and the last point of contact will be at g, where the line of action, or pressure line, is tangent to the base circles. Any part of the tooth face of the pinion that extends beyond a circle drawn through g is useless; in fact it will interfere with the radial portion of the larger gear unless the flank is undercut. This interference is shown at i. The

interference limits the permissible addendum length, and it is evident that, as the diameter of the pinion is decreased, the permissible addendum of the larger gear becomes smaller.

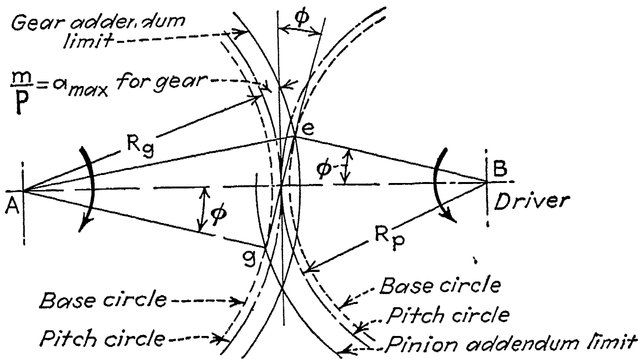


FIG. 267.

Let the addendum height be expressed in terms of the diametral pitch, making the addendum equal to m/P. From Fig. 267, the maximum outside radius of the gear A is

$$Ae = R_g + \frac{m}{P} = \sqrt{(Ag)^2 + (ge)^2}$$

and

$$R_g + \frac{m}{P} = \sqrt{R_g^2 \cos^2 \phi + (R_g + R_p)^2 \sin^2 \phi} \qquad (302)$$

but

$$R_p = \frac{n_p}{2P}, \qquad \text{and} \qquad R_g = \frac{n_g}{2P}$$

where n_p and n_g = the number of teeth in the pinion and gear, respectively.

Substituting these values in Eq. (302), expanding, and rearranging, it becomes

$$n_p^2 + 2n_pn_g = \frac{4m(n_g + m)}{\sin^2 \phi} \qquad (303)$$

This equation may be used to determine the smallest true involute pinion that will operate properly with a gear having n_g teeth, when the pressure angle and addendum ratio are known. Conversely, when n_p, the number of teeth on the pinion, is known,

the number of teeth on the largest gear with which it will operate without interference may be determined.

Example. Determine the smallest pinion that will operate with a 60-tooth gear when both have full-height $14\frac{1}{2}$-deg teeth. The addendum of full-height teeth is $1/P$; hence m is 1 Then

$$n_p^2 + 2 \times 60 n_p = \frac{4(60 + 1)}{0.2504^2}$$

from which

$$n_p = 26.6, \text{ say } 27 \text{ teeth}$$

279. Line of Action. The arc of action must be equal to or greater than the circular pitch, which for involute gears will be true when the line of action, or path of contact, is equal to or greater than $p \cos \phi$. In Fig. 267, the maximum line of action is

$$ge = (R_g + R_p) \sin \phi \geqq p \cos \phi \tag{304}$$

from which

$$p \leqq (R_g + R_p) \tan \phi$$

and

$$n_g + n_p \geqq \frac{2\pi}{\tan \phi} \tag{305}$$

This equation gives the number of teeth required on mating gears to obtain continuous action when the addenda of both gears are made as large as is possible without interference. For power transmission, the line of action should be greater than the minimum and at least equal to 1.4 times the circular pitch.

The theoretical length of the line of action of any pair of true involute gears is given by the equation

$$L_a = \left[\sqrt{\left(R_p + \frac{m_p}{P}\right)^2 - R_p^2 \cos^2 \phi} + \sqrt{\left(R_g + \frac{m_g}{P}\right)^2 - R_g^2 \cos^2 \phi} - (R_p + R_g) \sin \phi\right] \tag{306}$$

There will be interference when either radical in this equation is greater than $(R_p + R_g) \sin \phi$. In this case, the maximum line of action is determined by substituting $(R_p + R_g) \sin \phi$ for the larger radical.

The maximum number of teeth in action at one time is

$$n_a = \frac{L_a}{p \cos \phi} = \frac{PL_a}{\pi \cos \phi} \tag{307}$$

280. Standard Involute Gears. Composite System. It is desirable that all gears of any system will operate with all other gears of the same system. The accepted standard tooth form has an involute profile with a pressure angle of $14\frac{1}{2}$ deg, an addendum height of $1/P$, and a dedendum height of $1.157/P$. Interference when using small pinions may be prevented by undercutting the flanks, which weakens the teeth, or by modifying the tooth form at the tip. The latter method is preferable and is used in the composite system. The portion of the tooth profile near the pitch line is a true $14\frac{1}{2}$-deg involute, and the outer part of the face and the inner part of the flank are cycloidal in form Stand-

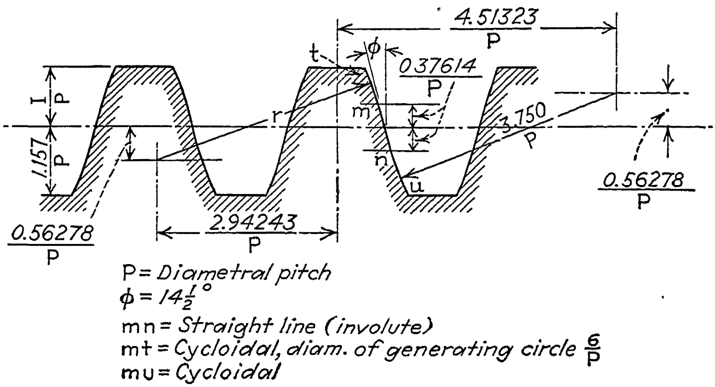


FIG. 268.—Basic rack for the $14\frac{1}{2}$-deg. A.G.M.A. composite tooth system.

ard proportions of the basic rack for this system are shown in Fig. 268. Although not a true involute gear, this is the accepted "standard $14\frac{1}{2}$-deg. full-height involute system" used for gears cut with formed cutters. The smaller nonspecialized manufacturers produce gears, cut with formed cutters, on milling machines, gear hobbers, and gear shapers.

281. Generated-tooth System. True involute profiles are obtained by the gear-generating process, using a straight-sided rack as the basic form. Small pinions have undercut flanks and do not have an arc of contact long enough to insure continuous driving action. In the $14\frac{1}{2}$-deg generated system, the smallest pair of equal gears with full-height teeth that will operate together have 14 teeth each, and a 32-tooth pinion is the smallest that will operate properly without interference with all gears, including a rack. To obtain full involute action with pinions

having less than 32 teeth, it is necessary to increase the outside diameter of the pinion and to decrease the outside diameter of the gear by the same amount, while keeping the whole depth the same. The amount of increase and decrease is given by the formula

$$\text{Diameter increment} = \frac{2 - n_p \sin^2 \phi}{P} \tag{308}$$

for $14\frac{1}{2}$-deg pinions having from 8 to 31 full-height teeth. For 20-deg pinions having 8 to 17 full-height teeth, the same correction should be made

TABLE 78.—GEAR-TOOTH PROPORTIONS

Item	Symbol	Full-depth systems: $14\frac{1}{2}$ deg, 20 deg, $22\frac{1}{2}$ deg, cycloidal, and composite		A.G.M.A. 20-deg, stub, helical, herringbone	
		Diametral pitch	Circular pitch	Diametral pitch	Circular pitch
Addendum	a	$1/P$	$0.3183p$	$0.8/P$	$0.2546p$
Dedendum, minimum	b	$1.157/P$	$0.3683p$	$1/P$	$0.3183p$
Working depth	h_w	$2/P$	$0.6366p$	$1.6/P$	$0.5092p$
Total depth, minimum	h_t	$2.157/P$	$0.6866p$	$1.8/P$	$0.5729p$
Pitch diameter	D	N/P	$0.3183Np$	N/P	$0.3183Np$
Outside diameter	D_o	$(N + 2)/P$	$0.3183(N + 2)p$	$(N + 1.6)/P$	$0.3183(N + 2)p$
Tooth thickness on the pitch line, basic	t	$1.5708/P$	$0.5p$	$1.5708/P$	$0.5p$
Clearance, minimum	c	$0.157/P$	$0.05p$	$0.2/P$	$0.0628p$
Fillet radius		$1.5c$	$1.5c$	$1.5c$	$1.5c$

Note: Diametral pitch used up to 1 diametral pitch inclusive. Circular pitch used for 3 in circular pitch and over. Diametral pitch varies by increments of $\frac{1}{4}$ from 1 to 3; $\frac{1}{2}$ from 3 to 4; 1 from 4 to 12; and 2 from 12 to 50.

In these systems the smallest pinion that will operate without interference with all gears, including a rack, has the following numbers of teeth: $14\frac{1}{2}$-deg full-height, 32 teeth; 20-deg full-height, 18 teeth, $22\frac{1}{2}$-deg full-height, 12 teeth; $14\frac{1}{2}$-deg composite, 12 teeth; 20-deg stub, 14 teeth. These values for full-height gears may be decreased when the addenda are modified according to Eq. (308).

282. Stub-tooth Systems. Since interference in full-height gears prevents the use of small pinions without modification, the stub, or shortened tooth, is often used. There are two accepted proportions for stub gears, the A.S.A. or A.G.M.A. standard, and the Fellows stub. In both systems the pressure angle is 20 deg. In the A.S.A. system, the addendum height is

0.8/P and the dedendum 1/P. In the Fellows system, the tooth is designated by two diametral-pitch numbers, thus 2-3, 3-4, 4-5, 6-8, etc. The first figure is the diametral pitch used in computing the pitch diameter, circular pitch, and the number of teeth. The second figure is the diametral pitch used to compute the tooth height. The Fellows stub tooth, because of its early adoption in the automotive industries, is generally used with small teeth, whereas the A.S.A. system is used with the larger teeth. It is preferable that one standard, namely the A.S.A., should be adopted for all gears.

283. Special Tooth Systems. It is not always necessary or desirable that the gears be made interchangeable, and several systems* have been developed to suit the particular needs of the industry employing them. In these systems, the addenda, dedenda, pressure angle, and center distances are chosen for each installation to give the best operating conditions and the strongest teeth. Space does not permit a complete discussion of these special systems.

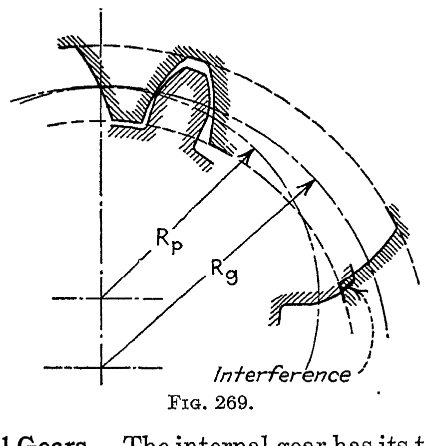


FIG. 269.

284. Internal Gears. The internal gear has its teeth cut on the inside of the rim and is used when the direction of rotation of both gears must be the same, and when short-center distances are required. The profile of the internal gear tooth is the same as that of the tooth space of a spur gear of the same pitch diam-

* Discussions of the Maag system, the range-cutter, proportional-center-distance, and the variable-center-distance systems may be found in Earle Buckingham, "Spur Gears," McGraw-Hill Book Company, Inc.

eter. The addendum of the internal gear is inside the pitch circle and, to prevent interference, is generally limited in length so that the tip of the tooth passes through the interference point. If full-height teeth are used, there will be fouling or interference between the tip of the pinion and the tip of the internal gear teeth, as shown in Fig. 269. To prevent this action, the gear should have at least 12 more teeth than the pinion when $14\frac{1}{2}$-deg full-height teeth are used. With 20-degree stub-tooth gears, the gear should have at least seven more teeth than the pinion.

285. The Strength of Spur Gears. The ability of any gear to transmit power may be limited by the strength of the teeth acting as beams or by the surface fatigue of the material, the load-carrying capacity being determined by the lower of these. The fatigue and ensuing wear will be discussed in a later paragraph.

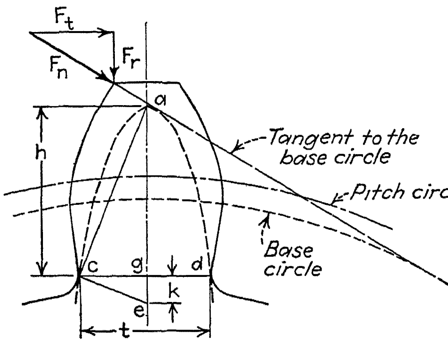


FIG. 270.

Each tooth may be considered to be a cantilever beam loaded as shown in Fig. 270. In the case of cast teeth not rigidly and accurately mounted, the load may be concentrated at an outer corner of the tooth; but with machine-cut teeth well mounted, the load may be considered to be distributed across the active face of the tooth.

The force F_n acting between the tooth surfaces is normal to the surface and in Fig. 270 is shown in the position producing the highest stress, *i.e.*, when contact is just beginning or ending. The normal force may be replaced by components F_t and F_r, acting perpendicular and parallel to the center line of the tooth. The radial component produces compression stress in the tooth,

which is usually disregarded in gear design. The tangential component causes bending stresses, which are used as the basis of the design of the tooth for strength. The maximum bending stress may be located and computed as follows: Through the point a in Fig. 270 draw a parabola (shown in dash lines) tangent to the tooth curves at c and d. This parabola represents the outline of a beam of uniform stress, and therefore the maximum stress in the actual tooth will be at the points of tangency c or d. This stress is

$$s = \frac{Mc}{I} = \frac{6F_t h}{ft^2}$$

from which

$$F_t = sf\frac{t^2}{6h}$$

Both t and h are dependent upon the size of the tooth and its profile; hence the equation may be written in the form

$$F_t = s_w fpy = s_w f\frac{\pi y}{P} = \frac{s_w fY}{P} \qquad (309)$$

where s_w = working stress, psi.
f = face width, in.
y and Y = form factors.
p and P = circular and diametral pitch, respectively.

This equation was first developed by Wilfred Lewis and is used, with some modifications, for the determination of the strengths of all types of gears.

For convenience in gear computations, values of the form factor for various tooth systems are given in Table 79. With cast teeth the inaccuracies of tooth outline eliminate the necessity of using an accurate form factor in the Lewis equation, and y is taken as 0.054 for all gears with cast teeth. To determine the form factor for any tooth profile not given in the tables draw the tooth outline to a large scale, locate the points where the inscribed parabola is tangent to the tooth outline and scale the distances t and h. The points of tangency may be located without drawing in the parabola. In Fig. 270,

$$\frac{t}{2k} = \frac{2h}{t}$$

TABLE 79.—FORM FACTORS FOR THE LEWIS EQUATION

Number of teeth	Cycloidal, 14½-deg involute, composite, and generated		20-deg involute				Fellows 20-deg stub teeth													
			Full depth		A.G.M.A. stub		4–5		5–7		6–8		7–9		8–10		9–11		10–12	
	y	Y	y	Y	y	Y	y	Y	y	Y	y	Y	y	Y	y	Y	y	Y	y	Y
10	0.056	0.176	0.064	0.201	0.083	0.261														
11	0.061	0.192	0.072	0.226	0.092	0.289														
12	0.067	0.210	0.078	0.245	0.099	0.311	0.096	0.302	0.111	0.348	0.102	0.320	0.100	0.314	0.096	0.302	0.100	0.314	0.093	0.292
13	0.071	0.223	0.083	0.261	0.103	0.324	0.101	0.318	0.115	0.361	0.107	0.336	0.106	0.332	0.101	0.317	0.104	0.327	0.098	0.308
14	0.075	0.236	0.088	0.276	0.108	0.340	0.105	0.330	0.119	0.374	0.112	0.352	0.111	0.348	0.106	0.332	0.108	0.339	0.102	0.320
15	0.078	0.245	0.092	0.289	0.111	0.349	0.108	0.339	0.123	0.386	0.115	0.364	0.115	0.361	0.110	0.346	0.111	0.348	0.105	0.330
16	0.081	0.255	0.094	0.295	0.115	0.361	0.111	0.348	0.126	0.396	0.119	0.374	0.118	0.370	0.113	0.355	0.114	0.354	0.109	0.340
17	0.084	0.264	0.096	0.302	0.117	0.368	0.114	0.358	0.129	0.405	0.122	0.383	0.121	0.380	0.116	0.364	0.116	0.366	0.111	0.349
18	0.086	0.270	0.098	0.308	0.120	0.377	0.117	0.368	0.131	0.411	0.124	0.390	0.124	0.390	0.119	0.374	0.119	0.374	0.114	0.358
19	0.088	0.277	0.100	0.314	0.123	0.387	0.119	0.374	0.133	0.414	0.127	0.398	0.127	0.398	0.122	0.383	0.121	0.380	0.116	0.364
20	0.090	0.283	0.102	0.320	0.125	0.393	0.121	0.380	0.135	0.425	0.129	0.405	0.129	0.405	0.124	0.390	0.123	0.386	0.118	0.371
21	0.092	0.289	0.104	0.327	0.127	0.399	0.123	0.386	0.137	0.431	0.131	0.411	0.131	0.411	0.126	0.396	0.125	0.392	0.120	0.377
23	0.094	0.296	0.106	0.333	0.130	0.408	0.126	0.396	0.141	0.441	0.134	0.422	0.135	0.422	0.129	0.407	0.128	0.402	0.123	0.387
25	0.097	0.305	0.108	0.339	0.133	0.417	0.129	0.405	0.143	0.449	0.137	0.432	0.138	0.432	0.133	0.417	0.130	0.409	0.126	0.396
27	0.100	0.311	0.111	0.349	0.136	0.427	0.132	0.414	0.146	0.458	0.140	0.440	0.140	0.440	0.135	0.425	0.133	0.417	0.129	0.405
30	0.102	0.320	0.114	0.358	0.139	0.436	0.135	0.425	0.149	0.468	0.143	0.449	0.144	0.452	0.138	0.433	0.136	0.427	0.132	0.415
34	0.104	0.326	0.118	0.371	0.142	0.446	0.139	0.438	0.152	0.478	0.147	0.460	0.148	0.465	0.142	0.447	0.139	0.436	0.136	0.426
38	0.107	0.335	0.122	0.383	0.145	0.455	0.141	0.443	0.155	0.487	0.150	0.471	0.150	0.471	0.145	0.455	0.141	0.443	0.139	0.437
43	0.110	0.345	0.126	0.396	0.147	0.465	0.144	0.452	0.158	0.496	0.153	0.481	0.153	0.481	0.148	0.465	0.144	0.452	0.141	0.443
50	0.112	0.352	0.130	0.408	0.151	0.474	0.147	0.461	0.161	0.506	0.156	0.490	0.156	0.490	0.151	0.471	0.147	0.461	0.144	0.452
60	0.114	0.358	0.134	0.421	0.154	0.484	0.150	0.471	0.164	0.515	0.159	0.500	0.159	0.500	0.154	0.483	0.150	0.471	0.148	0.469
75	0.116	0.364	0.138	0.434	0.158	0.496	0.154	0.484	0.167	0.525	0.162	0.509	0.162	0.509	0.157	0.493	0.153	0.480	0.151	0.474
100	0.118	0.371	0.142	0.446	0.161	0.506	0.158	0.496	0.171	0.536	0.166	0.521	0.166	0.521	0.160	0.503	0.156	0.490	0.154	0.484
150	0.120	0.376	0.146	0.459	0.165	0.518	0.162	0.509	0.174	0.546	0.170	0.534	0.169	0.531	0.164	0.515	0.160	0.503	0.158	0.496
300	0.122	0.383	0.150	0.471	0.170	0.535	0.167	0.525	0.179	0.562	0.174	0.548	0.172	0.542	0.168	0.527	0.165	0.518	0.163	0.512
Rack	0.124	0.390	0.154	0.484	0.175	0.550	0.173	0.543	0.184	0.578	0.179	0.562	0.176	0.553	0.172	0.540	0.170	0.534	0.168	0.528

and

$$t^2 = 4hk$$

Then

$$y = \frac{t^2}{6hp} = \frac{4hk}{6hp} = \frac{2k}{3p} \tag{310}$$

Equation (309) indicates that the load which the gear tooth will carry is minimum when y is minimum, a condition that is found when c is the point of tangency of the parabola and the tooth outline. To determine the minimum value of k without drawing the parabola, select any point c near the narrowest part of the tooth, draw ac and then draw ce perpendicular to ac, draw cg perpendicular to the tooth center line, and scale eg. Repeat for several points close to c. A few trials will determine the minimum value of eg, or k.

286. Working Stress in Gear Teeth. The permissible working stress s in the Lewis equation depends upon the material, the heat-treatment, the accuracy of the machine work, and the pitch-line velocity. Safe working stresses for common gear materials operating at very low velocities are usually assumed to be one-third the ultimate strength. Representative values are given in Table 80.

Slight inaccuracies in the tooth profile and tooth spacing, the fact that the teeth are not absolutely rigid, variations in the applied load, and repetitions of the loading cause impact and fatigue stresses that become more severe as the pitch-line velocity increases. To allow for these additional stresses, it is customary to introduce a velocity factor into the Lewis equation. When V is the pitch-line velocity in fpm.

$$F_t = \frac{s_w f Y}{P}\left(\frac{600}{600 + V}\right) \quad \text{for ordinary industrial gears} \tag{311}$$

operating at velocities up to 2,000 fpm;

$$F_t = \frac{s_w f Y}{P}\left(\frac{1{,}200}{1{,}200 + V}\right) \quad \text{for accurately cut gears} \tag{312}$$

operating at velocities up to 4,000 fpm; and

$$F_t = \frac{s_w f Y}{P}\left(\frac{78}{78 + \sqrt{V}}\right) \quad \text{for precision gears cut with a} \tag{313}$$

high degree of accuracy and operating at velocities of 4,000 fpm and over.

These equations are for gears operating under steady-load conditions. When operating more than 10 hr per day and when subjected to shock, the permissible tangential load should be reduced according to the factors in Table 81.

Table 80.—Safe Beam Stress or Static Stress of Materials for Gears
(Values of s_w for use in the Lewis equations)

Material	Safe stress s_w	Ultimate strength s_u	Yield stress s_y
Cast iron, ordinary	8,000	24,000	
Cast iron, cast teeth	4,500	24,000	
Cast iron, good grade	10,000	30,000	
Semisteel	12,000	36,000	
Cast steel	20,000	65,000	36,000
Cast steel, cast teeth	7,500	65,000	36,000
Forged carbon steel:			
S.A.E. 1020 casehardened	18,000	55,000	30,000
S.A.E.			
1030 not treated	20,000	60,000	33,000
1035 not treated	23,000	70,000	38,000
1040 not treated	25,000	80,000	45,000
1045 not treated	30,000	90,000	50,000
1045 hardened	30,000	95,000	60,000
1050 hardened	35,000	100,000	60,000
Alloy steels:			
Ni, S.A.E. 2320, casehardened	50,000	100,000	80,000
Cr-Ni, S.A.E. 3245, heat-treated	65,000	120,000	100,000
Cr-Van, S.A.E. 6145, heat-treated	67,500	130,000	110,000
Manganese bronze, S.A.E. 43	20,000	60,000	30,000
Gear bronze, S.A.E. 62	10,000	30,000	15,000
Gear bronze, cast teeth	6,000		
Phosphor bronze, S.A.E. 65	12,000	36,000	20,000
Aluminum bronze, S.A.E. 68	15,000	65,000	25,000
Rawhide	6,000		
Fabroil	6,000		
Bakelite	6,000	18,000 bending	
Micarta	6,000	18,000 bending	

For any set of operating conditions, there are many combinations of pitch, face width, and number of teeth that will satisfy the Lewis equation. However, well-proportioned gears should have a face width of from $3p$ to $4p$, or approximately $10/P$. Space requirements may require narrower teeth with a coarser

pitch in installations such as automobile change gears, and turbo-reduction gears have much wider faces.

TABLE 81 —SERVICE FACTORS

Type of load	Type of service		
	8–10 hr per day	24 hr per day	Intermittent, 3 hr per day
Steady	1 00	0 80	1 25
Light shock	0 80	0 65	1 00
Medium shock	0 65	0 55	0 80
Heavy shock	0 55	0.50	0 65

Note: These factors are for completely enclosed gears well lubricated with the correct grade of oil For nonenclosed gears, grease lubricated, use 65 per cent of the tabulated values.

Example. A compressor running at 300 rpm is driven by a 20-hp, 1,200-rpm motor through a pair of $14\frac{1}{2}$-deg full-height gears. The center distance is 15 in., the motor pinion is to be forged steel (S A.E 1045), and the driven gear is to be cast steel. Assume medium shock conditions. Determine the diametral pitch, the face width, and the number of teeth on each gear

Solution. The pitch diameters will be 6 and 24 in., respectively The pitch-line velocity will be

$$V = \frac{\pi \times 6 \times 1{,}200}{12} = 1{,}885 \text{ fpm.}$$

The shock factor from Table 81 is 0.65; hence the design tangential force at the pitch line will be

$$F_t = \frac{20 \times 33{,}000}{0.65 \times 1{,}885} = 538 \text{ lb}$$

If both gears were to be made of the same material, only the weaker pinion would have to be considered In this example, the pinion may be considered in order to approximate the dimensions, after which any adjustments required by the gear can be made. For trial purposes assume a face width equal to $10/P$ and a value of Y equal to 0.30. The permissible stress at low speeds is 30,000 psi for the pinion. Substituting in the Lewis equation

$$538 = \frac{30{,}000 \times 10 \times 0\ 30}{P^2}\left(\frac{600}{600 + 1{,}885}\right)$$

from which

$$P^2 = 40\ 3$$

and

$$P = 6.35$$

This value suggests the use of a standard diametral pitch of 6, with 36 teeth on the pinion and 144 teeth on the gear. Values of Y are 0 330 and 0.374, respectively. Then the face width required is

$$f = \frac{538 \times 6}{30{,}000 \times 0\ 33} \left(\frac{2{,}485}{600}\right) = 1.35 \text{ in.} \qquad \text{for the pinion}$$

and

$$f = \frac{538 \times 6}{20{,}000 \times 0.374} \left(\frac{2{,}485}{600}\right) = 1.78 \text{ in.} \qquad \text{for the gear}$$

A face width of $1\frac{3}{4}$ in. is probably satisfactory.

287. Dynamic Loads on Gear Teeth. For the more important installations where the operation is continuous, equations based on the work of Earle Buckingham are recommended by the A.G.M.A. and other engineering societies. The complete analysis of this method and the derivations of the equations may be found in the works of Mr. Buckingham.* Small machining errors and the deflection of the teeth under load cause periods of acceleration, inertia forces, and impact loads on the teeth with an effect similar to that of a variable load superimposed on a steady load (see Art. 76). The total maximum instantaneous load on the tooth, or dynamic load, is

$$F_d = F_t + F_i = F_t + \frac{0.05V(Kf + F_t)}{0.05V + \sqrt{Kf + F_t}} \qquad (314)$$

where F_d = total equivalent load applied at pitch line, lb.
F_t = tangential load required for power transmission, lb.
F_i = increment load (variable load), lb.
K = a factor depending upon machining errors.

To determine the proper value of K, the maximum errors for the various classes of gear cutting must be known. These errors are given in Table 82. The class of gear cutting required depends upon the speed of operation, and noise is a good measure of the accuracy required. The maximum errors that will permit reasonably quiet operation at different pitch-line velocities are given in Fig. 271. For extremely quiet operation, even these

* BUCKINGHAM, EARLE, Chairman of the Committee, Proposed Recommended Practice of the A.G.M.A. for Computing the Allowable Tooth Loads on Metal Spur Gears, presented at the annual meeting of the A.G M.A., Cleveland, May 12, 1932.

BUCKINGHAM, EARLE, "Spur Gears," McGraw-Hill Book Company, Inc.

errors must be reduced. Knowing the class of gear cutting required, the probable error is selected from Table 82, and K from Table 83.

The maximum dynamic load F_d as determined by Eq. (314), when used in the Lewis equation (309) should give stresses with

TABLE 82.—MAXIMUM ERROR IN ACTION BETWEEN GEARS

Diametral pitch	Class 1, industrial	Class 2, accurate	Class 3, precision
1	0 0048	0 0024	0 0012
2	0 0040	0 0020	0.0010
3	0 0032	0.0016	0 0008
4	0 0026	0.0013	0 0007
5	0 0022	0 0011	0 0006
6 and finer	0 0020	0.0010	0 0005

Courtesy A.G.M.A.

Class 1, industrial gears cut with formed cutters.
Class 2, gears cut with great care.
Class 3, very accurate cut and ground gears.

a reasonable margin of safety below the flexural endurance limit of the material and of course never in excess of the yield stress. With steady power transmission, the stress may be $0.80s_{ef}$, for pulsating loads $0.75s_{ef}$, and for rapidly fluctuating and shock

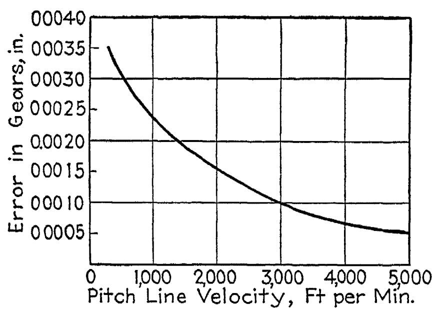


FIG. 271.—Maximum permissible error in gears at various speeds.

loads, $0.65s_{ef}$. These utilization factors are subject to the designer's experience with gears under similar service conditions. Table 84 gives values of the flexural endurance limit for some common gear materials.

288. Design of Spur Gears for Wear. The wear is dependent upon the materials used, the curvature of the tooth surfaces, the finish, the lubrication, and the amount of sliding action on the tooth surfaces. Pitting is a form of wear that occurs chiefly near the pitch line and is probably caused by fatigue failure of the material just under the surface where the pressure between two curved surfaces produces the maximum shear stress, as pointed

TABLE 83.—VALUES OF THE DYNAMIC FACTOR K

Materials	Tooth form	Errors in gears					
		0.0005	0.001	0.002	0.003	0.004	0.005
Cast iron and cast iron	14½-deg	400	800	1,600	2,400	3,200	4,000
Cast iron and steel	14½-deg	550	1,100	2,200	3,300	4,400	5,500
Steel and steel	14½-deg	800	1,600	3,200	4,800	6,400	8,000
Cast iron and cast iron	20-deg, full depth	415	830	1,660	2,490	3,320	4,150
Cast iron and steel	20-deg, full depth	570	1,140	2,280	3,420	4,560	5,700
Steel and steel	20-deg, full depth	830	1,660	3,320	4,980	6,640	8,300
Cast iron and cast iron	20-deg, stub	430	860	1,720	2,580	3,440	4,300
Cast iron and steel	20-deg, stub	590	1,180	2,360	3,540	4,720	5,900
Steel and steel	20-deg, stub	860	1,720	3,440	5,160	6,880	8,600

Courtesy A.G.M.A.
Bronze has approximately the same values of K as cast iron

out in Art. 205. When the material below the surface develops a fatigue crack, the sliding action of the tooth surfaces will pull the surface material out, forming a pit. Wear may take the form of scratches or scores caused by pitted material or dirt carried in the lubricant. Failure of the lubrication may cause the surfaces to overheat and seize. Abrasion and seizing can be practically eliminated if sufficient lubricant of the correct grade is supplied. Wear may occur in soft materials by the sliding action of the teeth pushing the metal toward the pitch line where the sliding action becomes pure rolling and then reverses in direction. This action produces a hump near the pitch line and causes excessive noise.

Since pitting is due to fatigue failure, and abrasion and piling are due to soft material, it is evident that the load limit for wear is determined by the surface endurance limit of the material, the curvature of the surfaces, and the relative hardness of the surfaces. When mating gears are of different materials, the harder will mechanically work-harden the softer, raising its endurance limit which for steels seems to increase in direct proportion to

TABLE 84.—FATIGUE LIMITS OF GEAR MATERIALS

Material	Brinell hardness number	Flexural endurance limit s_{ef}	Surface endurance limit s_{es}
Gray cast iron	160	12,000	90,000
Semisteel	200	18,000	90,000
Phosphor bronze	100	24,000	90,000
Steel	150	36,000	50,000
	200	50,000	70,000
For steel:	240	60,000	86,000
$s_{ef} = 250 \times$ Brinell number	250	61,250	90,000
For 400 Brinell number and	280	70,000	102,000
above, use $s_{ef} = 100{,}000$	300	75,000	110,000
$s_{es} = 400 \times$ Brinell number	320	80,000	118,000
$- 10{,}000$	350	85,000	130,000
	360	90,000	134,000
	400	100,000	150,000
	450		170,000
	500		190,000
	550		210,000
	600		230,000
Nonmetallic			32,000

the Brinell hardness. The pinion should always be the harder to allow for work-hardening of the gear, to preserve the involute profile, to allow for greater abrasive wear on the pinion, and to decrease the possibility of seizing.

According to Buckingham, the load limit for wear is expressed by the equation

$$F_w = \frac{D_p f s_{es}^2 \sin \phi}{1.4} \left(\frac{2n_g}{n_p + n_g} \right) \left(\frac{1}{E_p} + \frac{1}{E_g} \right) \tag{315}$$

where s_{es} is the surface endurance limit from Table 84, n is the number of teeth, and E is the modulus of elasticity, the subscripts g and p referring to the gear and pinion, respectively. The value of F_w should not be less than the permissible value of the dynamic load F_d, from Eq. (314).

289. Illustrative Example. Determine whether or not the gears in the example on page 383 are satisfactory as far as the dynamic and wear loads are concerned.

Solution. For a pitch-line velocity of 1,885 fpm, the permissible error e from Fig. 271 is 0.0017 in. For a 6-pitch gear, from Table 82 a Class 2 gear would have an e of 0 0010 in. and a Class 1 gear would have an e of 0 0020 in. Since the permissible error is 0.0017 in., a Class 2 gear must be used with a permissible error e of 0 001 in. From Table 83, the value of K is 1,600 for 14½-deg involute steel and steel gears when e is equal to 0.001 in. The force transmitted is

$$F_t = \frac{20 \times 33{,}000}{1{,}885}$$
$$= 350 \text{ lb.}$$

Then from Eq. (314)

$$F_d = 350 + \frac{0.05 \times 1{,}885(1{,}600 \times 1.75 + 350)}{0.05 \times 1{,}885 + \sqrt{1{,}600 \times 1\ 75 + 350}} = 2{,}325 \text{ lb}$$

Substituting F_d for F_t in the Lewis equation (309)

$$s = \frac{F_t P}{fY} = \frac{2{,}325 \times 6}{1.75 \times 0.33} = 24{,}100 \text{ psi} \qquad \text{for the pinion}$$

From Table 84, s_{ef} is 36,000 psi for steel with a Brinell number of 150. For the pinion the dynamic stress is $\frac{24{,}150}{36{,}000} = 0.671$ of s_{ef} and is satisfactory.

$$s = \frac{2{,}325 \times 6}{1.75 \times 0.374} = 21{,}310 \text{ psi} \qquad \text{for the gear}$$

From Table 2, s_{ef} is 30,000 psi for medium cast steel. Then the dynamic stress in the gear is $\frac{21{,}300}{30{,}000} = 0.710$ of s_{ef}. This indicates that the dynamic load is not excessive, and that the gears could withstand fluctuating and shock loads.

Check the gears for wear, using Eq. (315)

$$F_w = \frac{6 \times 1\ 75 \times (50{,}000)^2 \times 0.25}{1.4}\left(\frac{2 \times 144}{36 + 144}\right)\left(\frac{1}{30{,}000{,}000} + \frac{1}{30{,}000{,}000}\right)$$
$$= 500 \text{ lb}$$

which is less than $F_d = 2{,}325$ lb.

Since F_w should not be less than F_d, it would be necessary to heat-treat the gears to an average Brinell number of 300, which gives a surface endurance limit of 110,000 psi Then

$$F_w = \frac{6 \times 1.75(110{,}000)^2 \times 0.25}{1.4}\left(\frac{2 \times 144}{36 + 144}\right)\left(\frac{1}{30{,}000{,}000} + \frac{1}{30{,}000{,}000}\right)$$
$$= 2{,}420 \text{ lb}$$

which is greater than $F_d = 2{,}325$ lb.

290. Gears with Cast Teeth. The preceding discussions apply to gears with machine-cut teeth. Gears used in the cheaper grades of machinery where the speeds are low and smooth action not important often have the teeth cast and not machined. The Lewis equation may be used to determine the strength of these gears. The inaccuracies of cast teeth make it unnecessary to use an accurate outline factor, and Y may be assumed to be 0.17 for all gears. The strength of the teeth is expressed by the equation

$$F_t = \frac{s_w f Y}{P} = \frac{0.17 s_w f}{P} = 0.054 s_w p f \qquad (316)$$

The working stress, s_w, may be taken from Table 80 without applying a velocity factor. f is approximately equal to $2.5p$. Standard circular pitches for cast teeth vary by $\frac{1}{8}$-in. increments from $\frac{1}{2}$ to $1\frac{1}{2}$ in., by $\frac{1}{4}$-in. increments from $1\frac{1}{2}$ to 3 in., and by $\frac{1}{2}$-in. increments from 3 to 4 in.

291. Nonmetallic Spur Gears. Gears made of rawhide, laminated fabric, and phenolic-resin materials, such as Bakelite and Micarta, are frequently used to reduce noise. Rawhide and laminated materials are not rigid and should be reinforced at both ends by metal flanges. To avoid charring by the heat of friction, rawhide gears should not be operated at pitch-line velocities greater than 2,500 fpm.

The permissible tangential force on these gears is

$$F_t = \frac{s_w f Y}{P}\left(\frac{150}{200 + V} + 0.25\right) \qquad (317)$$

Well-proportioned nonmetallic gears should have a face width from $9.5/P$ to $12.5/P$. The dynamic-load equation (314) does not apply to nonmetallic gears. A fair estimate of the dynamic load is obtained by dividing the transmitted load F_t by

the velocity factor $\frac{150}{200 + V} + 0.25$. Equation (315) may be used to check these gears for wear. The modulus of elasticity E for nonmetallic gears is approximately 1,000,000 psi.

292. Helical Gears. In helical gears, the teeth are cut in the form of a helix about the axis of rotation, one gear having a right-hand helix, and the mating gear having a left-hand helix.

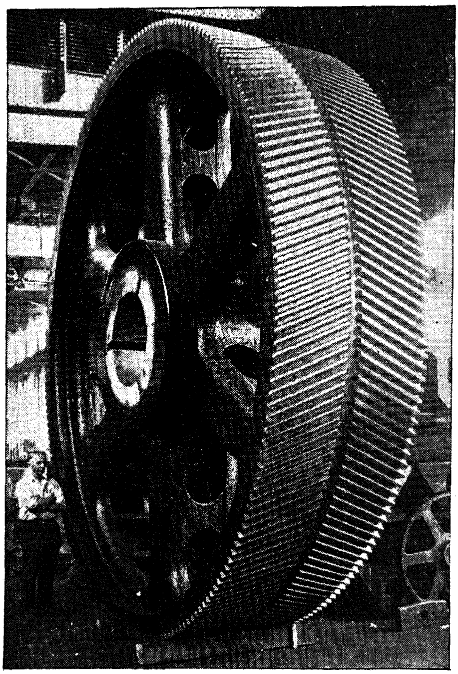


FIG. 272.—Staggered-tooth herringbone gear, "Wuest" type. (*Courtesy Mesta Machine Company.*)

When straight-tooth spur gears begin to engage, the contact theoretically extends across the entire tooth on a line parallel to the axis of rotation. It has already been shown that this sudden application of load produces high impact stresses and excessive noise at high speeds. When helical gears begin to mesh, contact occurs only at the point of the leading edge of the tooth, gradually extending along a diagonal line across the tooth as the gears rotate. The gradual engagement and load application reduce the noise and the dynamic stress so that helical

gears may be operated at higher speeds and can sustain greater tangential loads than straight-tooth spur gears of the same size. Pitch-line speeds of 4,000 to 7,000 fpm are common with automobile and turbine gears, and speeds of 12,000 fpm have been successfully used.

In order that contact may be maintained across the entire active face of the gear, the minimum width must be

$$f_{a\ \min} = \frac{p}{\tan \psi} \tag{318}$$

where ψ = helix angle, deg.

The A.G.M.A.'s recommended practice requires a minimum face 15 per cent greater than this for single helical gears.

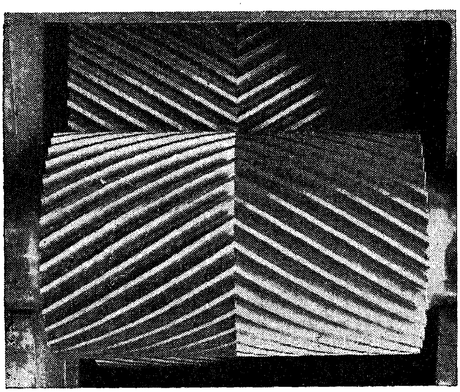


FIG. 273.—Sykes type continuous herringbone gear. (*Courtesy Farrel-Birmingham Company.*)

293. Herringbone Gears. Two helical gears having opposite helixes will eliminate the end thrust if mounted on the same shaft, and if both sets of teeth are cut on a single gear blank a double-helical or herringbone gear results. With the older methods of gear cutting, a groove must be left at the center to provide clearance for the cutters. The development of the Sykes gear-shaper now permits the use of continuous teeth as shown in Fig. 273.

294. Proportions of Helical and Herringbone Gears. The addenda and dedenda are usually the same as those of the A.G.M.A. 20-deg stub-tooth gear as given in Table 78. For-

merly, the pressure angle was standardized at 20 deg when measured in the plane of rotation, and the helix angle at 23 deg. Helical gears are not interchangeable, however, and the A.G.M.A.'s recommended practice (October, 1929) gives the following proportions

Pressure angle in the plane of rotation, ϕ, maximum	25 deg
minimum	15 deg 23 min
Helix angle, ψ, maximum	45 deg
minimum	20 deg
Addendum, a, maximum	$1/P$
minimum	$0.7/P$
Clearance, c, maximum	$0.3/P$
minimum	$0.157/P$
Dedendum, b	$a + c$

The maximum pressure angle combined with the maximum helix angle gives a normal pressure angle of about 18 deg and 15 min. The minimum pressure angle combined with the minimum helix angle gives a normal pressure angle of about 14½ deg. Helix angles of 15 deg 15 min, and 23 deg are in common use for single helical gears, and 30 and 45 deg for herringbone gears, the larger angles being used with the higher pitch-line velocities.

Recommended practice gives the pressure angles in the plane of rotation, even when standard hobs are used to form the teeth. The relation between the normal circular pitch and the real or normal pitch is

$$p_n = p \cos \psi$$

and

$$P_n = \frac{P}{\cos \psi} \tag{319}$$

The relation between the pressure angles is

$$\tan \phi_n = \tan \phi \cos \psi \tag{320}$$

The cutter for helical gears should be selected on the basis of the normal pitch, which should be standard, and on the virtual radius of the normal helix. However, Fellows cutters are based on a standard pitch in the plane of rotation.

$$\text{Number of teeth for which cutter is selected} = \frac{n}{\cos^3 \psi} \tag{321}$$

295. Strength of Helical Gears. The strength of helical gears is determined by the equation

$$F_t = \frac{s_w f Y}{P}\left(\frac{78}{78 + \sqrt{V}}\right). \tag{322}$$

The form factor may be taken from Table 79 when a standard profile in the plane of rotation is used. For any other profile, the method outlined in Art. 285 must be used. The active face width should be from $12.5/P$ to $20/P$ for helical gears, and $20/P$ to $30/P$ for herringbone gears. The width should be less than four times the pinion diameter to prevent excessive deflection, although widths of five diameters have been used. For general industrial gears, widths from 2 to $2\frac{1}{2}$ diameters are desirable.

296. Dynamic Loads on Helical Gears. The method outlined in Art. 287 may be used with helical gears when Eq. (314) is modified as follows

$$F_d = F_t + \frac{0.05V(Kf \cos^2 \psi + F_t) \cos \psi}{0.05V + (Kf \cos^2 \psi + F_t)^{\frac{1}{2}}} \tag{323}$$

297. Wear on Helical Gears. Ordinary industrial gears may be designed for wear by applying the service factors from Table 81 in the Eq. 322. High-speed reduction gears for turbines are commonly designed to carry pitch-line loads of 100 lb per in. of face per in. of diameter, and loads as high as 320 lb have been successfully carried.

The better grades of helical gears should be checked for wear by the method outlined in Art. 288. For this purpose, Eq. (315) is modified as follows:

$$F_w = \frac{D_p f s_{es}^2 \sin \phi}{1.4 \cos^2 \psi}\left(\frac{2n_g}{n_p + n_g}\right)\left(\frac{1}{E_p} + \frac{1}{E_g}\right) \tag{324}$$

Considerable research has been carried on in the last few years in connection with wear. In an article, Determining Capacity of Helical and Herringbone Gearing,* W. P. Schmitter presented a very complete analysis with recommended equation for use in gear design. This work should be carefully read by any designer interested in this type of gearing. The following

* *Mach. Design*, June and July, 1934.

equation developed by Schmitter for use in rating helical gear speed reducers is used in modified form in the tentative specifications of the A.G.M.A. and the A.P.I. for gear speed-reducers.

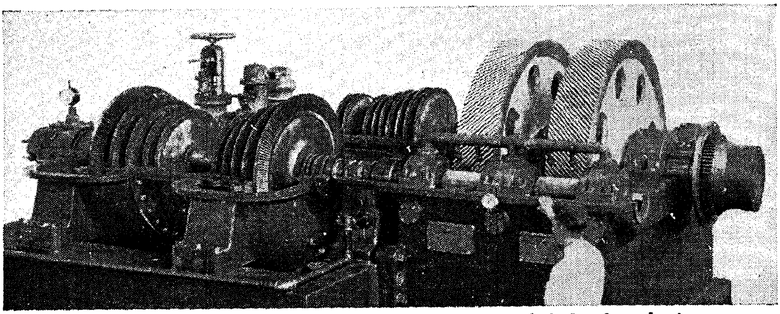


FIG. 274.—Steam turbine with right- and left-hand helical reducing gears.

The horsepower capacity is expressed by the equation

$$\text{hp} = \frac{C_m \times C_q \times C_c \times C_r \times C_v \times C_i \times D_p^2 f N}{126{,}000} \tag{325}$$

where $C_m = (s_s/958)^2$ = material factor.

s_s = allowable working shear stress, equal to the endurance limit s_{es} or to $s_y/\sqrt{FS}$ depending on which gives the smaller value, psi.

FS = factor of safety, assumed to be $2\frac{1}{3}$.

$C_q = \dfrac{0.9L_a}{p \cos \phi \cos \psi}$ = contact-length factor.

L_a = length of the line of action [see Eq. (306)], in.

$C_c = \sin \phi \cos \phi$ = curvature factor.

$C_r = \dfrac{n_g}{n_p + n_g}$ = ratio factor.

$C_v = \dfrac{78}{78 + \sqrt{V}}$ = velocity factor.

C_i = 0.64 for gears up to 2 in. face width.
= $0.667 - 0.0135f$ for gears from 2 to 18 in. face width.
= 0.425 for gears over 18 in. face width.

D_p = pitch diameter of pinion, in.

f = face width, in.

N = rpm of pinion.

The factor C_i is called the inbuilt factor, and is used to provide for the increasing errors as the gear size increases. These

errors increase partly on account of machining difficulties and partly because the mountings for the larger gears cannot be made so accurately or so rigid as those of small gears.

When the gear proportions are not accurately known and for the more general applications, the product of C_c and C_g may be assumed to be 0.4.

This equation gives the rating for uniform load conditions assuming 8 hr per day service. For other conditions of service apply the factors from Table 81.

298. Proportions of Gears. To complete the gear design, the first step is to determine the shaft size. If the pinion teeth are cut integral with the shaft, the root diameter should be slightly larger than the required shaft diameter. When a solid pinion is keyed to the shaft, the minimum pitch diameter is approximately

$$D_{\min} = 2 \times \text{bore} + \frac{0.25}{P} \tag{326}$$

The Nuttall Works of the Westinghouse Electric and Manufacturing Company recommend that the minimum thickness of metal between the keyway and the root circle shall be

$$t_{\min} = \frac{1}{P}\sqrt{\frac{n_p}{5}} \tag{327}$$

The outside diameter of the hubs of larger gears should be 1.8 times the bore for steel, 2 times the bore for cast iron, and 1.65 times the bore for forged steel or light service. The hub length should be at least $1\frac{1}{4}$ times the bore for light service and never less than the face width of the gear.

Small gears may be built with a web joining the rim to the hub. The web thickness should be from $1.6/P$ to $1.9/P$. The larger gears are provided with arms: four arms for split gears under 40 in. in diameter; six arms for gears up to 120 in. in diameter; eight arms for larger gears. These arms are assumed to be cantilever beams loaded at the pitch line, with the load equally distributed to all arms. The design load is the stalling load, or load that will develop the maximum stress in the teeth at zero velocity. Hence

$$F_o = \frac{s_w f Y}{P} \tag{328}$$

and

$$\frac{I}{c} = \frac{F_o D}{2n_a s_w} \tag{329}$$

where I/c = section modulus of arm section, in.^{3}
F_o = stalling load, lb.
n_a = number of arms.
s_w = permissible working stress in tension, psi.

Arms of various cross sections are used as shown in Fig. 275, and the elliptical arm is generally used except on very large and wide gears. Assuming the usual elliptical arm with the major axis twice the minor axis, the major axis at the outside of the hub will be

$$h = 4\sqrt[3]{\frac{I}{\pi c}} = \sqrt[3]{20.4\,\frac{I}{c}} \tag{330}$$

The arms are usually tapered toward the rim about $\frac{3}{4}$ in. per ft.

The minimum thickness of the rim below the root circle is generally taken to be equal to the tooth thickness at the pitch line. This rule should be used only as a rough check since it does not allow for the differences in support furnished by different numbers of arms. According to the Nuttall Works, this thickness should be

$$t_r = \frac{1}{P}\sqrt[3]{\frac{n_g}{2n_a}} \tag{331}$$

where n_g = number of teeth in gear.

Other dimensions may be made as indicated in Fig. 275. In the final design of the gear, certain dimensions must be modified to prevent sudden changes in the thickness of adjoining parts. Unless large fillets are provided and sudden changes in section are avoided, there is danger of weakening, or even of breakage, due to shrinkage stresses during the cooling of the casting.

299. Gear Mountings and Bearings. Gear mountings and bearings should be placed so that the bending deformation will be reduced to a minimum. It is often advisable to place a center bearing between the two sets of teeth on right- and left-handed helical gears. The deflection caused by bending or twist should not exceed 0.001 in. when measured over the length of the pinion. All computations for deformation are based on the

pitch diameter. Although there is no axial thrust on herringbone gears, the pinion (or the gear) should be mounted so that it can float axially to prevent binding.

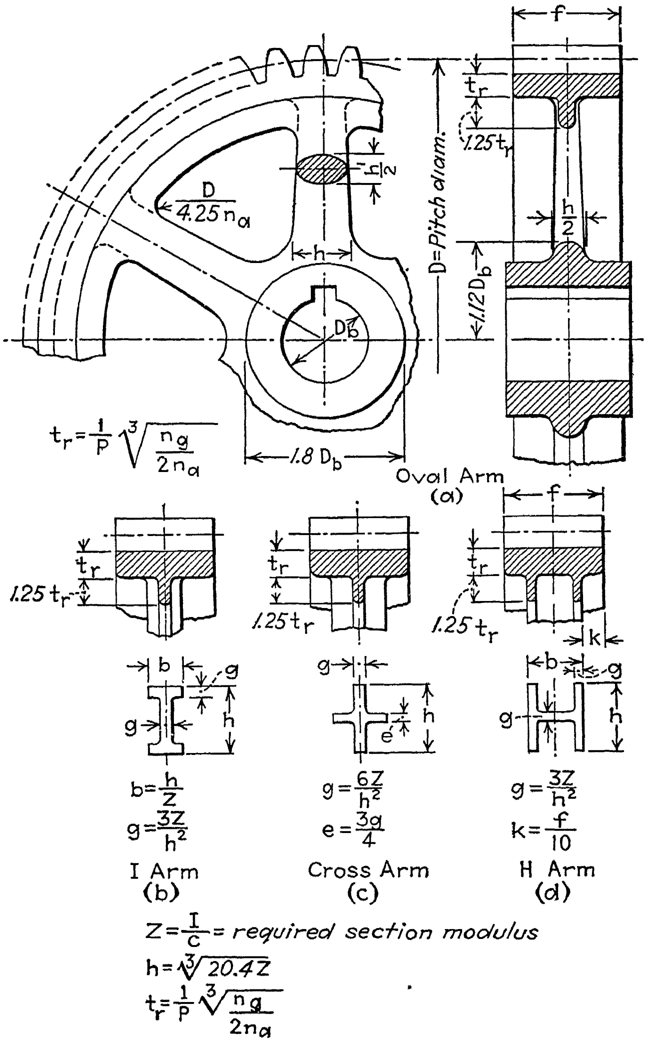


FIG. 275.

The load transmitted to the bearings of plain spur gears is the normal pressure between the tooth surfaces, or

$$F_n = \frac{F_t}{\cos \phi}$$

and the reaction on each bearing is

$$F_A = \frac{F_n b}{a + b}$$

and

$$F_B = \frac{F_n a}{a + b} \tag{332}$$

where a and b are the distances from the gear to the bearings.

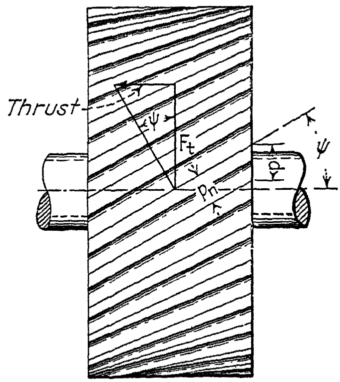


FIG. 276.

With helical gears there is an additional load caused by the end thrust at the pitch line of the gear. The end thrust, or axial load, is from Fig. 276

$$F_a = F_t \tan \psi \tag{333}$$

and the couple resisting this comprises the reactions

$$F_{Ay} = F_{By} = \frac{F_a D}{2(a + b)} \tag{334}$$

Note that F_{Ay} and F_{By} act in opposite directions and are not in the same plane with the reactions due to the normal pressure. The total reaction on each bearing is the vector sum of the reactions due to the normal pressure and the end thrust.

There is no end thrust on herringbone gears so that the bearing reactions are due to the normal pressures only.

300. Efficiency of Gears. It is generally assumed that the friction loss in gear teeth depends on the tooth profile, pitch-line velocity, surface finish, and lubrication. However, when there is sufficient lubrication to prevent overheating and scoring, the friction appears to be practically independent of the velocity. The finish of the tooth surface is the most important factor in the efficiency of gears. Gears cut in accordance with good commercial practice have an efficiency of 98 per cent or more. When lubrication is poor the efficiency may drop as low as 95 per cent. The power loss in the supporting bearings must be considered in addition to the loss in the teeth themselves.

301. Lubrication. To obtain the maximum life, the gears must be supplied with a generous supply of the proper lubricant. The lubricant must maintain an oil film between the teeth and must also carry away the heat of friction, especially from the pinion, which, having more contacts per minute, tends to heat faster than the larger gear. The lubricant must be thin enough to penetrate the space between the teeth and heavy enough that the pressure will not break the oil film. Oil should be kept clean, since grit and metal dust carried in suspension in the oil will cause abrasive action on the tooth surfaces. With proper lubrication and correct alignment of the bearings, a good pair of gears will have an indefinite life.

302. Gear Ratios and Gear Trains. Although large gear ratios have been obtained, it is customary to limit the reduction to 6:1 for spur gears, and 10:1 for helical and herringbone gears. For larger reductions two or more pairs of gears are used. In general, the permissible stress in the second and subsequent pairs of gears is about 75 per cent that allowed in the first driving pair.

CHAPTER XIX

BEVEL, WORM, AND SPIRAL GEARS

All gears previously discussed have had teeth cut on cylindrical pitch surfaces. When the shafts intersect, the pitch surfaces are conical, and the gears are called bevel gears. The shafts may intersect at any angle, and the bevel gears may have external or internal contact, and they may have straight or spiral teeth. Unless otherwise stated, a pair of bevel gears is assumed to be straight toothed and to have the axes intersect at right angles.

303. Forming Bevel-gear Teeth. The strength, wearing qualities, smoothness of action, and noise depend in large measure on the tooth profile and the method of manufacture. The teeth may be cast, cut with a rotating formed milling cutter, a template planer, or a generating planer. Cast teeth are used only for rough work where the speeds are low and noise is not objectionable. Milled teeth are used on many industrial gears because most jobbing shops have a milling machine, whereas only shops specially equipped for gear making have bevel-gear planers and generators. No formed milling cutter can produce properly shaped teeth, since all tooth elements should converge to the apex of the pitch cone. Hence milled teeth must be hand filed and honed to fit the teeth of the mating gear. Such gears are not interchangeable, but must be used in fitted pairs. Gear planers and shapers, using reciprocating tools whose path passes through the apex of the pitch cone, form teeth with the proper converging profile.

The tooth profile at the large end of the gear should theoretically be laid out on a sphere. The profile laid out on the surface of the back cone differs only slightly from the spherical profile, and the cone can be developed into a plane surface on which to study the tooth form and the tooth action. The profile of the tooth at the large end is therefore the same as the profile of a spur gear laid out on a pitch radius equal to the back-cone radius. The number of teeth on this imaginary spur gear is

called the virtual number, or formative number, of teeth and is found by the equation

$$n_v = \frac{n_g}{\cos \gamma_p} \tag{335}$$

where n_v = virtual number of teeth.
n_g = actual number of teeth.
γ_p = pitch cone angle.

The virtual number of teeth, not the actual number of teeth, must be used in selecting the proper cutters and in all computations for the strength of bevel gears (Gleason system excepted).

304. Proportions of Bevel Gears. Industrial gears were formerly made with standard full-height $14\frac{1}{2}$-deg involute teeth with the same proportions as given in Table 78 for spur gears. These had undercut teeth on pinions of low tooth numbers.

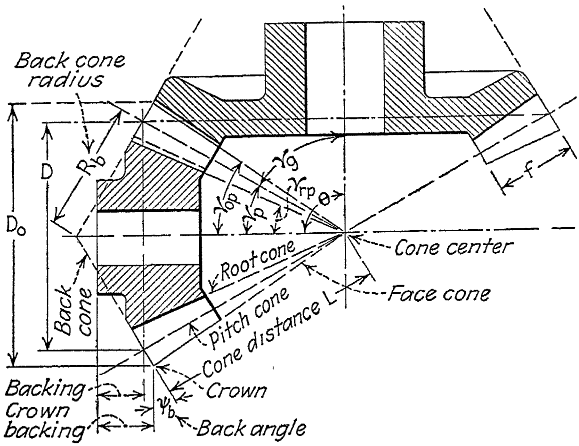


FIG. 277.

To prevent this undercutting, the use of long and short addendum teeth was introduced for pinions having less than 32 teeth with the $14\frac{1}{2}$-deg pressure angle, and less than 18 teeth with the 20-deg pressure angle. The total working depth with this system is taken as $2/P$, and the addendum of the pinion is made 0.7 of the working depth. Proportions for this system are given in Table 85.

The Gleason Company developed a system, applicable to generated and spiral-tooth bevel gears, which was later

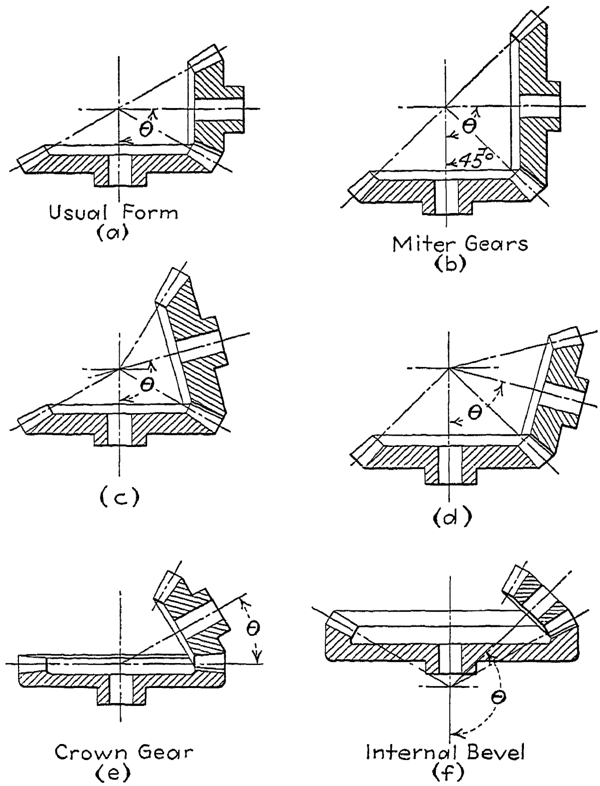


FIG. 278.—Bevel-gear arrangements.

adopted as the A.G.M.A.'s recommended practice. In this the pressure angle varies with the number of teeth on the pinion and with the gear ratio. The addenda are chosen so that the sliding action during approach is slightly less than that during

TABLE 85.—PROPORTIONS OF LONG- AND SHORT-ADDENDUM BEVEL GEARS

Item	Pinion (driver)	Gear (driven)
Addendum	$1.4/P$	$0.6/P$
Dedendum	$0.7571/P$	$1.5571/P$
Tooth thickness on the pitch line,		
14½-deg	$1.778/P$	$1.364/P$
20-deg	$1.862/P$	$1.280/P$

TABLE 86.—PROPORTIONS FOR STRAIGHT-TOOTH BEVEL GEARS*
(Gleason Generated System)

Gear Ratios	Pressure Angle, deg
14 or more teeth in the pinion	$14\frac{1}{2}$
13–13 to 13–24	$17\frac{1}{2}$
13–25 and higher	$14\frac{1}{2}$
12–12 and higher	$17\frac{1}{2}$
11–11 to 11–14	20
11–15 and higher	$17\frac{1}{2}$
10–10 and higher	20

* To be used only when the shafts are at right angles, and when the pinion is the driver and has at least 10 teeth.

$$\text{Working depth} = \frac{2}{P} \qquad \text{Total depth} = \frac{2.188}{P}$$

$$\text{Addendum of gear} = \frac{\text{addendum from table}}{P}$$

$$\text{Addendum of pinion} = \frac{2}{P} - \text{addendum of gear}$$

$$\text{Dedendum of gear} = \frac{2.188}{P} - \text{addendum of gear}$$

$$\text{Dedendum of pinion} = \frac{2.188}{P} - \text{addendum of pinion}$$

Gear ratios		Add., In.	Gear ratios		Add., In.	Gear ratios		Add., In.	Gear ratios		Add., In.
From	To		From	To		From	To		From	To	
1 00	1.00	1.000	1 15	1.17	0.880	1.42	1.45	0.760	2.06	2 16	0 640
1 00	1 02	0.990	1.17	1.19	0.870	1.45	1.48	0.750	2.16	2.27	0 630
1 02	1.03	0.980	1.19	1.21	0.860	1.48	1.52	0.740	2.27	2.41	0 620
1 03	1.04	0.970	1.21	1.23	0.850	1.52	1.56	0.730	2.41	2.58	0 610
1.04	1.05	0.960	1.23	1.25	0.840	1.56	1.60	0.720	2.58	2.78	0 600
1 05	1 06	0.950	1.25	1.27	0.830	1.60	1.65	0.710	2.78	3 05	0 590
1 06	1 08	0.940	1.27	1.29	0.820	1.65	1.70	0.700	3.05	3.41	0 580
1 08	1.09	0.930	1.29	1.31	0.810	1.70	1.76	0.690	3.41	3 94	0 570
1.09	1.11	0.920	1 31	1.33	0 800	1.76	1.82	0.680	3 94	4 82	0.560
1.11	1.12	0.910	1.33	1.36	0.790	1.82	1.89	0.670	4.82	6 81	0 550
1 12	1.14	0.900	1 36	1.39	0.780	1 89	1 97	0 660	6 81	∞	0.540
1.14	1 15	0 890	1.39	1.42	0.770	1.97	2.06	0.650			

* Courtesy A.G.M A.

recession, thus obtaining smoother action and quieter operation. The recommended proportions for straight bevel gears are given in Table 86, and for spiral bevel gears in Table 87.

TABLE 87.—PROPORTIONS FOR SPIRAL BEVEL GEARS*
(Gleason Generated System)

Gear Ratios	Pressure Angle, deg
12 or more teeth in the pinion.	$14\frac{1}{2}$
11–11 to 11–19	$17\frac{1}{2}$
11–20 and higher.	$14\frac{1}{2}$
10–10 to 10–24	$17\frac{1}{2}$
10–25 and higher.	$14\frac{1}{2}$

* To be used only when the shafts are at right angles, and when the pinion is the driver and has at least 10 teeth.

$$\text{Working depth} = \frac{1.700}{P} \qquad \text{Total depth} = \frac{1.888}{P}$$

$$\text{Addendum of gear} = \frac{\text{addendum from table}}{P}$$

$$\text{Addendum of pinion} = \frac{1.700}{P} - \text{addendum of gear}$$

$$\text{Dedendum of gear} = \frac{1.888}{P} - \text{addendum of gear}$$

$$\text{Dedendum of pinion} = \frac{1.888}{P} - \text{addendum of pinion}$$

Gear ratios		Add., In.	Gear ratios		Add., In.	Gear ratios		Add., In.	Gear ratios		Add., In.
From	To		From	To		From	To		From	To	
1.00	1.00	0.850	1.15	1.17	0.750	1 41	1.44	0.650	1.99	2.10	0.550
1.00	1.02	0.840	1.17	1.19	0.740	1.44	1 48	0 640	2.10	2.23	0.540
1.02	1 03	0.830	1.19	1.21	0.730	1 48	1.52	0.630	2.23	2.38	0.530
1.03	1 05	0.820	1.21	1.23	0.720	1.52	1.57	0.620	2.38	2.58	0.520
1 05	1.06	0.810	1.23	1.26	0.710	1 57	1 63	0.610	2.58	2.82	0.510
1.06	1 08	0.800	1.26	1.28	0.700	1.63	1.68	0.600	2.82	3.17	0.500
1.08	1.09	0.790	1.28	1 31	0.690	1.68	1.75	0.590	3.17	3.67	0.490
1.09	1.11	0.780	1.31	1.34	0.680	1.75	1.82	0.580	3.67	4.56	0 480
1 11	1 13	0.770	1.34	1.37	0 670	1 82	1.90	0 570	4 56	7.00	0.470
1.13	1.15	0 760	1.37	1.41	0 660	1.90	1.99	0 560	7.00	∞	0.460

* Courtesy A.G.M.A.

305. Strength of Bevel Gears. Since the size of the tooth and the force per unit of face length vary across the face of the tooth, the Lewis equation must be modified for use with bevel gears.

In Fig. 279, consider a very short length of tooth, dl, over which the force may be considered as uniform in intensity. The

force acting on this portion of the tooth is dF, and the Lewis equation becomes

$$dF = sp_l y\, dl$$

where p_l is the circular pitch at distance l from apex of pitch cone and the turning moment about the axis produced by this force is

$$r\, dF = rsp_l y\, dl$$

The tooth thickness, the circular pitch, and the radius r at

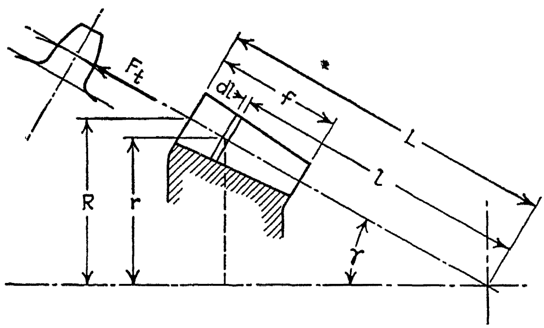


FIG. 279.

any point are proportional to the distance from the apex of the pitch cone. Hence

$$r\, dF = \frac{Rls}{L} \times \frac{pl}{L} \times y\, dl = Rspy \left(\frac{l}{L}\right)^2 dl$$

By integration of this equation, the total turning moment is

$$T = \frac{Rspy}{L^2} \int_{L-f}^{L} l^2\, dl = Rspfy \left(1 - \frac{f}{L} + \frac{f^2}{3L^2}\right)$$

The force that, applied at the pitch line (large end of the gear), will produce this torque is

$$F_t = sfpy \left(1 - \frac{f}{L} + \frac{f^2}{3L^2}\right) \tag{336}$$

The width of the tooth face in bevel gears is limited to one-third the length of the cone distance; *i.e.*, the maximum value of f is $L/3$. Hence the last term in the parenthesis will never be greater than $\frac{1}{27}$ and can be disregarded without appreciable error. By elimination of this term, the usual form of the Lewis

TABLE 88.—FORM FACTORS Y FOR BEVEL GEARS
(Gleason Generated System)
Straight-tooth bevel gears

Number of teeth in pinion	Gear ratios														
	1 00 to 1 25	1 25 to 1 50	1.50 to 1 75	1 75 to 2 00	2 00 to 2 25	2 25 to 2 50	2 50 to 2 75	2 75 to 3 00	3 00 to 3 25	3.25 to 3 50	3.50 to 3 75	3 75 to 4 00	4 00 to 4 50	4 50 to 5 00	5 00 to ∞
10	0 231	0 260	0 280	0 294	0 305	0 315	0 324	0 332	0 340	0 347	0 353	0 358	0 365	0 371	0 377
11	0 268	0 264	0 273	0 286	0 296	0 303	0 309	0 315	0 320	0 324	0 328	0 332	0 336	0 340	0 342
12	0 248	0 265	0 281	0.295	0 308	0.318	0 328	0.335	0 341	0 345	0 348	0 351	0 353	0 355	0 356
13	0 264	0 278	0 291	0 280	0 278	0 286	0 291	0 295	0 298	0.299	0 301	0 303	0 305	0 307	0 310
14	0 242	0 254	0 263	0 272	0 281	0 288	0 294	0 299	0 304	0.307	0 310	0 313	0 316	0 318	0 319
15	0 248	0 258	0 266	0.274	0.283	0 290	0.296	0.301	0 305	0.308	0 312	0 315	0 318	0 319	0 320
16	0 252	0 261	0 269	0 277	0 285	0 292	0 298	0 304	0 308	0.312	0 314	0 317	0 319	0 321	0 323
17–18	0 257	0 265	0.273	0.281	0 288	0 295	0 302	0 307	0 311	0 315	0.318	0 320	0 322	0 325	0 326
19–21	0 265	0 272	0 279	0 286	0.294	0 300	0.307	0 312	0 317	0.320	0 324	0 326	0 328	0 330	0 332
22–25	0 274	0 281	0 288	0.295	0.301	0.307	0 314	0 319	0 324	0 327	0 331	0 332	0 335	0 337	0 338
26–30	0 284	0 291	0.297	0 304	0.310	0 317	0 322	0 327	0 332	0.336	0 339	0 342	0 344	0 346	0 347

Spiral-tooth bevel gears

Number of teeth in pinion	1 00 to 1 25	1 25 to 1 50	1.50 to 1 75	1 75 to 2 00	2 00 to 2 25	2 25 to 2 50	2 50 to 2 75	2 75 to 3 00	3 00 to 3 25	3.25 to 3 50	3.50 to 3 75	3 75 to 4 00	4 00 to 4 50	4 50 to 5 00	5 00 to ∞
11	0 316	0 335	0 343	0 325	0 327	0 333	0 338	0 344	0 350	0 356	0 361	0 367	0 375	0 384	0 390
12	0 298	0 318	0 333	0 343	0 351	0 357	0 363	0 368	0 372	0 377	0 379	0 381	0 384	0 386	0 388
13	0 302	0 320	0 334	0 343	0 351	0 358	0 365	0 371	0 376	0 381	0 384	0 386	0 388	0 391	0 393
14	0 306	0 322	0 334	0 345	0.354	0 362	0 369	0 374	0 378	0.382	0 386	0 389	0 391	0 393	0 395
15	0 314	0 330	0 342	0 352	0.360	0 368	0.374	0 380	0 385	0.389	0.392	0 394	0 397	0 399	0 402
16	0 322	0 335	0 347	0 358	0 367	0 374	0 381	0.386	0 390	0.394	0 397	0 400	0 402	0 404	0 406
17–18	0 329	0 343	0 354	0 364	0.373	0 382	0 389	0.394	0.398	0 400	0 403	0 406	0 407	0 409	0 410
19–21	0 339	0 351	0 362	0 373	0 382	0.389	0 396	0 401	0 405	0 407	0 410	0 411	0 412	0 414	0 415
22–25	0 351	0 363	0.373	0 382	0 391	0.398	0 403	0 407	0 410	0 412	0.413	0 414	0 415	0 417	0 418
26–30	0 364	0 374	0.384	0.393	0.399	0.404	0.407	0.410	0.412	0 414	0.415	0.416	0 417	0.418	0 419

Courtesy A G.M.A.

equation for bevel gears is obtained.

$$F_t = sfpy\left(\frac{L-f}{L}\right) = \frac{sfY}{P}\left(\frac{L-f}{L}\right) \tag{337}$$

where p or P = pitch at large end.

F_t = equivalent tangential force at large end, lb.

y or Y = form factor for the virtual number of teeth.

The stress to be used in this equation is

$$s = s_w\frac{600}{600+V} \quad \text{for full-depth teeth finished with formed cutters} \tag{338}$$

and

$$s = s_w \frac{1{,}200}{1{,}200 + V} \qquad \text{for the Gleason generated system}$$

The velocity to be used is that at the large end. Form factors for the Gleason system are given in Table 88. In this case note that the form factor is selected from the table for the actual number of teeth on the pinion and not for the virtual number of teeth.

306. Design of Bevel Gears for Wear. To keep the wear within reasonable limits, the power transmitted must be based on

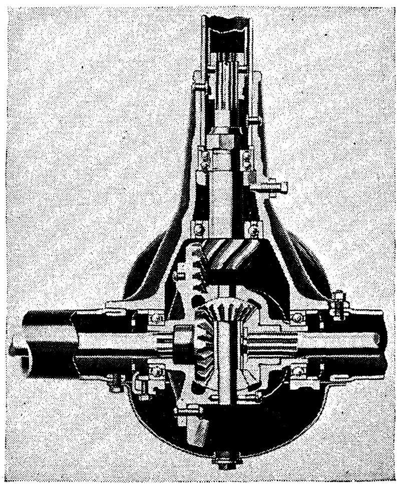


FIG. 280.—Straight- and spiral-tooth bevel gears in an automobile differential.

the materials, their heat-treatment, and the service conditions. The A.G.M.A. recommends that the permissible tooth load be limited by the formula

$$F_t = 375 C_m C_s f \sqrt{\frac{n_p}{P}} \qquad \text{for straight bevel gears} \tag{339}$$

and

$$F_t = 470 C_m C_s f \sqrt{\frac{n_p}{P}} \qquad \text{for spiral bevel gears} \tag{340}$$

where C_m is a material factor from Table 89, and C_s is a service factor from Table 90.

This equation is based on casehardened steel in both gears, continuous operation at rated load with no shock, 50 per cent starting overload, 60 per cent momentary overload, good lubrication, and proper mounting on rigid bearings.

The dynamic loading and the wear loads may be checked by Eqs. (314) and (315) for spur gears, using the virtual numbers of

TABLE 89.—MATERIAL FACTOR FOR EQS. (339) AND (340)

Material: Pinion	Material: Gear	Factor C_m
Soft steel	Cast iron	0.33
Casehardened steel	Heat-treated steel	0.50
Casehardened steel	Casehardened steel	1.00

TABLE 90.—SERVICE FACTORS FOR EQS. (339) AND (340)

Type of Drive	Factor C_s
Air compressor	0 75
Airplane	1.00–1.50
Air separator	1.00
Blowers and fans	1 00
Centrifugal extractors	1.00
Coal- and rock-screen drive	1 00
Coal and rock crushers	0 50
Conveyors	0.65–1 00
Electric tools (portable)	0 75
Hoisting machinery	1 00–1.30
Machine tools:	
Direct motor drive	0.65
Belt drive	1 00
Intermittent belt drive	1.30
Mining machinery	0.75
Pneumatic tools	0.75
Pulverizers (coal and cement)	1.00
Pumps:	
Centrifugal	1.00
Reciprocating	0 65
Speed reducers	1.00
Well-drilling machinery	0.75
Woodworking machinery	1.00

teeth for n_p and n_g, the pitch-line velocity at the large diameter, and F_t as the equivalent tangential force at this velocity.

Well-proportioned bevel gears have a face width from about $6/P$ to $10/P$ but never exceeding $L/3$.

307. Bearing Loads on Bevel Gears. Most bevel gears are mounted so that they overhang their supports. This condition, together with the thrust loads, makes the bearing loads much

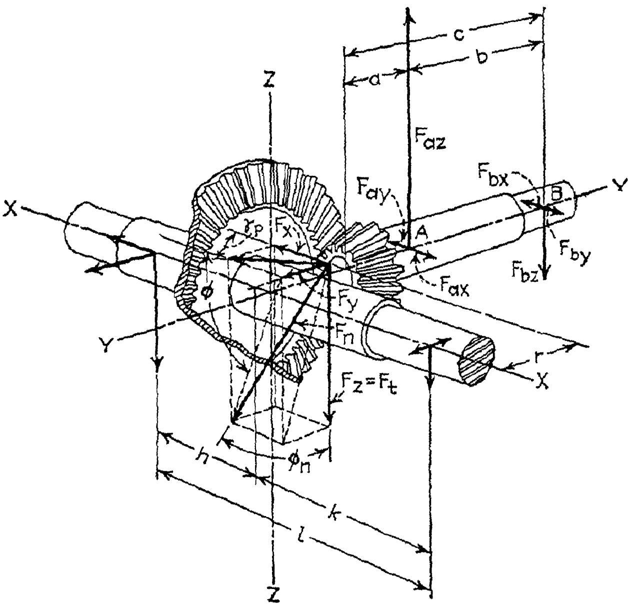


FIG. 281.

more severe than in spur gearing. In Fig. 281 a pair of straight bevel gears is shown with the loads exerted by the pinion on the gear tooth indicated. The force F_n is the actual force between the teeth. The forces acting on the pinion tooth are the reverse of those shown in the figure. An examination of the figure shows that

$$F_z = F_t = F_n \cos \phi_n \tag{341}$$

$$F_x = F_n \sin \phi_n \cos \gamma_p = F_t \tan \phi_n \cos \gamma_p$$

and

$$F_y = F_n \sin \phi_n \sin \gamma_p = F_t \tan \phi_n \sin \gamma_p$$

The force F_t is the turning force tangential to the pitch cone. The force F_x is the end thrust on the gear and a radial force

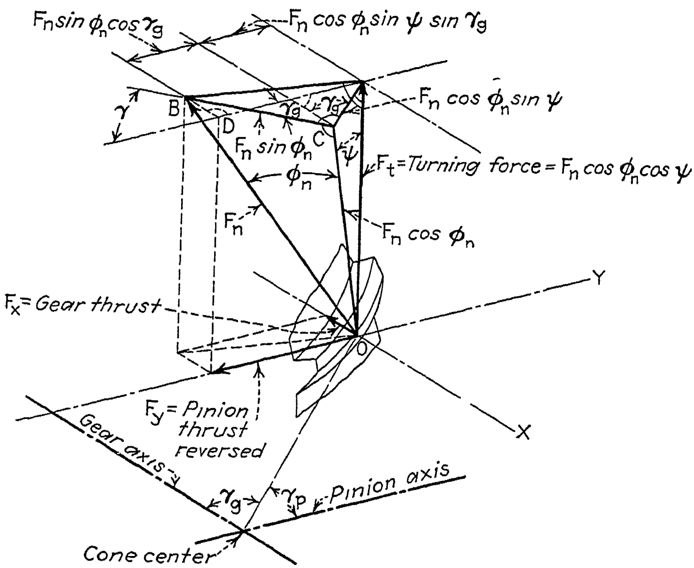


FIG 282.—Forces acting on a spiral bevel gear.

TABLE 91 —AXIAL THRUST ON SPIRAL BEVEL GEARS

Driving pinion		Axial thrust (+ indicates that the gear, or pinion, is forced away from the cone center)
Spiral	Rotation viewed toward the cone center	
RH	Clockwise	$\left[+\frac{\tan \phi_n \sin \gamma}{\cos \psi} - \tan \psi \cos \gamma\right] F_t$ for the driver
LH	Counter-clockwise	$\left[+\frac{\tan \phi_n \sin \gamma}{\cos \psi} + \tan \psi \cos \gamma\right] F_t$ for the driven
RH	Counter-clockwise	$\left[+\frac{\tan \phi_n \sin \gamma}{\cos \psi} + \tan \psi \cos \gamma\right] F_t$ for the driver
LH	Clockwise	$\left[+\frac{\tan \phi_n \sin \gamma}{\cos \psi} - \tan \psi \cos \gamma\right] F_t$ for the driven

ψ = spiral angle of pinion.
γ = pitch angle of pinion.
ϕ_n = tooth-pressure angle in plane normal to the cone element.
F_t = turning force, lb.

on the pinion. The force F_y is a radial force on the gear and the end thrust on the pinion. The actual forces on the bearings are indicated in the figure and are found in the same manner as described for worm gears in Art. 315.

Note that the forces are assumed to be concentrated at the center of pressure on the tooth surface, which is practically at the center of the face. In the Lewis equation, the value of F_t was the equivalent force concentrated at the large end of the gear. Hence the values in the Lewis equation must be multiplied by the factor $D/(D - f \sin \gamma)$ to obtain the value of F_t in the bearing force equations.

The end thrust is much more serious in spiral bevel gearing than in straight bevel gearing, and the thrust reverses in direction when the rotation of the gears reverses. Hence spiral bevel gears must be mounted with thrust bearings capable of carrying thrust in both directions. In addition to the thrust of a straight bevel tooth there is an axial force due to the spiral form of the tooth. The total end thrust on the pinion, from Fig. 282, is

$$F_y = \frac{F_t \tan \phi_n \sin \gamma}{\cos \psi} \pm F_t \tan \psi \cos \gamma \tag{342}$$

the plus or minus sign depending on the direction of the spiral and the direction of rotation of the gear. Thrust or axial force is assumed to be plus when it forces the gear, or pinion, away from the cone center. The spiral angle ψ is measured as shown in Fig. 283.

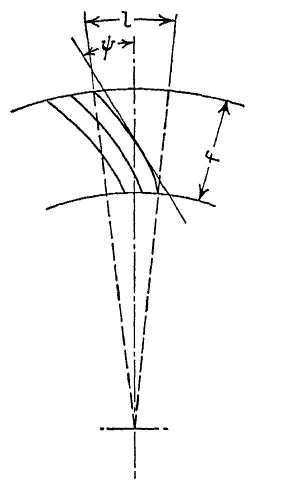


FIG. 283.

For convenience, the end thrusts are given in Table 91 for the different combinations of spiral and rotation. In determining the rotation, look at the gear or pinion toward the cone center.

308. Hypoid Gears.* These gears are approximations of hyperboloidal gears, *i.e.*, gears whose pitch surfaces are hyperbo-

* STEWART, A L., and WILDHABER, E., The Design and Manufacture of Hypoid Gears, *Jour. S.A.E.*, Vol. 18, June, 1926.

CANDEE, A. H., Large Spiral and Hypoid Gears, *Trans. A.S.M.E.*, Vol. 51, 1928.

loids of revolution. As shown in Fig. 284, they resemble spiral bevel gears, but the axes do not intersect. The advantages of this type of gear are somewhat smoother action and the possi-

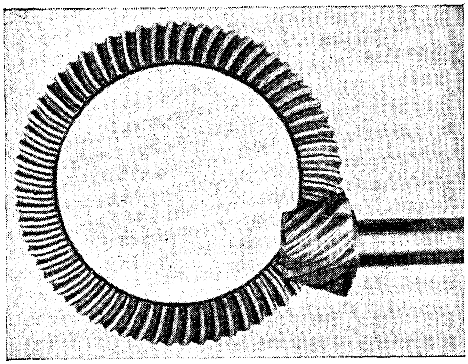


FIG. 284.—Hypoid gear and pinion. (*Courtesy Gleason Works.*)

bility of extending the shafts past each other so that bearings can be used on both sides of the gear and the pinion.

309. Worm Gears. The maximum gear ratio advisable with helical gearing is about 10:1. For larger ratios, a gear train or double reduction should be used, or a worm and worm gear may be used. Worm and gear sets with ratios from 10:1 up to 100:1 are regularly employed, and ratios as high as 500:1 have been used. The worm and worm wheel is a special case of helical gearing with non-parallel axes, the axes being at right angles.

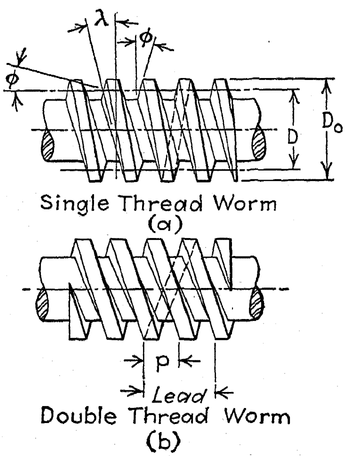


FIG. 285.

The worm may be cut with a single, double, triple, or quadruple thread as shown in Fig. 285. The velocity ratio does not depend on the diameters but is found by dividing the number of teeth on the gear by the number of threads on the worm, a 40-tooth gear and a double-thread worm having a velocity ratio of 20. The lead of the worm is the distance from any point on one thread to the corresponding point on the next turn of the same thread, measured parallel to the axis.

It is also the distance that a thread advances for one complete revolution of the worm. The lead angle λ is the angle between the tangent to the pitch helix and the plane of rotation. This angle is the complement of the helix angle as used with helical gears.

The lineal pitch p, or the circular pitch, is the distance from any point on one thread to the corresponding point on the adjacent thread, measured parallel to the worm axis. Note that, for a single-thread worm, the lineal pitch is the same as the lead; on a double-thread worm, the pitch is one-half the lead, etc.

The pitch diameter of the worm gear is the diameter measured on the central plane.

The throat diameter is the outside diameter of the worm gear measured on the central plane.

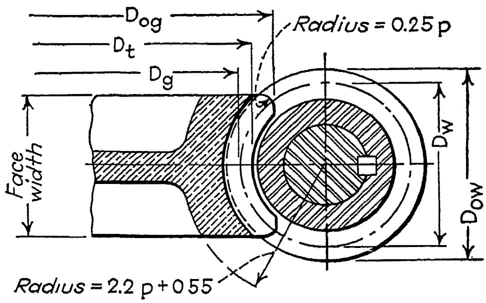


FIG. 286.—A.G.M.A. standard worm form.

310. Proportions of Worms. The standard thread form is a straight-sided tooth, with a pressure angle of $14\frac{1}{2}$ deg for single and double threads, and 20 deg for triple- and quadruple-threaded worms, the angle in all cases being measured perpendicular to the pitch helix. The worm threads should be formed with a straight-sided milling cutter whose diameter is not less than the outside diameter of the worm or greater than $1\frac{1}{4}$ times the outside diameter. The A.G.M.A. recommends that the standard linear pitches shall be $\frac{1}{4}$, $\frac{5}{16}$, $\frac{3}{8}$, $\frac{1}{2}$, $\frac{5}{8}$, $\frac{3}{4}$, 1, $1\frac{1}{4}$, $1\frac{1}{2}$, $1\frac{3}{4}$, and 2 in. Other standard dimensions are given in Table 92. The A.G.M.A standard is intended for general industrial worms and does not cover worms of very large or small pitch, worms of more than four threads, worms of gear ratios over 100:1, or worms cut directly on the shaft or where the use justifies greater refinements in the design.

In the design of worms not covered by this standard, the following recommendations of the Brown and Sharpe Manufacturing Company* are useful. Since the velocity ratio does not depend on the worm diameter, the diameter is limited only by the shaft size required to transmit the power without excessive

TABLE 92.—PROPORTIONS OF A.G.M.A. STANDARD INDUSTRIAL WORMS AND GEARS

Item	Symbol	Single and double		Triple and quadruple	
		Worm	Gear	Worm	Gear
Normal pressure angle	ϕ_n	$14\frac{1}{2}$ deg	$14\frac{1}{2}$ deg	20 deg	20 deg
Linear pitch	p	St'd	St'd	St'd	St'd
Pitch diameter	D_w, D_g	$2.4p + 1.1$	$0.3183pN_g$	$2.4p + 1.1$	$0.3183pN_g$
Addendum	a	$0.318p$	....	$0.286p$	
Whole depth	h_t	$0.686p$	....	$0.623p$	
Outside diameter	D_o	$3.036p + 1.1$	$D_t + 0.4775p$	$2.972p + 1.1$	$D_t + 0.3183p$
Throat diameter	D_t	.....	$D_g + 0.636p$	.....	$D_g + 0.572p$
Normal tooth thickness	t_n	$0.5p \cos \lambda$	.	$0.5p \cos \lambda$	
Face length	.	$p\left(4.5 + \frac{N}{50}\right)$	.	$p\left(4.5 + \frac{N}{50}\right)$	
Face width	.	....	$2.38p + 0.25$	.....	$2.15p + 0.20$
Top round		$0.05p$	. .	$0.05p$	
Hub diameter	. .	$1.664p + 1$	$1.875 \times$ Bore	$1.726p + 1$	$1.875 \times$ Bore
Hub extensions	.	p	$0.25 \times$ Bore	p	$0.25 \times$ Bore
Bore, maximum		$p + 0.625$	......	$p + 0.625$	

N = number of teeth on the worm gear.

deformation. This company recommends that the minimum pitch diameter be

$$D_w = 2.35p + 0.4 \text{ in.} \qquad \text{for worms cut on the shaft} \tag{343}$$

and

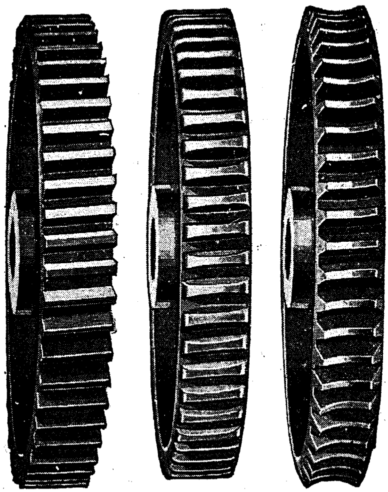
$$D_w = 2.40p + 1.1 \text{ in.} \qquad \text{for worms bored to fit over the shaft} \tag{344}$$

The following pressure angles are recommended: with lead angles up to 12 deg, $14\frac{1}{2}$ deg; with lead angles up to 20 deg, 20 deg; with lead angles up to 25 deg, $22\frac{1}{2}$ deg; and with lead angles greater than 25 deg, 25 deg. For worms mating with gears having 24 teeth or more, the 20-deg pressure angle is

* "Treatise on Gearing," Brown and Sharpe Manufacturing Company, Providence, R.I.

recommended. Pressure angles of 30 deg are common in automotive gears and in industrial reduction units. The larger pressure angles are used with the larger lead angles because of the difficulty of machining the high-lead threads. The threads are usually formed by a revolving milling cutter or a hob; and if the lead is large and the pressure angle small, the interference of the cutter will undercut the flanks of the worm.

311. Proportions of Worm Gears. Worm gears are made in three general types shown in Fig. 287. The straight-faced gear is simply a helical gear; and since it has only point contact with the worm thread, it is used for only very light loads. The hobbed straight-faced gear is cut with a hob, after which the outer surface is turned. This form is used for light loads and indexing wheels. The concave-faced gear is cut with a hob of the same pitch diameter as the mating worm so that the teeth fit the contour of the worm and present a larger contact area. The concave face is the accepted standard form and is used for all heavy services and general industrial uses.



(a) (b) (c)

FIG. 287.—(a) Straight-face; (b) hobbed straight-face; (c) concave face.

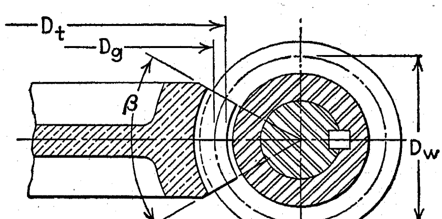


FIG. 288.

The permissible width of gear face is limited by the fact that hob-cut gears tend to become pointed near the ends of wide gears. For this reason, the sides of the gear are usually beveled

as shown in Fig. 288. According to O. F. Shepard,* the extreme value of the face angle is

$$\tan\frac{\beta}{2} = \frac{\tan\phi_n}{\tan\lambda} \tag{345}$$

where β = face angle.
ϕ_n = normal pressure angle.
λ = lead angle.

The face angle ranges from 60 to 90 deg, 60 deg being the usual angle. The proportions of worm gears to mate with A.G.M.A. standard industrial worms are given in Table 92.

312. Design of Worm and Gear for Strength. Since the teeth of the worm gear are always weaker than the worm threads, the strength may be determined by applying the Lewis equation to the gear. Because of the sliding action between the worm and gear teeth, the dynamic forces are not so severe as in the regular forms of gearing. Including the velocity factor, the Lewis equation for worm gears is

$$F_t = s_w p f y \left(\frac{1{,}200}{1{,}200 + V_g}\right) \tag{346}$$

where F_t = tangential pitch-line load on the gear, lb.

The other symbols have the same meaning as before. The value of the form factor taken from Table 79 for full-height spur teeth is not strictly correct, but will give results on the safe side.

The strength of the worm gear is usually not the determining factor in the design of a worm drive, but the strength should be checked before the design is finally approved.

313. Design of Worm and Gear for Wear. The wear and the heating are usually the determining factors in the successful application of a worm and gear. Although many methods of design have been proposed, there is no generally accepted method, and the methods proposed by various authorities do not agree. Until research work now under way suggests a rational method of design, each manufacturer must use one of the empirical methods, backed by experience with the particular type of worm and gear used.

Lubrication is important in the determination of the wearing qualities, since a thin or low-viscosity lubricant will be squeezed

* *Trans. A.G.M.A.*, Vol. 11, p. 201.

out, and metal-to-metal contact will result in high friction and overheating. If it is assumed that the proper grade of lubricant

TABLE 93.—PERMISSIBLE SURFACE PRESSURES s_c FOR USE IN THE WEAR FORMULA (347)

Material		Number of teeth in the gear							
Worm	Gear	10	20	30	40	50	60	70	80 and over
0.20C steel, untreated	Cast iron	75	225	425	750	900	1,080	1,250	1,350
0.40C steel, untreated	Bronze, S.A.E. 63, sand cast	112	340	625	1,075	1,350	1,625	1,900	2,000
0.40C steel, heat-treated, ground	Bronze, S.A.E. 63, sand cast	170	510	940	1,600	2,000	2,425	2,850	3,000
0.10C alloy steel, carburized, hardened, ground	Bronze, S.A.E. 65, sand cast	225	675	1,250	2,150	2,700	3,250	3,800	4,000
	Bronze, S.A.E. 65, chill cast	310	930	1,725	2,950	3,700	4,500	5,250	5,500
	Nickel bronze, sand cast	375	1,125	2,980	3,600	4,500	5,450	6,350	6,700
	Nickel bronze, chill cast	450	1,350	2,500	4,300	5,400	6,500	7,600	8,000

Values tabulated are for 14½-deg pressure angles.
Multiply by 1.05 for 20-deg pressure angle.
Multiply by 1.10 for 30-deg pressure angle.

is used, the capacity of the worm and gear may be determined from the formula

$$F_w = A \cos \lambda \left(\frac{600}{600 + V_w}\right) \frac{s_c}{C_s} \tag{347}$$

where F_w = permissible tangential load on gear, lb.
A = projected area of tooth, sq in.
s_c = permissible working compressive stress from Table 93, psi.
V_w = pitch-line velocity of worm, fpm.
C_s = a service factor from Table 94.

The projected area of the tooth is

$$A = h_w \frac{D_w}{2} \frac{\beta}{57.3} \tag{348}$$

where h_w = working depth of worm, in.
β = face angle, deg.

TABLE 94.—SERVICE FACTORS FOR WORM GEARS

Type of Service	Factor C_s
Intermittent with light shock	1 0–1 5
Continuous with medium shock:	
line shafts, crushers, etc.	1.5–2.0
Continuous with heavy shock:	
reciprocating pumps, paper and rubber mills, etc	2 0–2.5
Continuous with frequent and very heavy shocks:	
main-line drives, steel mills, etc	2.5–3 0

TABLE 95.—SAFE BEAM STRESS OR STATIC STRESS OF MATERIALS FOR WORM GEARS
(For use with the Lewis equation)

Material	Safe stress s_w	Ultimate strength s_u	Yield stress s_y
Cast iron, good grade	10,000	30,000	
Semisteel	12,000	36,000	
Cast steel	20,000	60,000	27,000
Manganese bronze, S.A.E. 43	20,000	70,000	25,000
Bronze, S.A.E. 63, leaded gun metal	8,000	30,000	12,000
Phosphor bronze, S.A.E. 65	15,000	35,000	20,000

Bronze S.A.E. 63 is preferred with unhardened worms, and S A.E. 65 for chilling to harden for use with worms of great accuracy and hardness.

Different authorities give allowable stresses ranging from 65 to 150 per cent of the tabulated values. The tabulated values represent average practice

314. Materials for Worms and Gears. Experience indicates that the gear should be made of softer material than the worm,

and that the worm should be hard and carefully finished. Worms are universally made of hardened steel, and the gear is generally made of bronze. Cast-iron gears with soft-steel worms are used for light loads and installations where the service required is unimportant. Representative materials are given in Table 95.

315. Bearing Pressures. A typical worm and gear is shown in Fig. 289. In this figure, let the pressure between the teeth

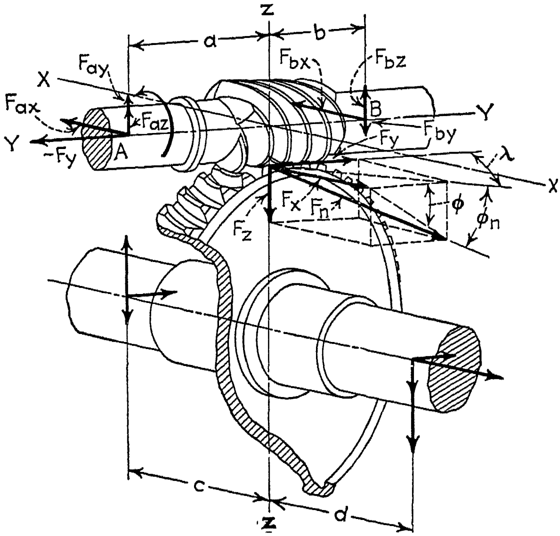


FIG. 289.

and normal to the tooth surface be F_n. The sliding action between the surfaces acts chiefly along the tangent to the pitch helix. The friction force is fF_n. Considering the forces acting on the gear, we have

$$
\begin{aligned}
F_x &= F_n \cos \phi_n \sin \lambda + fF_n \cos \lambda = F_n (\cos \phi_n \sin \lambda + f \cos \lambda) \\
F_y &= F_n \cos \phi_n \cos \lambda - fF_n \sin \lambda = F_n (\cos \phi_n \cos \lambda - f \sin \lambda) \\
&= \frac{F_x (\cos \phi_n \cos \lambda - f \sin \lambda)}{\cos \phi_x \sin \lambda + f \cos \lambda} \qquad (349) \\
F_z &= F_n \sin \phi_n = \frac{F_x \sin \phi_n}{\cos \phi_n \sin \lambda + f \cos \lambda}
\end{aligned}
$$

The force F_x is the tangential turning force on the worm and the end thrust on the gear. The force F_y is the tangential turning force on the gear and the end thrust on the worm. The force F_z is a separating force tending to force the worm and gear apart.

Consider the worm only, with bearings centered at A and B. The force F_z produces the forces F_{az} and F_{bz} acting on the bearings parallel to the Z axis. The magnitude of these bearing forces is

$$F_{az} = F_z\left(\frac{b}{a+b}\right) \tag{350}$$

and

$$F_{bz} = F_z\left(\frac{a}{a+b}\right) \tag{351}$$

Similarly, the force F_x produces the forces F_{ax} and F_{bx} acting on the bearings and parallel to the X axis.

The force F_y produces the forces F_{ay} and F_{by} equal but opposite in direction and parallel to the Y axis. Then

$$F_{ay} = F_{by} = F_y\left(\frac{D_w}{2(a+b)}\right) \tag{352}$$

The total force on each bearing is the vector sum of the three component forces just determined. The bearing forces on the gear are found in the same manner.

316. Efficiency of the Worm and Gear. The efficiency is the ratio of the work output of the gear to the work input of the worm. The work done by the worm per minute is

$$W_i = F_x V_w$$

and the work output of the gear per minute is

$$W_o = F_y V_g = F_y V_w \tan \lambda$$

Hence the efficiency is

$$\begin{aligned}\text{Eff} &= \frac{F_y \tan \lambda}{F_x} = \frac{\tan \lambda\ (\cos \phi_n \cos \lambda - f \sin \lambda)}{\cos \phi_n \sin \lambda + f \cos \lambda} \\ &= \frac{\tan \lambda\ (\cos \phi_n - f \tan \lambda)}{\cos \phi_n \tan \lambda + f}\end{aligned} \tag{353}$$

A critical study of this equation brings out some interesting things that must be considered in high-efficiency drives. For a given pressure angle, the efficiency depends upon the lead angle and the coefficient of friction. If the equation is differentiated and equated to zero, it is found that the efficiency is maximum when

$$\tan \lambda = \sqrt{1 - f^2} - f \tag{354}$$

Different values of the coefficient of friction result in the efficiency curves shown in Fig. 290. The curve of maximum efficiency obtained with each coefficient of friction is plotted in the same figure. Note that for any coefficient of friction, the efficiency curve rises rapidly with increasing lead angle, remains fairly constant over a wide range, and then drops off rapidly at high values of the lead angle. Small lead angles generally indicate inefficient worm drives; hence single-thread worms are seldom used except where high gear ratios are required, or where the worm is to be self-locking. A self-locking worm

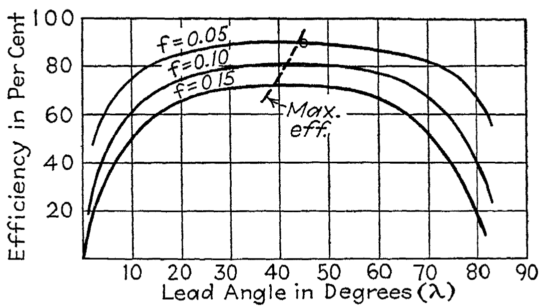


FIG. 290.—Relation between lead, coefficient of friction, and efficiency of worm drives.

is one in which the gear can not drive the worm. To secure this condition, the tangent of the lead angle must be less than the coefficient of friction.

The coefficient of friction varies with the lubricant used, the finish of the tooth surfaces, and the rubbing velocity. With the best modern worm-and-gear materials and very careful machining, it is possible to obtain a coefficient of friction of 0.02; ordinary industrial worms have a coefficient of about 0.05 if well lubricated; with low speeds and indifferent lubrication, the coefficient may be as high as 0.15.

The coefficient of friction for a casehardened, ground, and polished steel worm mating with a bronze gear, lubricated with oil similar to 600*W*, is given approximately by the formula

$$f = \frac{2.3}{18V_r^{0.6}} \tag{355}$$

where V_r is the rubbing velocity (up to 3,000 fpm). This formula assumes an operating temperature of 160 F. The rubbing

velocity is

$$V_r = \frac{V_w}{\cos \lambda} = \frac{\pi D_w N_w}{12 \cos \lambda} = \frac{0.262 D_w N_w}{\cos \lambda} \tag{356}$$

where D_w = worm pitch diameter, in.
N_w = rpm of the worm.

The rubbing velocity for ordinary industrial worms is limited to 1,200 fpm, although 3,000 fpm for well-designed, hardened, and ground worms is practical A few installations have operated successfully at 6,000 fpm.

With the lower viscosity oils and lower grade machine work, the coefficient of friction may be 25 per cent higher than indicated by Eq. (355). These coefficients and the corresponding

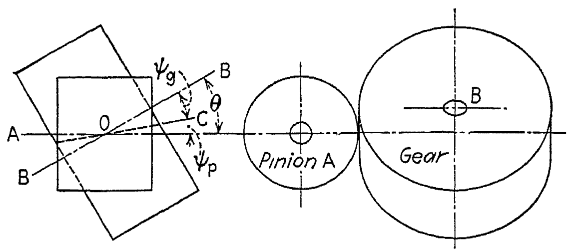


FIG. 291.

efficiencies do not include the losses in the bearings. With proper alignment and good bearings, the bearing loss may be neglected.

317. Heat Radiation from Worm Drives. Considerable trouble may be experienced if the housing does not have sufficient radiating capacity. All the heat generated by friction must be dissipated through the oil to the housing and thence to the atmosphere. Most oils lose their lubricating properties at temperatures around 200 F, and the operating temperature of the worm should be limited to 180 F. The heat to be radiated per minute is

$$H = \frac{F_n f V_w}{778 \cos \lambda} \qquad \text{in Btu} \tag{357}$$

where F_n = force normal to the tooth surface, lb.

The radiating capacity of the bearing depends on its construction. The ordinary housing will radiate about 1.8 Btu per °F per hr per sq ft of surface. If the housing is exposed to good air

currents, its radiating capacity will be increased. In some installations it is necessary to circulate the oil through coolers, to provide cooling coils inside of the housing, or to provide radiating fins on the housing.

318. Spiral Gears. Helical gears are sometimes used to transmit power between shafts that are not parallel and do not intersect. The helices may be of the same or opposite hand, and in the general case the helix angles of the two gears are not the same. By reference to Fig. 291 it is seen that

$$\theta = \psi_p + \psi_g \qquad \text{for helices of the same hand} \qquad (358)$$

and

$$\theta = \psi_p - \psi_g \qquad \text{for helices of the opposite hand} \qquad (359)$$

The angular-velocity ratio is not inversely proportional to the pitch diameters, as in spur gears, but is given by the equation

$$\frac{N_p}{N_g} = \frac{D_g \cos \psi_g}{D_p \cos \psi_p} \qquad (360)$$

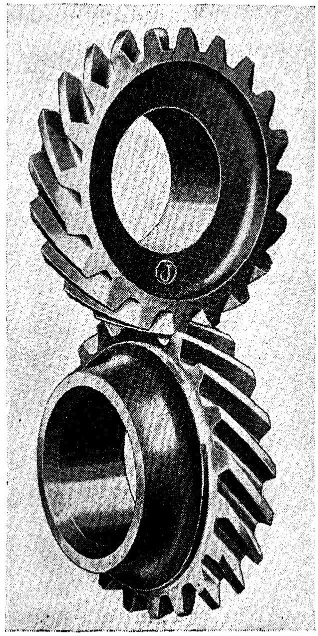


FIG. 292.—Identical helical gears used on shafts at right angles. (*Courtesy W. A. Jones Foundry and Machine Company.*)

With nonparallel axes, the normal pitches must be the same, and the pitches measured in the plane of rotation may or may not be the same. A special case occurs when the axes are at right angles and the velocity ratio is unity. In this case the gears will be identical and the helix angles will be 45 deg. These gears, often miscalled spiral gears, are used to drive feed mechanisms on machine tools, camshafts, water pumps on small internal-combustion engines, and similar units that require only small amounts of power. This type of gearing should not be used to transmit heavy power loads since contact occurs only where the common normal to the axes cuts the tooth surface.

CHAPTER XX

CYLINDERS, PIPES, AND TUBES

The variety of applications of cylinders in machine design and the wide range of materials used have led to different methods of design, depending upon the material, the type of cylinder, the service conditions, and many factors which must be considered in addition to the strength requirements. In small cylinders, strength, wear, and corrosion resistance are probably the most important considerations. In large cylinders distortion caused by their own weight may be serious; hence rigidity is a vital requirement. Initial distortion and variations in wall thickness are more serious in cylinders subjected to external pressure than in cylinders subjected to internal pressure only. In steam apparatus, stills, cookers, and similar apparatus, the effects of temperature on the permissible working stress and on the distortion of the cylinder become important. The design of cylinders for many uses has been more or less standardized, and in the following paragraphs the more important theories and standard formulas are presented.

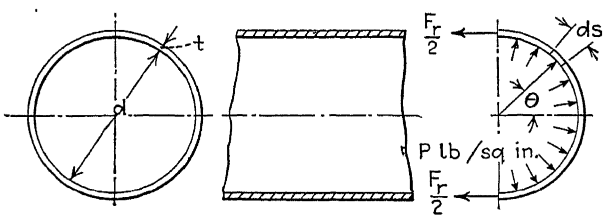


FIG. 293.

319. Thin Cylinders. A cylinder in which the ratio of the wall thickness to the inside diameter is less than 0.07 may be called a thin cylinder. In Fig. 293, such a cylinder is shown cut by an imaginary plane through its axis. When an internal pressure is applied, the total force acting on the half cylinder and tending to rupture it along the cutting plane is

$$F = \int pL\, ds \cos\theta = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} pLr \cos\theta\, d\theta = 2pLr = pLd$$

and the total resisting force in the cylinder walls cut by the plane is

$$F_r = 2tLs_t$$

For equilibrium, the rupturing force and the resisting forces must be equal. Hence, assuming uniform stress distribution

$$pLd = 2tLs_t$$

and

$$s_t = \frac{pd}{2t} \tag{361}$$

where p = internal pressure, psi.
d = inside diameter, in.
t = wall thickness, in.
L = length, in.
s_t = tensile stress, psi.

When there is a seam or joint in the cylinder, the joint efficiency e must be considered, and

$$s_t = \frac{pd}{2et} \tag{362}$$

Imagine a plane passed through the cylinder perpendicular to the axis. The total force tending to rupture the cylinder along this plane and the corresponding resisting force are

$$F = \frac{\pi d^2}{4} p$$

and

$$F_r = \pi d t s_t$$

from which

$$s_t = \frac{pd}{4t} \tag{363}$$

This is the stress in the cylinder wall parallel to its axis, and is one-half the tengential stress, as found in Eq. (361).

Internal pressure on a thin vessel that is not truly cylindrical tends to make it become cylindrical. However, if the material is fairly rigid it will resist this action and bending stresses of unknown magnitude will be induced in the wall.

320. Thin Spheres with Internal Pressure. The stress in the wall of a sphere is the same as the stress in a cylinder in the direction parallel to the axis; hence Eq. (363) applies to spheres.

321. Long Thin Tubes with Internal Pressure. Small tubes and pipes generally have wall thicknesses much greater than

TABLE 96.—VALUES OF s TO BE USED IN EQS. (364) AND (365)

Material	Spec. No.	For temperatures in °F not to exceed							
		−20 to 650	700	750	800	850	900	950	1000
Seamless medium-carbon steel	S-18	12,000	12,000	10,400	8,300	6,350	4,400	2,600	
Seamless low-carbon steel	S-18	9,400	9,000	8,600	7,150	5,850	4,400	2,600	
Fusion-welded steel*	S-1	11,000	10,400	9,500	8,000	6,300	4,400	2,600	
Fusion-welded steel:									
Grade B*	S-2	10,000	9,600	9,000	7,500	6,000	4,400	2,600	
Grade A*.	S-2	9,000	8,800	8,400	6,900	5,700	4,400	2,600	
Lap-welded steel	S-18	9,000	8,800	8,400	6,900	5,700	4,400	2,600	
Butt-welded steel.	S-18	9,000	8,800	8,400					
Lap-welded wrought iron	S-19	8,000	7,650	6,900					
Butt-welded wrought iron	S-19	8,000	7,650	6,900					
Seamless alloy steel, Grade Pl	S-45	11,000	11,000	11,000	10,750	10,500	10,000	8,000	5,000

The thickness subtraction factor in Eqs. (364) and (365) limits the actual working stress to a value less than s in order to allow for cross strains and mechanical or thermal stresses that cannot be determined accurately

* Fusion-welded in accordance with Pars P-101 to P-111, inclusive, A.S.M.E. Boiler Construction Code.

that required by the thin-cylinder equation, since with threaded joints the effective metal below the threads is only about one-half the wall thickness, and also some allowance must be made for flexure stresses due to out-of-round, for non-uniform thickness, for corrosion, and for wear under service conditions. In some conditions of service, flexure stresses are also set up when the supports are far apart, and when the installation is such that temperature changes may cause bending.

Iron and steel pipes are made from plates, formed while hot, with the longitudinal joint either butt-welded or lap-welded. Seamless tubing is formed by drawing and has no longitudinal joint. Iron and steel pipes have been thoroughly standardized with respect to wall thickness and with respect to the steam or hydrostatic pressure that may be safely carried. The nominal

diameter of standard weight pipe up to 14 in. is the approximate inside diameter; for 14 in. and above, the nominal diameter is the outside diameter. The extra metal required in extra-strong and double extra-strong pipes is added to the inside, the outside diameters of all pipes of the same nominal diameter being the same. Dimensions of pipes and pipe fittings, and the permissible pressures for each size may be found in tabulated form in any mechanical engineer's handbook or pipe manufacturer's catalog.

The A.S.M.E. Boiler Code requires that the permissible steam pressure on steel and iron pipes from $\frac{1}{4}$ to 5 in. in diameter, shall be

$$p = \frac{2s}{d_o}(t - 0.065) - 125 \tag{364}$$

and for pipes of nominal diameters over 5 in.

$$p = \frac{2s}{d_o}(t - 0.1) \tag{365}$$

where p = working pressure, psi.
t = wall thickness, in.
d_o = actual outside diameter of the pipe, in.
s = permissible working stress from Table 96, psi.

Note that these equations are modifications of Barlow's thick-cylinder equation [Eq. (396)] with a thickness subtraction factor to allow for out-of-round and nonuniform thickness. When the pipe is pierced with holes for tubes, the maximum stress in the ligaments between the holes must not exceed the values in the table, and the holes must not pierce the weld.

Tubes are designated by their outside diameter, and may be obtained in a number of wall thicknesses, the thickness being designated by a gauge number. The A.S.M.E. Boiler Code gives the following formulas for the permissible pressure in wrought-iron and steel tubes for water-tube boilers.

$$p = \left(\frac{t - 0.039}{d_o}\right) 18{,}000 - 250 \tag{366A}$$

$$p = \left(\frac{t - 0.039}{d_o}\right) 14{,}000 \tag{366B}$$

$$p = \left(\frac{t - 0.039}{d_o}\right) 10{,}600 \tag{366C}$$

Formula (366*A*) applies to seamless tubes at all pressures, to welded steel tubes at pressures below 875 psi, and to lap-welded wrought-iron tubes at pressures below 358 psi.

Formula (366*B*) applies to welded steel tubes at pressures of 875 psi and above.

Formula (366*C*) applies to lap-welded wrought-iron tubes at pressures of 358 psi and above. These formulas are modifications of the thin cylinder equation.

322. Engine and Press Cylinders. Large cylinders for engine and power-press service are most frequently made of cast iron, owing to the comparative ease in casting the complicated shapes required. High-pressure and hydraulic-press cylinders are included in the thick-cylinder class discussed in Arts. 327 to 335. Steam-engine cylinder-design practice may be used as a guide for similar low-pressure cylinders. The wall thickness of such cylinders in present-day practice is given by the formula

$$t = \frac{pd}{2s_t} + 0.3 \text{ in.} \tag{367}$$

where s_t is taken as 1,250 psi for ordinary grades of cast iron.

323. Openings in Cylindrical Drums. Many cylindrical vessels subjected to internal pressure must have openings provided in the shell. A rule providing for such openings in cylinders of ductile material, based on data secured through experience with a large number of installations having unreinforced openings and giving satisfactory service over long periods of time, is proposed by D. S. Jacobus.* The largest permissible opening is given by the formula

$$d_o = 2.75 \sqrt[3]{dt(1.0 - K)} \tag{368}$$

where K = the ratio of the stress in the solid plate to one-fifth the minimum tensile strength of the steel used in the shell, or

$$K = \frac{pd}{2t} \times \frac{5}{s_u}$$

The maximum diameter of the unreinforced hole should be limited to 8 in. and should not exceed $0.6d$.

* JACOBUS, D. S., Openings in Cylindrical Drums, *Mech. Eng.*, May, 1932, p. 368.

324. Thin Tubes with External Pressure. The equations developed in the preceding paragraphs are not applicable to cylinders subjected to external pressure, since these cylinders will collapse at apparent stresses much below those required for direct compression failure of the material. For this reason, experimental data must be used as the basis of empirical design formulas.

In 1906 Prof. R. T. Stewart presented formulas based on four years of research at the McKeesport Works of the National Tube Company. The formulas are based on results obtained with Bessemer steel tubes, but other tubes showed very little variation from these results. It should be noted that the length does not enter into the formulas, and Prof. Stewart concludes that the tube length between transverse joints that tend to maintain the circular form has no influence upon the collapsing pressure of commercial lap-welded steel tubes so long as the length is not less than six times the tube diameter. Very short tubes are subject to the supporting effect of the end connections.

Stewart's formulas for the collapsing pressure on Bessemer steel tubes are

$$p_{cr} = 50{,}210{,}000 \left(\frac{t}{d_o}\right)^3 \tag{369}$$

when p_{cr} is less than 581 psi, or when t/d_o is less than 0.023, and

$$p_{cr} = 86{,}670 \frac{t}{d_o} - 1{,}386 \tag{370}$$

when p_{cr} is greater than 581 psi, or t/d_o is greater than 0.023. It is seldom that external pressure is used with tubes having a ratio t/d_o less than 0.023 and the values given by the two equations are nearly the same for these thin tubes.

Experiments conducted at the University of Illinois by Prof. A. P. Carman led to the following formulas for the collapsing pressure of brass tubes:

$$p_{cr} = 25{,}150{,}000 \left(\frac{t}{d_o}\right)^3 \tag{371}$$

when t/d_o is less than 0.03, and

$$p_{cr} = 93{,}365 \frac{t}{d_o} - 2{,}474 \tag{372}$$

when t/d_o is greater than 0.03.

All the formulas are for the collapsing pressure, *i.e.*, the ultimate strength of the tubes, and must be modified to suit the service conditions. The factor of safety to be used should not be less than 3 for the most favorable conditions and should be increased to 6 when there is a possibility of loss of life. When corrosion, weakening due to heating, and other service conditions reduce the collapsing resistance of the tube, the factor of safety should be increased in proportion to the weakening effect. For example, external pressure tubes in boilers are designed with an apparent factor of safety of from 8 to 10.

325. Short Tubes with External Pressure. The cylinder heads and the end connections tend to stiffen the tubes and prevent their collapse. When the length is less than six diameters, the strengthening effect must be considered. The best known experimental work on the crushing strength of short tubes is that of Sir William Fairbourn who, in 1858, developed the following formula from which many more recent formulas have been deduced:

$$p_{cr} = 9{,}675{,}600 \frac{t^{2.19}}{Ld_o} \tag{373}$$

For very short tubes, it is possible that the material will fail by direct compression or crushing before the collapsing pressure is reached; hence the crushing stress should be computed from the equation

$$s_c = \frac{p_o d_o}{2t} \tag{374}$$

In certain types of boilers, the furnace is inside the flues, this construction requiring large-diameter flues with the water pressure on the outside. The flues may be plain or corrugated cylinders, or they may be strengthened by stiffening rings, several types of construction being shown in Fig. 294. Seamless flues and welded flues from 5 to 18 in. in diameter may be designed with Eqs. (369) and (370), using a factor of safety of 5. Rules for the construction of the other types are given in the A.S.M.E. Boiler Code, Arts. P-240 to P-244.

326. Unfired Pressure Vessels with External Pressure. There are a number of formulas in use for the design of cylindrical vessels for steam cookers, digesters, stills, vacuum tanks, and

similar applications, and the results have not always been reliable. Designers interested in this type of pressure vessel are referred to recent technical papers on the subject.* Prob-

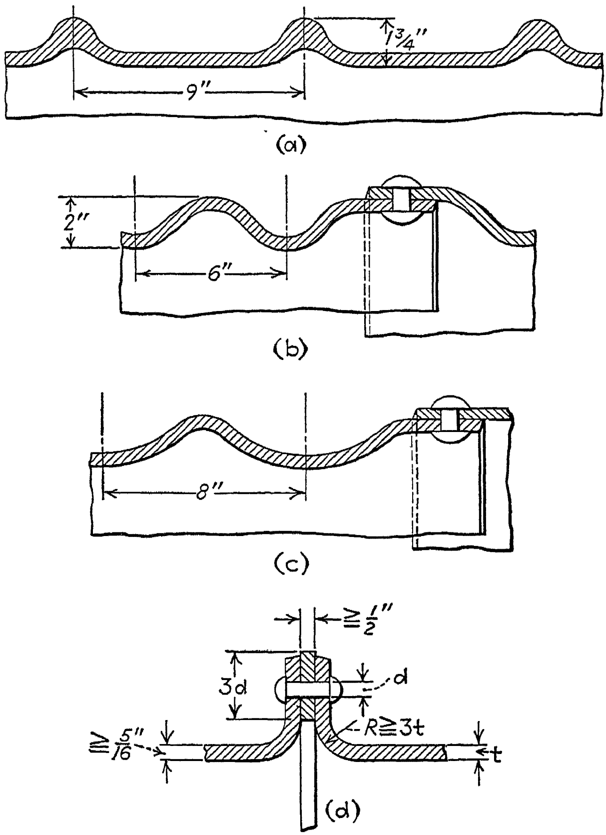


FIG. 294.—(a) Purves furnace; (b) Fox furnace; (c) Morison furnace; (d) Adamson ring. (*From A.S.M.E. Boiler Code.*)

ably the most reliable formula for the critical pressure or collapsing pressure of large cylinders is

* Proposed Rules for the Construction of Unfired Pressure Vessels Subjected to External Pressure, *Mech. Eng.*, April, 1934, p. 245. Also Rules for Construction of Unfired Pressure Vessels, A S M.E. Boiler Construction Code.

SAUNDERS, H. F , and D. F. WINDENBURG, Strength of Thin Cylindrical Shells under External Pressure, *Trans. A.S.M.E.*, 1931.

$$p_{cr} = \frac{2.6E(t/d_o)^{2.5}}{\frac{L}{d_o} - 0.45\left(\frac{t}{d_o}\right)^{0.5}} \tag{375}$$

where L = maximum distance between supports or stiffening rings, in.

To obtain safe working pressures, the critical pressure should be at least five times the working pressure, and in no case should the vessel be designed for less than 15 psi working pressure. When the stiffeners are close together, *i.e.*, when L/d is small, the crushing stress may reach the yield stress before collapsing

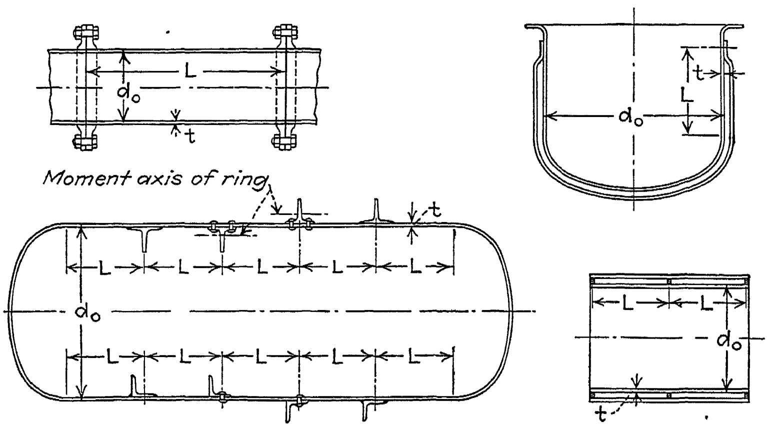


FIG. 295.—Diagrammatic representation of variables for design of unfired cylindrical vessels subjected to external pressure. (*Mechanical Engineering, April,* 1934.)

occurs, and, to avoid this condition, the design should be checked by Eq. (374), keeping the crushing stress at least 15 per cent below the yield stress. In the preliminary design, the shell thickness may be estimated by this equation, and the spacing of the stiffener rings determined by substitution of this value in Eq. (375).

Stiffening rings, several types of which are shown in Fig. 296, must extend entirely around the circumference and may be riveted or welded to the shell. The moment of inertia of the stiffening rings is obtained from the equation

$$I = \frac{p_{cr}d^3L}{24E} \tag{376}$$

Since the stiffening rings should be able to support a greater load than the shell proper, it is advisable to use a value of p_{cr} for the rings about 10 per cent larger than that of the shell proper. The heads of these vessels are usually dished heads, the design of which is explained in Art. 341, Chap. XXI.

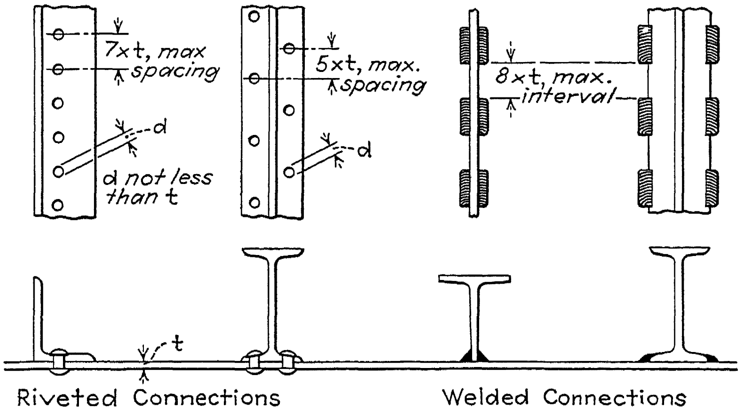


FIG. 296.—Methods of attaching stiffening rings to the shell of unfired cylindrical vessels subjected to external pressure. (*Mechanical Engineering*, *April*, 1934.)

327. Thick Cylinders. In the discussion of thin cylinders, the stress was assumed to be uniformly distributed through the entire wall thickness. This assumption is approximately true for very thin walls only. In the case of a cylinder with internal pressure, the stress varies from a maximum at the inner surface to a minimum at the outer surface, as indicated in Fig. 297. Several theories of stress distribution have been developed, and several of those most commonly used are discussed in the following paragraphs. The following notation will be used:

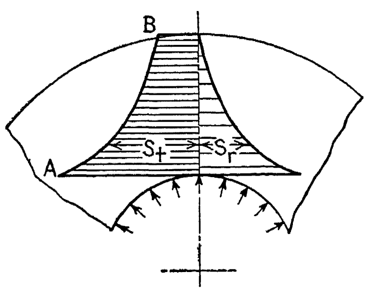


FIG. 297.

d_i = inside diameter, in.
d_o = outside diameter, in.
d_c = diameter of the contact surface in compound cylinders, in.
r = radius at any point in the wall, in.
s_a = axial stress, psi.

s_r = radial stress, psi.
s_t = tangential stress, psi.
p_i = internal pressure, psi.
p_o = external pressure, psi.
p_c = pressure at the contact surface of compound cylinders, psi.
m = Poisson's ratio of lateral contraction.
t = wall thickness, in.

328. Lamé's Equations. In the general case, there will be pressure applied to the cylinder both on the inside and on the outside. In Fig. 298, consider the cylinder to be cut by a plane

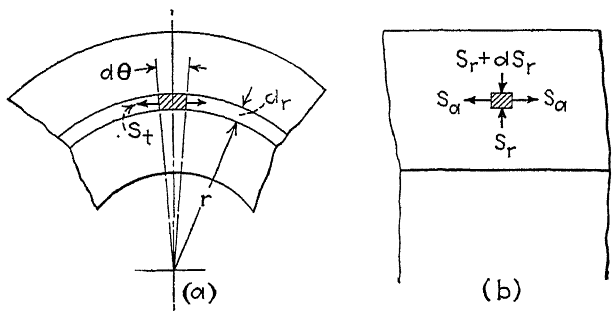


FIG. 298.

perpendicular to the axis and at some distance from the end wall so as to eliminate the constraining action of the end wall. In this plane, consider an annular ring of radius r and thickness dr. The axial stress may be considered to be uniform over the wall thickness. Hence

$$s_a = \frac{\dfrac{\pi d_i^2}{4} p_i - \dfrac{\pi d_o^2}{4} p_o}{\dfrac{\pi d_o^2}{4} - \dfrac{\pi d_i^2}{4}} = \frac{p_i d_i^2 - p_o d_o^2}{d_o^2 - d_i^2} \tag{377}$$

Now consider a small portion of wall material of unit axial length, of radial thickness dr, and of tangential thickness $r\,d\theta$, as shown in Fig. 298. This small body is held in equilibrium by axial, radial, and tangential stresses, s_a, s_r, and s_t as shown. The total deformation in the axial direction due to these stresses is

$$\Delta_a = \frac{(s_a + ms_r - ms_t)}{E}$$

But Δ_a, s_a, m, and E are constant, hence

$$s_r - s_t = \text{a constant} = C \tag{378}$$

The radial stresses on the small body under consideration are s_r at the inner surface, and $(s_r + ds_r)$ at the outer surface.

The tangential stress on this body is found from the thin-cylinder formula. Hence

$$s_t = \frac{pd}{2t} = \frac{-(s_r + ds_r) \times 2(r + dr) + 2rs_r}{2dr}$$

Expanding, collecting the terms, and neglecting the infinitesimal product $ds_r \times dr$, we have

$$\frac{dr}{r} = \frac{-ds_r}{s_r + s_t} = \frac{-ds_t}{2s_t + C}$$

since

$$s_r = C + s_t,$$

and

$$ds_r = ds_t$$

By integration,

$$2 \log r = \log \frac{B}{2s_t + C}$$

and

$$r^2 = \frac{B}{2s_t + C}$$

where B and C are constants. Then

$$s_t = -\frac{C}{2} + \frac{B}{2r^2} \tag{379}$$

and

$$s_r = \frac{C}{2} + \frac{B}{2r^2}$$

The constants B and C may be eliminated, since s_r equals p_i and s_t equals $C - p_i$, when r equals $d_i/2$, and s_r equals p_o and s_t equals $C - p_o$ when r equals $d_o/2$. By substitution of these values in Eq. (379), the tangential stress in the cylinder wall is

$$s_t = \frac{p_i d_i^2 - p_o d_o^2}{d_o^2 - d_i^2} + \frac{d_i^2 d_o^2}{4r^2}\left(\frac{p_i - p_o}{d_o^2 - d_i^2}\right) = a + \frac{b}{r^2} \tag{380}$$

and the radial stress is

$$s_r = \frac{p_i d_i^2 - p_o d_o^2}{d_o^2 - d_i^2} - \frac{d_i^2 d_o^2}{4r^2}\left(\frac{p_i - p_o}{d_o^2 - d_i^2}\right) = a - \frac{b}{r^2} \tag{381}$$

where a and b = constants for any given values of pressures and diameters.

Equations (380) and (381) are Lamé's general equations for the tangential and radial stresses at any radius r in the wall of a thick cylinder. These stresses are the maximum normal stresses in the wall at the radius r.

329. Lamé's Equations for Internal Pressure. The most common case dealt with in machine design is a cylinder subjected to internal pressure only. In this case, p_o is zero, and

$$s_t = \frac{p_i d_i^2}{4r^2}\left(\frac{4r^2 + d_o^2}{d_o^2 - d_i^2}\right) \tag{382}$$

and

$$s_r = \frac{p_i d_i^2}{4r^2}\left(\frac{4r^2 - d_o^2}{d_o^2 - d_i^2}\right) \tag{383}$$

Inspection of these equations shows that both stresses increase as the radius decreases; hence the maximum stresses are at the inner surface where r equals $d_i/2$. Then

$$s_{t\,\max} = \frac{p_i(d_o^2 + d_i^2)}{d_o^2 - d_i^2} \tag{384}$$

and

$$s_{r\,\max} = -p_i \tag{385}$$

the negative sign indicating that s_r is a compressive stress.

The usual design problem is to determine the wall thickness when the allowable working stress and the internal pressure are known. Substitution of $d_i + 2t$ for d_o in Eq. (384) leads to the more convenient form

$$t = \frac{d_i}{2}\left(\sqrt{\frac{s_t + p_i}{s_t - p_i}} - 1\right) \tag{386}$$

This equation is based on the maximum normal stress in the wall and is therefore applicable to brittle materials.

Maximum Shear in the Cylinder Wall. The maximum shear stress in the cylinder wall is found by the substitution of the

values of s_t and s_r from Eqs. (384) and (385) in Eq. (31). Hence

$$s_{s\,\max} = \frac{s_t - s_r}{2} = \frac{p_i d_o^2}{d_o^2 - d_i^2} \tag{387}$$

Since the maximum-shear theory holds that the shear stress is one-half the tension stress, the equivalent maximum tension in the cylinder wall is

$$s_{t\,\max} = \frac{2p_i d_o^2}{d_o^2 - d_i^2} \tag{388}$$

and

$$t = \frac{d_i}{2}\left(\sqrt{\frac{s_t}{s_t - 2p_i}} - 1\right) \tag{389}$$

This equation is applicable to ductile materials but is not considered to be so accurate as Clavarino's and Birnie's equations, since the effect of lateral contraction is neglected.

330. Clavarino's Equations for Closed Cylinders. In the preceding article, the effects of lateral deformations of the wall material were neglected, but, according to the maximum-strain theory, these lateral deformations affect the load-carrying capacity of the material. By the introduction of the effect of the lateral deformation, the equivalent axial, radial, and tangential stresses are

$$s'_a = s_a + ms_r - ms_t$$
$$s'_r = s_r + ms_a + ms_t$$

and

$$s'_t = s_t - ms_a + ms_r$$

When these values are used as the stresses acting on the body shown in Fig. 298, an analysis similar to that for Lamé's equations leads to the following general equations for the equivalent stresses:

$$s'_t = (1 - 2m)a + \frac{(1 + m)b}{r^2} \tag{390}$$

and

$$s'_r = (1 - 2m)a - \frac{(1 + m)b}{r^2} \tag{391}$$

where m is Poisson's ratio, and a and b have the same meaning as in Eq. (380). When a and b are evaluated, and when $d_i/2$

is substituted for r, the equation for the wall thickness is

$$t = \frac{d_i}{2}\left[\sqrt{\frac{s_t' + (1 - 2m)p_i}{s_t' - (1 + m)p_i}} - 1\right] \tag{392}$$

where s_t' = permissible working stress in tension, psi.

These equations are known as Clavarino's equations for closed cylinders and are applicable to cylinders having the ends closed or fitted with heads so that axial stress is produced in the wall.

331. Birnie's Equations for Open Cylinders. When the cylinder is open at the end so that no direct axial stress is possible, an analysis similar to that for Clavarino's equations leads to the following equations for equivalent stress:

$$s_t' = (1 - m)a + (1 + m)\frac{b}{r^2} \tag{393}$$

and

$$s_r' = (1 - m)a - (1 + m)\frac{b}{r^2} \tag{394}$$

The equation for the wall thickness is

$$t = \frac{d_i}{2}\left[\sqrt{\frac{s_t' + (1 - m)p_i}{s_t' - (1 + m)p_i}} - 1\right] \tag{395}$$

These equations are applicable to certain types of pump cylinders, rams, cannon, and similar cylinders where no axial stress is present. An examination of Eqs. (392) and (395) shows that Birnie's equation always gives greater values of t than does Clavarino's equation, therefore if there is any doubt as to whether the cylinder is open or closed, Birnie's equation should be used.

332. Barlow's Equation. In Lamé's equation for cylinders subjected to internal pressure only, write $d_i + 2t$ for d_o, and Eq. (384) becomes

$$s_t = \frac{p_i(2d_i^2 + 4td_i + 4t^2)}{4(d_it + t^2)}$$

When t is small compared to d_i, the term t^2 may be neglected, and

$$s_t = \frac{p_id_o}{2t} \tag{396}$$

This is Barlow's equation, which is similar to the thin-cylinder equation except that d_o replaces d_i. It is slightly more accurate and is commonly used in computing wall thicknesses for high-pressure oil and gas pipes.

TABLE 97.—VALUES OF s_t TO BE USED IN EQ. (396) FOR NONFERROUS SEAMLESS TUBES AND PIPES

Material	Spec. No.	For temperatures in F not to exceed						
		Sub-zero to 150	250	350	400	450	500	550
Muntz metal tubing and high brass tubing	S-24	5,000	4,000	2,500				
Muntz metal condenser tubes	S-47	5,000	4,000	2,500				
Red brass tubes	S-24	6,000	5,500	5,000	4,500			
Copper tubes	S-22	6,000	5,000	4,500	4,000			
Copper pipes	S-23	6,000	5,000	4,500	4,000			
Admiralty tubing	S-24	7,000	6,500	6,000	5,500	4,500		
Admiralty condenser tubes	S-47	7,000	6,500	6,000	5,500	4,500		
Steam bronze	S-41	6,800	6,300	5,800	5,400	5,000	4,200	3,300

Stresses in this table when used with Eq (396) are applicable only to diameters ½ to 6 in. outside diameter inclusive, and for wall thicknesses not less than No. 18 B.W.G. (0 049 in.)

Additional wall thickness should be provided where corrosion, or wear due to cleaning operations, is expected.

Where tube ends are threaded, additional wall thickness of $\frac{0.8}{\text{number of threads}}$ is to be provided.

Requirements for rolling or otherwise setting tubes in tube plates, may require additional wall thickness.

333. Changes in Cylinder Diameter Due to Pressure. Although the changes are relatively small, there are cases, such as press and shrink fits, where the changes in cylinder diameter must be known. The unit deformation is equal to the unit stress divided by the modulus of elasticity. Hence the total increase in cylinder diameter is

$$\Delta d = \frac{s_t}{E} d \tag{397}$$

The value of s_t may be found from Eq. (393) and the constants a and b in this equation may be determined from Eq. (380).

When the cylinder is subjected to internal pressure only, p_o becomes zero, and the maximum s_t is at r equal to $d_i/2$. Hence the increase in cylinder diameter due to internal pressure is

$$\Delta d_i = \frac{d_i}{E}\left[\frac{(1-m)d_i^2p_i}{d_o^2-d_i^2} + \frac{(1+m)d_i^2d_o^2p_i}{d_i^2(d_o^2-d_i^2)}\right] = \frac{p_id_i}{E}\left(\frac{d_o^2+d_i^2}{d_o^2-d_i^2}+m\right) \tag{398}$$

Similarly, the decrease in external diameter of a cylinder subjected to external pressure only is

$$\Delta d_o = \frac{p_od_o}{E}\left(\frac{d_o^2+d_i^2}{d_o^2-d_i^2}-m\right) \tag{399}$$

334. Compound Cylinders. When high internal pressures are to be sustained, it is good practice to make the cylinder from two or more annular cylinders with the outer ones shrunk onto the inner ones. This puts the inner cylinder in compression before the internal pressure is applied, and when the internal pressure is applied the resulting stress in the cylinder wall is much lower than if a single cylinder had been used.

Consider a cylinder open at the ends so that no axial stress is imposed. The tangential stress is the larger and is the stress to be considered in the design. The value of this tangential stress at any radius r, from Birnie's equation is

$$s_t = (1-m)\,\frac{d_i^2p_i - d_o^2p_o}{d_o^2-d_i^2} + (1+m)\,\frac{d_i^2d_o^2(p_i-p_o)}{4r^2(d_o^2-d_i^2)} \tag{400}$$

In Fig. 297 is shown a simple cylinder subjected to internal pressure only, and the variation of the tangential stress over the cross section of the wall is shown by the curve AB. The equation of this stress, found by making p_o equal to zero in Eq. (400) is

$$s_t = (1-m)\,\frac{d_i^2p_i}{d_o^2-d_i^2} + (1+m)\,\frac{d_i^2d_o^2p_i}{4r^2(d_o^2-d_i^2)} \tag{401}$$

In Fig. 299 is shown a compound cylinder with the outer cylinder shrunk on. In this case the inner cylinder is subjected to an external pressure caused by the shrinking of the outer cylinder, and the outer cylinder is subjected to an internal pressure. Call the unit pressure between the cylinders p_c.

In Eq. (400) substitute for p_i, p_o, d_o, and r the respective values zero, p_c, d_c, and $d_i/2$. Then the tangential stress at the inner

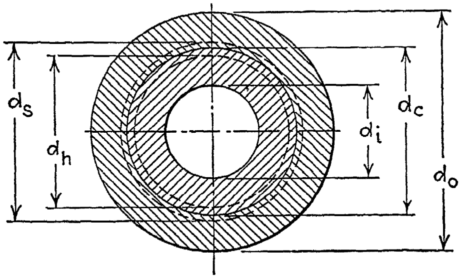


FIG. 299.

surface of the inner cylinder is

$$s_{t-i} = \frac{-2p_c d_c^2}{d_c^2 - d_i^2} \tag{402}$$

Similarly, the tangential stress at the outer surface of the inner cylinder is

$$s_{t-c} = -p_c\left[\frac{(d_c^2 + d_i^2)}{d_c^2 - d_i^2} - m\right] \tag{403}$$

and the tangential stress at the inner surface of the outer cylinder is

$$s_{t-c} = p_c\left(\frac{d_o^2 + d_c^2}{d_o^2 - d_c^2} + m\right) \tag{404}$$

Equations (403) and (404) give the stresses due to shrinking only. If internal pressure is applied to the cylinder there are stresses set up in addition to the shrinkage stresses. These additional stresses may be found for both cylinders when the proper values of r and p are substituted in Eq. (400).

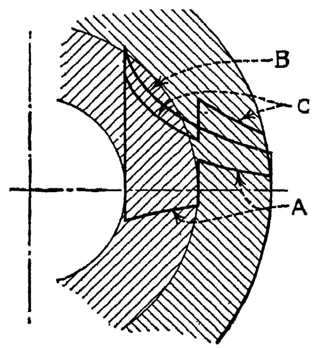


FIG. 300.

The final tangential stress distribution in the cylinder wall is shown in Fig. 300. Curve A shows the distribution due to shrinkage pressure only. Curve B shows the distribution due to internal pressure only. Curve C shows the final stress distribution, the stresses in curve C being the sum of the stresses in curves A and B.

335. Radial Pressure between the Cylinders. The radial pressure between the cylinders at the surface of contact depends on the modulus of elasticity of the materials, and on the difference between the outer diameter of the inner cylinder and the bore of the outer cylinder, before they are shrunk together.

Changes in the diameters of the two cylinders, in Fig. 299, due to pressure at their contact surface, are

$$\Delta d_s = d_s - d_c$$

and

$$\Delta d_h = d_c - d_h$$

from which

$$\Delta d_s + \Delta d_h = d_s - d_h = A$$

where A = the total shrinkage allowance, in.

The values of Δd_s and Δd_h in terms of the pressure p_c have been found in Eqs. (399) and (398). Hence

$$A = \frac{p_c d_s}{E_s}\left(\frac{d_s^2 + d_i^2}{d_s^2 - d_i^2} - m_s\right) + \frac{p_c d_h}{E_h}\left(\frac{d_o^2 + d_h^2}{d_o^2 - d_h^2} + m_h\right)$$

Without appreciable error, d_c may be substituted for d_s and d_h, since they differ by only a few thousandths of an inch. Then

$$\frac{A}{d_c} = p_c\left[\frac{d_c^2 + d_i^2}{E_s(d_c^2 - d_i^2)} + \frac{d_o^2 + d_c^2}{E_h(d_o^2 - d_c^2)} - \frac{m_s}{E_s} + \frac{m_h}{E_h}\right] \tag{405}$$

From this equation, the pressure between the cylinders, p_c, can be determined when the total shrinkage allowance is known, or the total allowance to produce the required pressure can be found.

In most compound cylinders, the outer and inner cylinders are made of the same material, and E_s and E_h are equal, and m_s and m_h are equal, and the last two terms in the parenthesis cancel out. After expansion of the terms and simplification

$$p_c = \frac{AE(d_c^2 - d_i^2)(d_o^2 - d_c^2)}{2d_c^3(d_o^2 - d_i^2)} \tag{406}$$

336. Selection of the Thick-cylinder Equation. The particular equation to be used depends upon the material used. The maximum-normal-stress theory applies to brittle materials such as hard steel, cast iron, and cast aluminum, and Eq. (380)

may be used. In general, the maximum-shear theory applies to ductile materials such as the low-carbon steels, brass, bronze, and aluminum alloys. Research indicates, however, that the maximum-strain theory agrees with experimental results on thick cylinders more closely than do the equations based on the maximum shear. Hence engineers favor the maximum-strain theory for all materials used in thick cylinders.

337. Working Stress in Thick Cylinders. In thick cylinders and compound cylinders, the maximum stress is present at the inner surface only, and the stress decreases rapidly toward the outer surface. Momentary overstresses are therefore not so serious as in some other machine members, since the material at the inner surface may flow slightly and readjust the stress distribution without causing failure. It is therefore permissible to use relatively high stresses, and, if no shock is present, 85 per cent of the yield stress may be considered satisfactory. Thick-walled cylinders of cast iron or cast steel are liable to contain defects and to be more unreliable than thin-walled cylinders; hence with cast cylinders it is often better to use high stresses in order to obtain better castings with thinner walls.

CHAPTER XXI

FLAT PLATES AND CYLINDER HEADS

338. Flat Heads. In many machines flat plates or slightly dished plates are used as diaphragms, pistons, cylinder heads, boiler heads, and the sides of rectangular tanks. These plates may be simply supported at the edges or at the center, or they may be more or less rigidly connected to the supporting member by means of bolts, welds, or rivets, or by casting integral with the support. The rigid stress analysis of such plates is difficult, and many of the equations developed have not been entirely verified by experimental evidence. The formulas generally used are based on the work of Bach and Grashof, and, since the derivations are complex and may be found in texts on stress analysis,* the derivations will not be presented here. Typical formulas employed are presented in Table 98. Results obtained by the use of these formulas are in general conservative.

The following symbols are used with the formulas in Table 98:

F = total load supported, lb.
p = load per unit area, psi.
s = unit stress, psi.
E = modulus of elasticity, psi.
a = length of the long side of a rectangular plate, in.
b = length of the short side of a rectangular plate, in.
r_o = outer radius of circular plate, in.
r_i = inner radius of plate with a concentric circular hole, in.
r = radius of circle over which the load is distributed, in.

* TIMOSHENKO and LESSELS, "Applied Elasticity," Chap. X, Westinghouse Night School Press.

SEELY, F. B., "Advanced Mechanics of Materials."

FISH, C. D., Stresses and Deformations in Flat Circular Cylinder Heads, A Mathematical Analysis, *Trans. A.S.M.E.*, Vol. 43, 1921, p. 615.

WAHL, A. M., and LOBO, G., JR., Stresses and Deflections in Flat Circular Plates with Central Holes, *Trans. A.S.M.E.*, 1930

WAHL, A. M., Design of Semi-circular Plates and Rings under Uniform External Pressure, *Trans. A.S.M.E.*, APM-54-28.

t = plate thickness, in.
y = maximum deflection in the plate, in.
m = reciprocal of the Poisson's ratio given in Table 15, page 59.

No plate can be supported with absolute rigidity, and the values of s and y obtained by the use of these equations are subject to modification according to the experience and judgment of the designer in determining the degree of rigidity. A plate or cylinder head held in place by evenly spaced bolts near its outer edge is usually considered to be freely supported, since the bolts are generally small enough to be considerably distorted when the load is applied. Heads cast integral with heavy cylinder walls approach the condition of rigid or fixed supports. These cast heads should be provided with generous fillets at the juncture with the walls to prevent the formation of crystalline cleavage planes at the corners when the casting is cooling. This weakening condition is illustrated in Fig. 301. Large corner

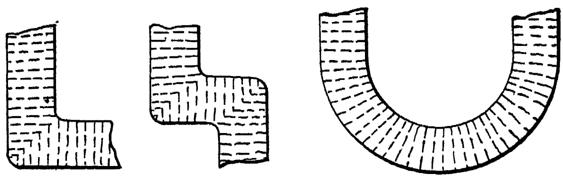


FIG. 301.

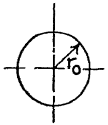
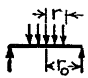
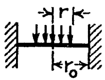
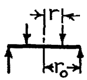
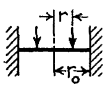
radii will prevent this action. Semispherical heads are the strongest forms to resist internal pressure; however, flat heads may be desirable and when used should be provided with generous corner radii and should be free from abrupt changes in thickness.

339. Bolted and Welded Heads.* The A.S.M.E. Boiler Code provides for the construction of unstayed steel heads, cover plates, blind flanges, etc., such as are shown in Fig. 302. The thickness of such flat plates is determined by the formula

$$t = d\sqrt{\frac{Kp}{s}} \tag{407}$$

* For the application of the maximum-strain theory to the design of cylinder and pipe flanges, see Waters and Taylor, The Strength of Pipe Flanges, *Trans. A.S.M E.*, 1927.

TABLE 98.—MAXIMUM STRESSES AND DEFLECTIONS IN FLAT PLATES

Form of plate	Type of loading	Type of support	Eq.	Total load F	Maximum stress s_{max}	Location of s_{max}	Maximum deflection y_{max}
	Distributed over the entire surface	Edge supported	(1)	$\pi r_o^2 p$	$s_r = s_t = \dfrac{-3F(3m+1)}{8\pi m t^2}$	Center	$\dfrac{3F(m-1)(5m+1)r_o^2}{16\pi E m^2 t^3}$
		Edge fixed	(2)	$\pi r_o^2 p$	$s_r = \dfrac{3F}{4\pi t^2}$	Edge	$\dfrac{3F(m^2-1)r_o^2}{16\pi E m^2 t^3}$
	Distributed over a concentric circular area of radius r	Edge supported	(3)	$\pi r^2 p$	$s_r = s_t = \dfrac{-3F}{2\pi t^2}\left[(m+1)\log_e\dfrac{r_o}{r} - (m-1)\dfrac{r^2}{4r_o^2} + m\right]$	Center	$\dfrac{3F(m^2-1)}{16\pi E m^2 t^3}\left[\dfrac{(12m+4)r_o^2}{m+1} - 4r^2\log_e\dfrac{r_o}{r} - \dfrac{(7m+3)r^2}{m+1}\right]$
		Edge fixed	(4)	$\pi r^2 p$	$s_r = \dfrac{3F}{2\pi t^2}\left(1 - \dfrac{r^2}{2r_o^2}\right)$ $s_r = s_t = \dfrac{-3F}{2\pi m t^2}\left[(m+1)\log_e\dfrac{r_o}{r} + (m+1)\dfrac{r^2}{4r_o^2}\right]$	Edge Center	$\dfrac{3F(m^2-1)}{16\pi E m^2 t^3}\left[4r_o^2 - 4r^2\log_e\dfrac{r_o}{r} - 3r^2\right]$ when r is very small (concentrated load) $\dfrac{3F(m^2-1)r_o^2}{4\pi E m^2 t^3}$
	Distributed on circumference of a concentric circle of radius r	Edge supported	(5)	$2\pi r p$	$s_r = s_t = \dfrac{-3F}{2\pi m t^2}\left[\dfrac{m-1}{2} + (m+1)\log_e\dfrac{r_o}{r} - (m-1)\dfrac{r^2}{2r_o^2}\right]$	All points inside the circle of radius r	$\dfrac{3F(m^2-1)}{2\pi E m^2 t^3}\left[\dfrac{(3m+1)(r_o^2-r^2)}{2(m+1)} - r^2\log_e\dfrac{r_o}{r}\right]$
		Edge fixed	(6)	$2\pi r p$	$s_r = s_t = \dfrac{-3F}{4\pi m t^2}\left[(m+1)\left(2\log_e\dfrac{r_o}{r} + \dfrac{r^2}{r_o^2} - 1\right)\right]$ $s_r = \dfrac{3F}{2\pi t^2}\left(1 - \dfrac{r^2}{r_o^2}\right)$	Center when $r < 0.31r_o$ Edge when $r > 0.31r_o$	$\dfrac{3F(m^2-1)}{2\pi E m^2 t^3}\left[\dfrac{1}{2}r_o^2 - r^2 - r^2\log_e\dfrac{r_o}{r}\right]$

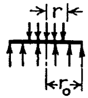
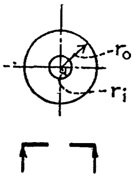
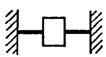
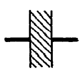
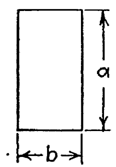
	Distributed over a concentric circular area of radius r	Uniform pressure over entire lower surface	(7)	$\pi r^2 p$	$s_r = s_t = \frac{-3F}{2\pi m t^2}\left[(m+1)\log_e \frac{r_o}{r} + \frac{m-1}{4}\left(1 - \frac{r^2}{r_o^2}\right)\right]$	Center	$\frac{3F(m^2-1)}{16\pi E m^2 t^3}\left[4r^2 \log_e \frac{r_o}{r} + 2r^2\left(\frac{3m+1}{m+1}\right) - r_o^2\left(\frac{7m+3}{m+1}\right) + \frac{(r_o^2 - r^2)r^4}{r^2 r_o^2} + \frac{r^4}{r_o^2}\right]$ when r is very small (concentrated load) $\frac{3F(m-1)(7m+3)r_o^2}{16\pi E m^2 t^3}$
	Distributed over the entire surface	Outer edge supported	(8)	$F = \pi(r_o^2 - r_i^2)p$	$s_t = \frac{-3p}{4mt^2(r_o^2 - r_i^2)}\left[r_o^4(3m+1) + r_i^4(m-1) - 4mr_o^2 r_i^2 - 4(m+1)r_o^2 r_i^2 \log_e \frac{r_o}{r_i}\right]$	Inner edge	$\frac{3F(m^2-1)}{2Em^2t^3}\left[\frac{r_o^4(5m+1)}{8(m+1)} + \frac{r_i^4(7m+3)}{8(m+1)} - \frac{r_o^2 r_i^2(3m+1)}{2(m+1)} + \frac{r_o^2 r_i^2(3m+1)}{2(m-1)}\log_e \frac{r_o}{r_i} - \frac{2r_o^2 r_i^4(m+1)}{(r_o^2 - r_i^2)(m-1)}\left(\log_e \frac{r_o}{r_i}\right)^2\right]$
	"	Outer edge fixed and supported	(9)	$F = \pi(r_o^2 - r_i^2)p$	$s_t = \frac{-3p(m^2-1)}{4mt^2}\left[\frac{r_o^4 - r_i^4 - \frac{1}{2}r_o^2 r_i^2 \log_e \frac{r_o}{r_i}}{r_o^2(m-1) + r_i^2(m+1)}\right]$	Inner edge	
	"	Outer edge fixed and supported, inner edge fixed	(10)	$F = \pi(r_o^2 - r_i^2)p$	$s_r = \frac{3p}{4t^2}\left[(r_o^2 + r^2) - \frac{4r_o^2 r^2}{r_o^2 - r^2}\left(\log_e \frac{r_o}{r}\right)^2\right]$	Inner edge	$\frac{3p(m^2-1)}{16Em^2t^3}\left[r_o^4 + 3r_i^4 - 4r_o^2 r_i^2 - 4r_o^2 r_i^2 \log_e \frac{a}{b} + \frac{16 r_o^2 r_i^4}{r_o^2 - r_i^2}\left(\log_e \frac{r_o}{r_i}\right)^2\right]$
	"	Inner edge fixed and supported	(11)	$F = \pi(r_o^2 - r_i^2)p$	$s_r = \frac{3p}{4t^2}\left[4r_o^4(m+1)\log_e \frac{r_o}{r_i} - \frac{-r_o^4(m+3) + r_o^4(m-1) + 4r_o^2 r_i^2}{r_o^2(m+1) + r_i^2(m-1)}\right]$	Inner edge	

TABLE 98.—MAXIMUM STRESSES AND DEFLECTIONS IN FLAT PLATES.—(*Continued*)

Form of plate	Type of loading	Type of support	Eq.	Total load F	Maximum stress s_{max}	Location of s_{max}	Maximum deflection y_{max}
	Uniform over entire surface	All edges supported	(12)	$F = abp$	$s_b = \dfrac{-0.75b^2p}{t^2\left(1 + 1.61\frac{b^3}{a^3}\right)}$	Center	$\dfrac{0.1422b^4p}{Et^3\left(1 + 2.21\frac{b^3}{a^3}\right)}$
	"	All edges fixed	(13)	$F = abp$	$s_b = \dfrac{0.5b^2p}{t^2\left(1 + 0.623\frac{b^6}{a^6}\right)}$	Center of long edge	$\dfrac{0.0284b^4p}{Et^3\left(1 + 1.056\frac{b^5}{a^5}\right)}$
	"	Short edges fixed, long edges supported	(14)	$F = abp$	$s_a = \dfrac{0.75b^2p}{t^2\left(1 + 0.8\frac{b^4}{a^4}\right)}$	Center of short edge	
	"	Short edges supported, long edges fixed	(15)	$F = abp$	$s_b = \dfrac{-b^2p}{2t^2\left(1 + 0.2\frac{a^4}{b^4}\right)}$	Center of long edge	

This table is abstracted from a table in "Handbook of Formulas for Strength of Materials" by Raymond J. Roark. Additional and more complete equations will be found in this book.

In this table m is the reciprocal of the value given in Table 1, p. 59. Positive sign for s indicates tension at upper surface and equal compression at lower surface, negative sign indicates reverse condition.

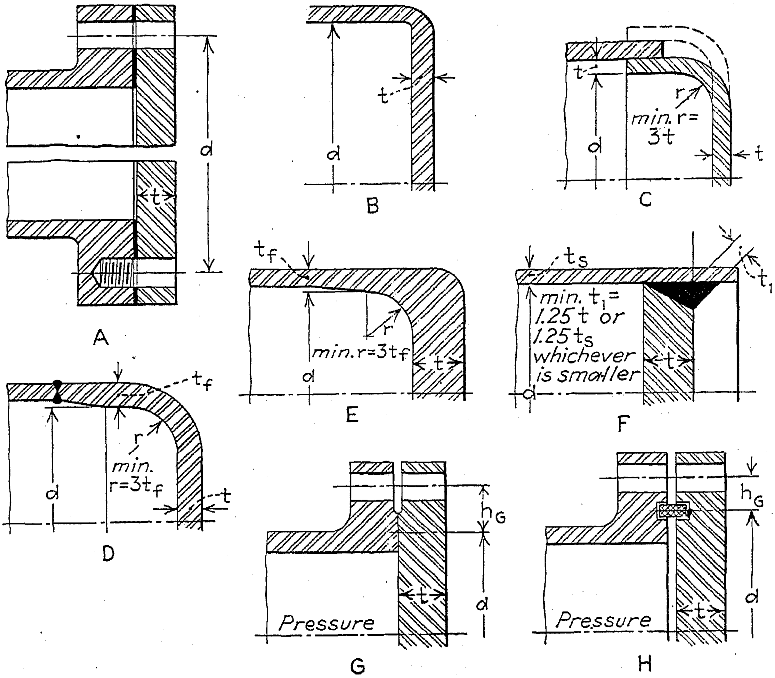


FIG. 302.—Solid, welded, and bolted heads. (*A.S.M.E. Boiler Code.*)

where K depends on the type of construction and has the following values:

(*A*) $K = 0.162$ for plates riveted or bolted rigidly to flanges.

(*B*) $K = 0.162$ for integral flat heads, where the diameter is less than 24 in., and t/d is equal to or greater than 0.05.

(*C*) $K = 0.30$ for flanged heads attached to the shell by lap joints.

(*D*)–(*E*) $K = 0.25$ for integral forged heads or heads butt-welded to pipes or shells.

(*F*) $K = 0.50$ for plates inside the shell, fusion-welded to meet the requirements of the respective class of fusion-welded vessel.

(*G*)–(*H*) $K = 0.30 + \dfrac{1.04Wh_g}{Hd}$ for plates bolted in such a manner that they tend to dish. In this,

W is the total bolt load in lb, H is the total hydrostatic force on the area bounded by the outside diameter of the gasket or contact area, and h_g and d are as shown in the figure.

340. Gaskets and Contact Pressures for Bolted Heads. The bolts must maintain a compression pressure on the contact surface greater than the hydrostatic pressure in the vessel, in order to maintain a leakproof joint. The contact pressure required depends upon the type of joint and the type of gasket material. Low-pressure, low-temperature gaskets are usually soft and extend over the entire flange face, with the bolt holes cut out. Such gaskets seldom require a unit contact pressure more than one or two times the unit working pressure. Wide ring gaskets, extending only to the inside of the bolts, are used on flat or raised-face flanges. Their use is limited to 300 psi working pressure, and 750 F. The gaskets may be made of soft compositions similar to rubber, or they may be thin hard-composition or soft metal. Contact-pressure ratios, in use at present, vary from 2 for the softer materials, to 4 for the harder materials. Tongued and grooved flange-faces are suitable for the higher pressures and temperatures, since the higher contact pressures may be obtained without unreasonable bolt sizes. The gaskets are made of both soft and hard materials, in widths from $\frac{1}{4}$ to 1 in. Narrow hard gaskets require the highest contact pressures. Contact-pressure ratios vary from 3 to 8 for narrow gaskets, and from 3 to 6 for wide gaskets.

341. Dished Heads. To obtain some of the desirable strength of the semispherical head, many cylinders are fitted with dished heads that are a portion of a sphere flanged to fit into the shell. For dished heads subjected to pressure on the concave side, the A.S.M.E. Boiler Code requires that the thickness shall be

$$t = \frac{8.33pR}{2s_u} \tag{408}$$

where p = internal pressure, psi.

R = inside radius of curvature of the head, in.

s_u = ultimate strength of material, psi.

Note that this equation is the same as Eq. (363) for spheres with a factor of safety of 8.33 included, this factor of safety

allowing for the higher stress at the curve joining the flange with the spherical segment. The corner radius must be at least $3t$ and not less than 0.06 times the inside shell diameter. The radius of curvature of the head must not be greater than the shell diameter, and when it is less than 0.8 times the shell diameter, the value of R in this equation must be taken as $0.8d$.

When the dished head is turned so that the pressure is on the convex side, the permissible pressure is only 60 per cent of that allowed by the Eq. (408); *i.e.*, for the same pressure the head thickness must be $1\frac{2}{3}$ times that required when the pressure is on the concave side.

When openings 6 in. or larger are used in the head, the thickness must be increased 15 per cent and never less than $\frac{1}{8}$ in. over the thickness as determined by the above equation.

342. Stayed Flat Surfaces. Flat plates of large area must be supported by stays to reduce the deformation and stress. According to Grashof, the maximum stress and deflection in an infinitely large plate subjected to a uniformly distributed load, and supported by stays equally spaced in both directions, are

$$s = 0.2275 \frac{a^2p}{t^2}$$

and

$$y = 0.0284 \frac{a^4p}{t^3E} \tag{409}$$

where a = pitch or distance between stays, in.

A modified form of this equation is given in the A.S.M.E. Boiler Code for use in determining the maximum allowable pressure on flat steel plates in pressure vessels. This form is

$$p = \frac{KT^2}{a^2} \tag{410}$$

where T is the plate thickness in sixteenths of an inch, a is the greatest pitch if the stays are not equally spaced in both directions, and K is a constant depending upon the type of stay. Values of K are

K = 112 for stays screwed through plates not over $\frac{7}{16}$ in. thick, with the ends riveted over.

K = 120 for stays screwed through plates over $\frac{7}{16}$ in. thick, with the ends riveted over.

K = 135 for stays screwed through plates and provided with single nuts outside the plate or with inside and outside nuts, but no washers.

K = 150 with heads not less than 1.3 times the stay diameter, screwed through the plates, or made with a taper fit and having heads formed before installing and not riveted over, these heads to have a true bearing on the plate.

K = 175 for stays with inside and outside nuts and outside washers, when the washer diameter is not less than $0.4a$, and the thickness not less than T.

343. Types of Stays. Stays are rods, plates, angles, or other structural shapes used to brace flat and curved surfaces for the purpose of reducing the stress and the distortion. Stay bolts are short rods, used to join parallel plates at regular intervals. They may be screwed through the plates and riveted over, or nuts may be used as indicated in the preceding paragraph. The body of the stay is generally turned to a diameter smaller than the root diameter of the threads to reduce local stresses in the threads, and in short stays a small hole is drilled from the ends to a point beyond the threaded portion. This hole acts as a telltale to indicate stay failures by allowing the leakage of steam or gas. On account of the unequal expansion of stayed surfaces and the relative motion between the plates, stay bolts are

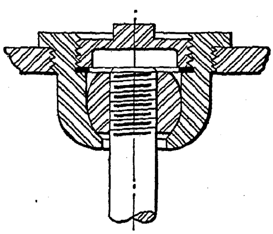


FIG. 303.—Flexible stay.

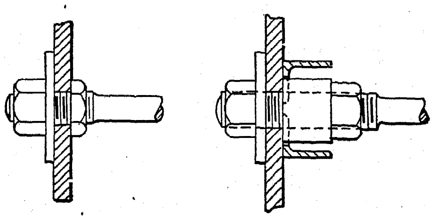


FIG. 304.—Through stays.

subjected to unknown bending stresses in addition to the direct tensile stress. To reduce these bending stresses, flexible stays have been developed, one form of which is shown in Fig. 303. Through stays joining surfaces at a considerable distance apart are generally spaced at least 16 in. apart to allow a man's passing

between them for inspection or for cleaning the stayed vessel. To prevent distortion of the plate between the stays, angles, tees, channels, and other structural shapes are riveted to the plate to act as stiffeners. The use of through stays is illustrated in Fig. 304.

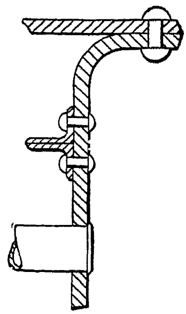


FIG. 305.—Angle brace.

Diagonal stays are used on long cylinders to form braces between the head and the shell. Two forms are shown in Fig. 306. The portion of the stay which is flared out to receive the rivets may fail by bending and should be checked by the beam formulas. The rivets are subjected to a small shear stress and to a direct tension stress. Since they are also subjected to unknown tension stresses due to the shrinkage of hot-driven rivets, it is customary to reduce the permissible tension stress to equal the permissible shear stress, which for ordinary steel rivets is 8,800 psi.

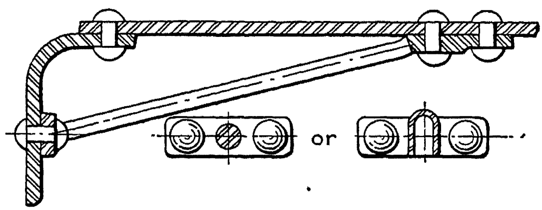


FIG. 306.—Diagonal stay.

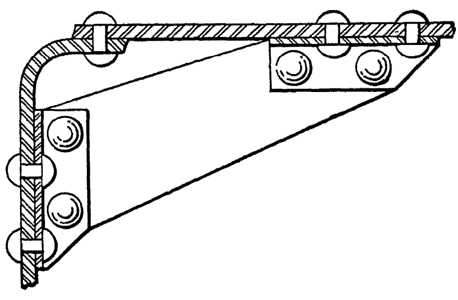


FIG. 307.—Gusset stay.

Gusset stays are diagonal stays made of flat plate material attached to the head and shell of the pressure vessel as shown in Fig. 307. Gusset stays may be flanged at the ends to receive the rivets, or they may be attached by means of angle irons.

344. Allowable Stress in Stays. The A.S.M.E. Boiler Code specifies that the maximum stress on stays used in pressure vessels shall be as given in Table 99.

TABLE 99.—MAXIMUM ALLOWABLE STRESSES IN STAYS AND STAY BOLTS

Type of stay	Stress, psi: For lengths between supports not exceeding 120 diam.	Stress, psi: For lengths between supports exceeding 120 diam.
a. Unwelded or flexible stays less than 20 diameters long, screwed through plates with ends riveted over	7,500	
b. Hollow steel stays less than 20 diameters long, screwed through plates with ends riveted over	8,000	
c. Unwelded stays and unwelded portions of welded stays, except as specified in (*a*) and (*b*)	9,500	8,500
d. Steel through stays exceeding $1\frac{1}{2}$ in. diameter	10,400	9,000
e. Welded portions of stays	6,000	6,000

A S M E. Boiler Code.

CHAPTER XXII

METAL FITS AND TOLERANCES

345. Machine Fits. When the various members of a machine are assembled, each must fit properly with its mating member. Journals, and other sliding parts, must be made so that they will move relative to the mating member, but must not be so loose that they will not function properly. On the other hand, when keys and similar devices are not advisable, machine members may be held rigidly together by means of press, force, or shrink fits. In the case of sliding fits, the journal or inner member is slightly smaller than the outer member in which it fits, *i.e.*, the parts are made with a clearance. In the case of force and shrink fits the inner member is made slightly larger than the hole into which it is assembled, that is, the parts have a slight interference.

To facilitate interchangeable manufacture and assembly, it is necessary that mating parts be machined with sufficient accuracy that hand fitting is not required. It is impossible to produce commercially a large number of parts of exactly the same dimensions; hence a manufacturing tolerance must be allowed, *i.e.*, a small permissible variation in size that will not prevent proper functioning of the assembled parts.

346. Definitions. The *nominal size* of a part is the approximate standard size but does not indicate any degree of accuracy in the machine work. The *basic size* is the exact theoretical size from which all limiting variations are made and is the minimum size of external members and the maximum size of internal members at which interference begins. The *allowance* is the intentional difference in dimensions of mating parts; or the minimum clearance space that is to be provided between mating parts. *Interference* is the amount by which the dimensions of mating parts overlap. The *tolerance* is the permissible variation in size, or manufacturing allowance. *Limits* are the extreme permissible dimensions of any part.

To illustrate the various terms, consider a $1\frac{1}{2}$-in. journal and bearing. The nominal size of each is $1\frac{1}{2}$ in. To allow proper functioning, the journal should be slightly smaller than the bore of the bearing. Hence an allowance of 0.0015 in. might be desirable, and the basic diameter for the bearing bore is 1.5000 in., and the basic diameter of the journal is 1.4985 in. If a manufacturing tolerance of 0.001 in. is permitted, the limit dimensions for the bearing bore are 1.5010 and 1.5000 in., and the limit dimensions for the journal are 1.4985 and 1.4975 in.

347. Classes of Fits. The amount of allowance and tolerance for any parts depends on the service for which they are intended. Many manufacturers have so-called standard fits although these may not be interchangeable, and any attempt to make them so would make the cost prohibitive. However, in production manufacturing, these parts must be made within certain limits, even if selective assembly is permissible.

Recommended tolerances and allowances for a number of classes of fits are tabulated in the A.S.A. Standards, B 4a-1925. These tables are too extensive for presentation here, but the formulas from which they are derived are given in Table 100.

The *loose fit* is intended for use where accuracy is not essential and where considerable freedom is permissible, such as in agricultural, mining, and general-purpose machinery.

The *free fit* is suitable for running fits where the speeds are in excess of 600 rpm, and the pressures in excess of 600 psi. This fit is suitable for shafts of generators, motors, engines, and some automotive parts.

The *medium fit* is suitable for running fits where the speeds are under 600 rpm, and the pressures under 600 psi. This fit is used for the more accurate machine-tool and automotive parts.

The *snug fit* is a zero-allowance fit and is the closest fit that can be assembled by hand. It is suitable where no perceptible shake is permissible and also where the parts are not to slide freely when under load.

The *wringing fit* is practically a metal-to-metal fit and is not interchangeable but is selective. Light tapping with a hammer is necessary to assemble the parts.

The *tight fit* has a slight negative allowance or metal interference, and light pressure is required to assemble the parts. This fit is suitable for semipermanent assembly, for long fits in

heavy sections, and for drive fits in thin sections. It is also suitable for shrink fits in light sections.

The *medium force fit* requires considerable pressure to assemble. This fit is suitable for press fits on locomotive wheels, car wheels,

TABLE 100.—FORMULAS FOR RECOMMENDED ALLOWANCES AND TOLERANCES

Class of fit	Method of assembly	Allowance	Selected average interference of metal	Hole tolerance	Shaft tolerance
1. Loose.	Strictly interchangeable	$0.0025\sqrt[3]{d^2}$	....	$0.0025\sqrt[3]{d}$	$0.0025\sqrt[3]{d}$
2. Free........	Strictly interchangeable	$0.0014\sqrt[3]{d^2}$	....	$0.0013\sqrt[3]{d}$	$0.0013\sqrt[3]{d}$
3. Medium....	Strictly interchangeable	$0.0009\sqrt[3]{d^2}$	....	$0.0008\sqrt[3]{d}$	$0.0008\sqrt[3]{d}$
4. Snug.......	Strictly interchangeable	0.0000	....	$0.0006\sqrt[3]{d}$	$0.0004\sqrt[3]{d}$
5. Wringing...	Selective assembly	....	0.0000	$0.0006\sqrt[3]{d}$	$0.0004\sqrt[3]{d}$
6. Tight.	Selective assembly	....	$0.00025d$	$0.0006\sqrt[3]{d}$	$0.0006\sqrt[3]{d}$
7. Medium force.......	Selective assembly	....	$0.0005d$	$0.0006\sqrt[3]{d}$	$0.0006\sqrt[3]{d}$
8. Heavy force or shrink...	Selective assembly	....	$0.001d$	$0.0006\sqrt[3]{d}$	$0.0006\sqrt[3]{d}$

d = diameter of fit in inches.

The hole is the base, or nominal, size. All hole tolerances are positive, and negative tolerance for hole is zero. Shaft tolerances are negative for working fits (with positive tolerance zero), and positive for force fits (with negative tolerance zero).

generator and motor armatures, and crank disks. It is also suitable for shrink fits on medium sections. This is the tightest fit recommended for cast-iron external members, since it stresses the cast iron to its yield stress.

The *heavy force* and *shrink fit* is used for steel external members that have a high yield stress. It will overstress cast-iron

external members. When the force fit requires impractical assembly pressure, the shrink fit should be used.

The shrink fits are used in the assembly of steel rims on cast-iron wheels, high-grade steel rims on cast gear spiders, aluminum-alloy heads on steel cylinders of aeronautic engines, and built-up large-bore guns. For heavy power transmission, keys are used in addition to the force fits; especially with shafts over 3 in. in diameter. The key is used as a locating guide during assembly, and also to maintain a tight connection, since the slight twist of the shaft may cause creep between the shaft and hub, so that the joint will gradually work loose.

348. Force Required for Press-fit Assembly. The total axial force required to assemble a force fit varies directly as the length of the external member, directly as the thickness of the external member, with the character and finish of the materials, and with the difference in diameters of the mating members. The total axial force in pounds may be approximated from the equation

$$F_a = \pi d L f p_c \tag{411}$$

where d = nominal diameter, in.
L = length of the external member, in.
f = coefficient of friction.
p_c = radial pressure between the two members, psi.

The value of the coefficient of friction varies greatly, ranging from 0.04 to 0.25, but averaging about 0.08. In the computation of the axial force required for assembly, the coefficient of friction should be taken high, say 0.10 or 0.125, and the lower values, 0.05 to 0.075, should be used in computing the holding power. For holding power in torsion, the coefficient is about 0.10 for press fits and 0.125 for shrink fits.

The radial pressure between the members is found from a consideration of the stresses acting in thick cylinders subjected to internal and external pressures. In Art. 335, it is shown that the unit radial pressure is

$$p_c = \frac{AE(d_c^2 - d_i^2)(d_o^2 - d_c^2)}{2d_c^3(d_o^2 - d_i^2)} \tag{412}$$

where A = total shrinkage allowance, in.
E = modulus of elasticity of material, psi.
d_o = outside diameter of external member, in.

d_c = nominal diameter of contact surfaces, in.

d_i = inside diameter of inner member, in.

If the internal and external members are not made of the same materials, Eq. (405), page 442, should be used to determine unit radial pressure p_c.

349. Stress Due to Force and Shrink Fits. The radial pressure p_c found from Eq. (412) or (405), is a direct compression stress at the outside of the internal member, and at the inside of the external member. The corresponding tangential stresses at the contact surfaces may be found by applying Eqs. (393) and (394), for thick cylinders with open ends.

When relatively thin bands are shrunk on heavy internal members, the stress on the internal member is usually disregarded, and the band is assumed to be a thin hoop with internal pressure. The shrinkage stress in the band is then found from the equation

$$s_t = E\delta = \frac{EA}{d_c} \tag{413}$$

This equation is only approximate, since it assumes that the inner member does not decrease in diameter.

Example. Determine the radial pressure between the hub and shaft, the axial force required to assemble, and the tangential stress in the hub, when a cast-iron hub of 4-in. outside diameter is pressed on a hollow steel shaft of 2-in. outside diameter and 1-in. inside diameter. The bore of the hub is 0.002 in. smaller than the shaft diameter. The modulus of elasticity of the steel is 30,000,000 and of the cast iron 10,000,000 psi. The hub and shaft are made of different materials; hence Eq. (405) is used to determine the radial pressure.

$$p_c = \frac{0.002}{2}\left(\frac{1}{\dfrac{4+1}{30 \times 10^6(4-1)} + \dfrac{16+4}{10 \times 10^6(16-4)} - \dfrac{0.30}{30 \times 10^6} + \dfrac{0.26}{10 \times 10^6}}\right)$$
$$= 4{,}200 \text{ psi}$$

The approximate force required to press the members together is found from Eq. (411).

$$F = \pi \times 2 \times 3.50 \times 0.1 \times 4{,}200 = 9{,}250 \text{ lb}$$

The maximum tangential stress in the hub is found from Eq. (404), since the hub may be considered as an open-end cylinder.

$$s_t = 4{,}200\left(\frac{16+4}{16-4} + 0.26\right) = 8{,}100 \text{ psi}$$

350. Shrink Links. To facilitate handling and shipping, flywheels, large gears, and some machine frames are made in sections. The separate parts are assembled either by the use of bolts or by the use of shrink links as shown in Figs. 308 and 309.

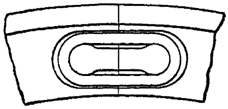


FIG. 308.

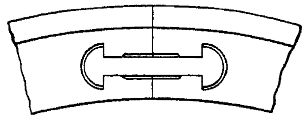


FIG. 309.

The cross-sectional area of the shrink link is computed from the total stress that must be transmitted through the wheel rim, assuming the load on the shrink link to be simple tension. The effective length of the link is made shorter than the mating slot, so that when the heated link is put in place and cooled, the stress induced in the link by shrinkage is slightly below the yield stress of the material. In the computation of the shrinkage allowance, it should be noted that the rim is stressed where the link bears against it. Professor C. D. Albert* shows that the average compressive stress in the part of the rim affected is

$$s_c = \frac{F}{\sqrt{a_b a_r}} \tag{414}$$

where F = total pull on the link, lb.

a_b = area of the bearing surface of the link, sq in.

a_r = area of contact between the rim segments, sq in.

The total shrinkage allowance for any desired stress in the link may be found as follows:

Let s_t = stress in the link, psi.

L_o = original length of the link, in.

L_f = final length of the link, in.

L = original length of the slot, in.

$A = L - L_o$, the shrinkage allowance, in.

a = cross-sectional area of the link, sq in.

Then

$$s_t = \frac{L_f - L_o}{L_o} E$$

* ALBERT, C. D., "Machine Design Drawing Room Problems," John Wiley & Sons, Inc.

But the total load on the link is

$$F = s_t a = \frac{L_f - L_o}{L_o} Ea$$

The compressive stress in the rim is

$$s_c = \frac{L - L_f}{L} E_r = \frac{F}{\sqrt{a_b a_r}} = \frac{(L_f - L_o)Ea}{L_o \sqrt{a_b a_r}}$$

and since L equals L_o, very nearly,

$$(L - L_f)E_r = \frac{(L_f - L_o)Ea}{\sqrt{a_b a_r}}$$

from which

$$L_o = \frac{L}{1 + \left(1 + \dfrac{aE}{E_r \sqrt{a_b a_r}}\right)\dfrac{s_t}{E}} \tag{415}$$

Example. Two steel shrink links, each having a cross-sectional area of $1\frac{1}{2}$ sq in. are used to connect two segments of a cast-iron flywheel. The contact area on the rim segments is 20 sq in. The bearing area under the head of each shrink link is $1\frac{9}{16}$ sq in. The original length of the slot in the wheel rim is 6 in. The modulus of elasticity of steel is 30,000,000 psi, and of cast iron 10,000,000 psi. The final stress in the shrink links is to be 30,000 psi.

The total pull on the two links is

$$P = 30{,}000 \times 1\ 5 \times 2 = 90{,}000 \text{ lb}$$

The average compression stress on the rim is

$$s_c = \frac{90{,}000}{\sqrt{2 \times 1.5625 \times 20}} = 36{,}000 \text{ psi}$$

The total decrease in the length of the slot is

$$L - L_f = \frac{36{,}000 \times 6}{10{,}000{,}000} = 0.0216 \text{ in.}$$

The final length of the slot is

$$L_f = L - 0.0216 = 6 - 0.0216 = 5.9784 \text{ in.}$$

The increase in the length of the link is

$$L_f - L_o = \frac{30{,}000}{30{,}000{,}000} L_o = 0.001 L_o$$

from which

$$L_f = 1.001L_o = 5.9784 \text{ in.}$$

and

$$L_o = 5\ 9724 \text{ in.}$$

which is the length of the lınk before heating and assemblıng the link and wheel segments

When this value is checked by Eq. (415),

$$L_o = \frac{6}{1 + \left(1 + \dfrac{3 \times 30 \times 10^6}{10 \times 10^6 \sqrt{2 \times 1.5625 \times 20}}\right) \dfrac{30{,}000}{30 \times 10^6}}$$
$$= \frac{6}{1 + (1 + 1.2 \times 3)0.001} = \frac{6}{1 + 0.0046} = 5\ 9725 \text{ in.}$$

351. Temperatures for Shrink Fits. In the assembly of shrink fits, the outer member is heated until it expands to an internal diameter (or length) several thousandths of an inch larger than the internal member. The actual temperature required should be checked to prevent injury to the metal from overheating.

In the case of the shrink link in the example of the preceding article, the length required after heating is $(L - L_o)$ plus a clearance. Allow 0.004 in. clearance, and the increase in length is

$$(6 - 5.9725) + 0.004 = 0.0315 \text{ in.}$$

Assume the original temperature to be 70 F, and the temperature required is

$$T = 70 + \frac{0.0315}{5.9725 \times 0.0000063} = 70 + 835 = 905 \text{ F}$$

where 0.0000063 = the coefficient of expansıon of steel.

CHAPTER XXIII

MISCELLANEOUS MACHINE MEMBERS

There are many machine members to which the principles developed in the preceding chapters may be applied in some modified form or in conjunction with other considerations. Limited space will permit the consideration of only a few typical members at this time.

352. Pistons. The piston of a steam or internal-combustion engine receives the impulse from the expanding steam or gas and transmits the energy through the connecting rod to the crank. In internal-combustion engines, the piston must also transmit a large amount of heat from the combustion chamber to the cylinder walls. With the older types of slow-speed engines having a low power output per cubic inch of cylinder volume, the design problems were not so serious as they now are with high-speed high-capacity engines. Modern automobile engines develop their maximum power at around 4,000 rpm; and at these speeds, heavy reciprocating pistons develop high inertia forces that are undesirable. Piston design must be considered from several viewpoints, such as: strength to resist the fluid and inertia forces; minimum weight; high-speed reciprocation without noise; bearing area sufficient to prevent undue wear; gas and oil sealing of the cylinder; dispersion of the heat of combustion; and resistance to thermal and mechanical distortion.

Pistons for steam engines are commonly made of cast iron or cast steel. Cast iron and cast steel are used for the larger stationary internal-combustion engines, and cast iron and aluminum alloys are used for the smaller high-speed internal-combustion engines. Cast iron and steel have the high strength required, good wearing qualities at the required temperatures, low thermal expansion, but relatively low thermal conductivity. The chief advantage of the aluminum alloys is their low weight. Their strength is good at low temperatures, but they lose about 50 per cent of their strength at temperatures above 600 F. Their ther-

mal expansion is about $2\frac{1}{2}$ times that of cast iron, and the resistance to abrasion is low at the high temperatures. However, aluminum has a very high thermal conductivity.

The most common form of piston for small steam engines is the box piston shown in Fig. 310, cast in one piece with internal stiffening ribs. In the larger sizes, built-up pistons consisting of a spider or body, a bull ring, a follower, and several piston

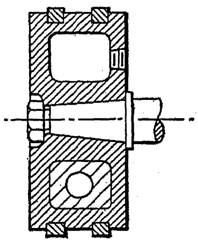


FIG. 310.—Box piston.

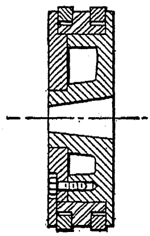


FIG. 311.—Built-up piston.

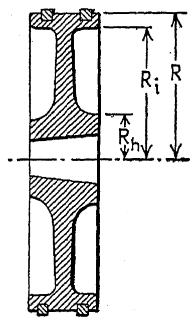


FIG. 312.—Plate piston.

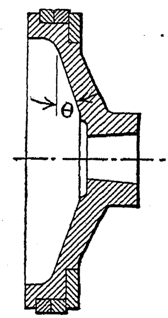


FIG. 313.—Conical plate piston.

rings, as shown in Fig. 311, are generally used. In marine engines and locomotive engines where light weight is a factor, conical plate pistons are used.

No exact method of stress analysis for pistons has been developed. For the preliminary layout, the following empirical rules may be used:

Width of face	0.3 to $0.5D$
Thickness of walls and ribs	
for low pressure	$\frac{D}{60} + 0.4$ in.
for high pressure	$\frac{D}{40} + 0.4$ in.
Hub diameter	$1.6 \times$ the piston-rod diameter
Width of piston rings	0.03 to $0.06D$
Thickness of piston rings	0.025 to $0.03D$

The stresses may be approximated by applying the flat-plate formulas of Table 98. The cylindrical outer wall may be assumed to be stiff enough to allow the flat walls to be considered as fixed at the inner and outer edges. Consider the load on the piston to be divided into two parts: a distributed load over the plate inside the outer cylindrical wall, *i.e.*, the area πR_i^2; and

the load on the outer wall, $p\pi(R^2 - R_i^2)$, distributed around the edge of the plate. Using flat-plate formulas (see Table 98), the first load gives the stress

$$s_1 = \frac{3p}{4t^2}\left(R_i^2 - 3R_h^2 + \frac{4R_h^2}{R_i^2 - R_h^2}\log_e \frac{R_i}{R_h}\right) \tag{416}$$

at the outer edge, and

$$s_2 = \frac{3p}{4t^2}\left(R_i^2 + R_h^2 - \frac{4R_i^2R_h^2}{R_i^2 - R_h^2}\log_e^2 \frac{R_i}{R_h}\right) \tag{417}$$

at the inner edge.

The second load acting on the rim gives the stress

$$s_3 = \frac{3p(R^2 - R_i^2)}{2t^2}\left(1 - \frac{2R_h^2}{R_i^2 - R_h^2}\log_e \frac{R_i}{R_h}\right) \tag{418}$$

at the outer edge, and

$$s_4 = \frac{3p(R^2 - R_i^2)}{2t^2}\left(1 - \frac{2R_i^2}{R_i^2 - R_h^2}\log_e \frac{R_i}{R_h}\right) \tag{419}$$

at the inner edge. The sum of the stresses at the outer edge, $s_1 + s_3$, or at the inner edge, $s_2 + s_4$, should not exceed the permissible stress of the material in tension. For cast iron, the factor of safety should be at least 8.

The stress in a box piston may be approximated in the same manner by neglecting the stiffening effect of the ribs and assuming that the loads are equally distributed between the two plates.

An empirical formula for the thickness of conical pistons is

$$t = 1.825\sqrt{\frac{pD}{s}}\sin\theta \tag{420}$$

Trunk pistons are used for small steam and internal-combustion engines and consist of a head to carry the cylinder pressure, a skirt to act as a bearing for the connecting-rod side thrust, a piston pin to connect it to the connecting rod, and piston rings to seal the cylinder. These pistons are used in single-acting engines and are open at one end. The head may be treated as a flat plate with a uniform load and rigidly supported at the outer edge. Using Eq. 2, Table 98, the head thickness is

$$t_h = \sqrt{\frac{3pD^2}{16s}}\ \text{in.} \tag{421}$$

where p = pressure, psi.
D = cylinder diameter, in.
s = permissible stress in tension, psi.

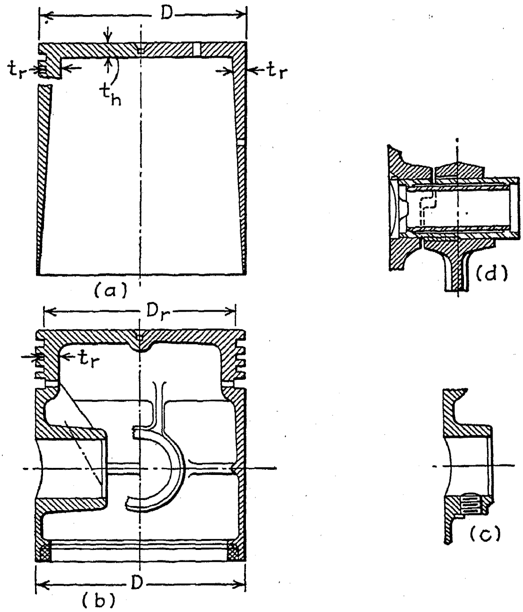


FIG. 314.—Trunk piston for small internal-combustion engine. (*a*) piston laid out for heat transfer. (*b*) piston modified for structural efficiency. (*c*) and (*d*) alternate boss and pin designs.

Empirical formulas are commonly used in the design of trunk pistons for automotive-type engines, the most common being

$$\text{Thickness of head} = 0.032D + 0.06 \text{ in.}$$

$$\text{Thickness of wall under the rings} = \text{thickness of head}$$

$$\text{Width of rings} = \frac{D}{20} \quad \text{for concentric rings}$$

$$= \frac{D}{27.5} \quad \text{opposite the joint of eccentric rings}$$

$$= \frac{D}{55} \quad \text{at the joint of eccentric rings}$$

$$\text{Length } L \text{ of piston} = D \text{ to } 1.5D$$

The length of the skirt below the ring section should be such that the side thrust from the connecting rod does not exceed 25 psi during the expansion stroke. The center of the piston pin should be from 0.02 to 0.04D above the center of the skirt to offset the turning effect of the friction. The diameter of the piston pin is determined by allowing a maximum bearing pressure of 2,500 psi with the maximum explosion pressure.

In internal-combustion engines, the heat flow through the head to the cylinder walls may determine the head thickness required. The head thickness for heat flow is

$$t_h = \frac{HD^2}{16c(T_c - T_e)A} = \frac{H}{12.56c(T_c - T_e)} \qquad (422)$$

where H = heat flowing through the head, Btu per hr.

D = cylinder diameter, in.

A = piston-head area, sq. in.

c = heat-conduction factor, Btu per sq. in. per in. length per hr per °F (2.2 for cast iron and 7.7 for aluminum).

T_c = temperature at center of head, °F.

T_e = temperature at edge of head, °F.

The heat flowing through the head may be estimated by the formula

$$H = KCw \times bhp$$

where C = higher heat value of fuel used, Btu per lb.

w = weight of fuel used, lb per bhp per hr.

bhp = brake horsepower of engine per cylinder.

K = constant representing that part of the heat supplied to the engine which is absorbed by the piston, approximately 0.05.

Experiments on successful piston designs indicate that $T_c - T_e$ is about 400 F for cast iron and T_c is 800 F. For aluminum pistons, $T_c - T_e$ will be about 130 F and T_c about 500 F. The wall thickness for ideal heat flow should taper from t_h at the head to zero at the open end.

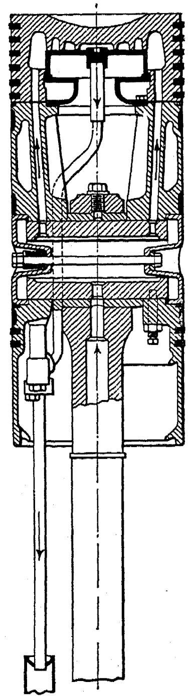


FIG. 315.—Water-cooled trunk piston for large Diesel engine. (*Courtesy Nordberg Mfg. Co.*)

The thickness at the ring section must be modified to maintain equivalent areas across the ring grooves. The area under the grooves should equal the edge area of the head since the same amount of heat is flowing. Then

$$\pi D t_h = \pi(D_r - t_r)t_r$$

from which

$$t_r = \tfrac{1}{2}(D_r \pm \sqrt{D_r - Dt_h}\,) \tag{423}$$

where t_r = thickness under the ring groove, in.
D_r = diameter of the bottom of the ring groove, in.

The root diameters of the ring grooves, allowing for ring clearances, should be

$$D_r = D - (2b + 0.006D + 0.020) \qquad \text{at the compression rings}$$

and

$$D_r = D - (2b + 0.006D + 0.060) \qquad \text{at the oil grooves}$$

where b = the radial depth of the ring section, in.

Small internal-combustion engines usually have three compression rings and one oil ring. Since the heat transfer from the rings to the cylinder walls is better than from the piston wall to the rings, it is better to use many narrow rings in preference to a few wide shallow rings. The piston, as laid out for heat transfer, must be modified for structural efficiency. Side thrust from the connecting rod is transmitted through the piston-pin bosses to the skirt, and a stiffening rib should be provided at the center line of the boss and should extend around the skirt to distribute this load and prevent distortion of the skirt. Short triangular ribs should extend from the lower edge of the ring section to the bosses to carry the gas pressure In some designs, these ribs extend across the head; but this may cause the head expansion to force the skirt out of round. At the open end, a small flange is provided for stiffening and to locate the piston during machining operations. In high-speed engines the inertia of the piston at the outer end of the stroke tends to throw the head and ring section away from the piston pins; and the wall section below the rings must be sufficient to sustain this inertia force.

353. Piston Rods. The piston rod is often designed as a column with pin ends, but most rods are short enough so that they

may be treated as members subjected to direct stress. Then

$$\frac{\pi D^2 p}{4} = \frac{\pi d^2 s_t}{4}$$

from which

$$d = D\sqrt{\frac{p}{s_t}} \tag{424}$$

where D = cylinder diameter, in.
d = rod diameter, in.
p = unbalanced pressure or difference between the steam-inlet pressure and the exhaust pressure, psi.

Since the stress is reversed, the factor of safety based on the ultimate strength should be at least 10 for double-acting engines and 8 for single-acting engines.

The rod ends forming the connections to the piston and crossheads are designed as indicated in Arts. 123 and 124.

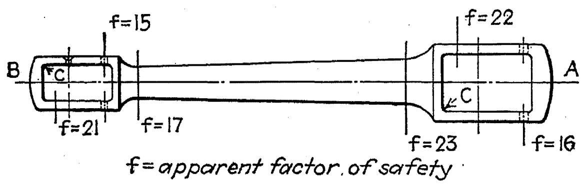


FIG. 316.—Solid connecting rod.

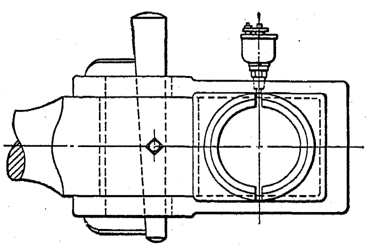


FIG. 317.—Open-strap and key design for low-speed engines.

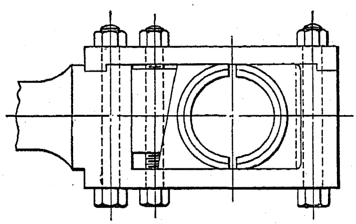


FIG. 318.—Open-side and strap design.

354. Connecting Rods. The best types of connecting rods for large engines are shown in Figs. 316 to 320. The solid rod, forged in one piece with the eyes at the ends machined out, is the strongest and most common form used with side-crank engines. Connecting rods for center-crank engines are made with the crank pin end in two or more pieces to facilitate assembly on the crankpin. Wedge blocks placed as shown in Fig. 318 have maxi-

mum accessibility for adjustment; the wear should be taken up in the same direction on both bearings to maintain the original rod length. Wedges should always be placed so that the bolts pull the wedges down when tightened, to reduce the danger of excessive end play and of the piston striking the cylinder head if the bolt should break.

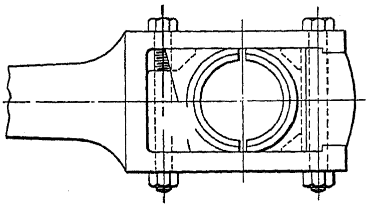
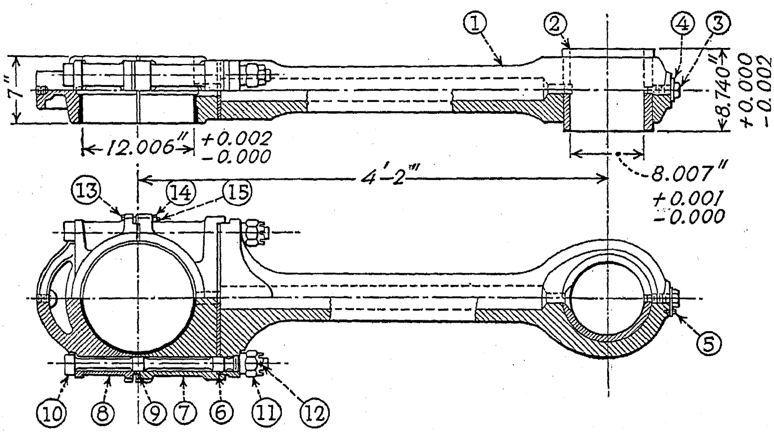


FIG. 319.—Open-end and block design.

The crankpin end is generally provided with a babbitt-lined bearing box. Bronze bearings give better service at the wrist pin, where the pressures are high and the space limited. Rods for single-acting engines using trunk pistons are usually not made adjustable on account of the inaccessibility.



SPECIFICATION FOR 1 SET

Part No.	Part	Drawing No.	Part No.	Part	Drawing No.
1	Conn. rod	VH-803-V	9	Laminated shim	VH-735-V
2	Bushing	VH-1149-U	10	Conn. rod bolt	VH-972-U
3	1" Spec. cap scr.	VH-1150-U	11	$1\frac{1}{2}$" castle nut	VH-972-U
4	Lock plate	VH-1150-U	12	$\frac{1}{4}$" cotter pin	VH-972-U
5	$\frac{5}{16}$" × $\frac{1}{2}$" c. scr.	VH-1150-U	13	$\frac{1}{2}$" × $3\frac{1}{2}$" bolt	VH-735-V
6	Compression pl.	VH-735-V	14	$\frac{1}{2}$" nut	VH-735-V
7	Top half bearing	VH-735-V	15	$\frac{3}{32}$" cotter pin	VH-735-V
8	Bottom half brg.	VH-735-V			

FIG. 320.—Marine-type connecting rod for large Diesel engine. (*Courtesy Nordberg Manufacturing Company.*)

The body of the rod is treated as a column with pin ends subjected to an additional bending load due to the inertia of the

oscillating rod. The stress due to the column action is

$$s = \frac{F_r}{A\left[1 - \frac{s_y}{4n\pi^2 E}\left(\frac{L}{k}\right)^2\right]} \tag{425}$$

where F_r = axial force on the rod, due to steam or gas pressure, corrected for inertia effects of the piston and other reciprocating parts, lb.

A = cross-sectional area, sq in.

s_y = yield stress of material, psi.

n = column-end factor (unity for pin ends).

E = modulus of elasticity, psi.

L = length center to center of pins, in.

k = radius of gyration of cross section, in.

The value of F_r from Fig. 322 is

$$F_r = \frac{F}{\sqrt{1 - \left(\frac{R \sin \theta}{L}\right)^2}} \tag{426}$$

where F = force transmitted from piston, lb.

R = crank radius, in.

θ = angle between the crank and the cylinder center line, measured from the head-end dead-center position.

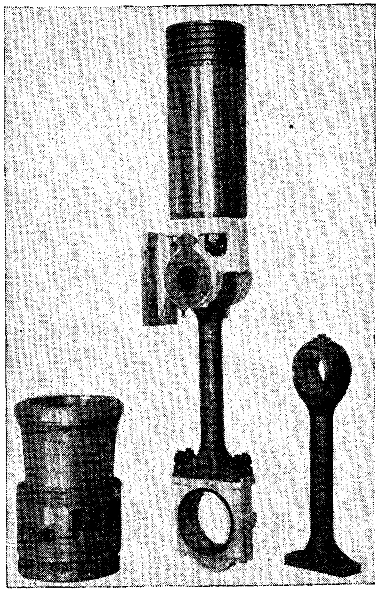


FIG. 321.—Piston, attached crosshead, connecting rod, and cylinder liner for large Diesel engine. (*Courtesy Nordberg Manufacturing Company.*)

Heavily loaded steam engines have the cutoff at half stroke or later, and the maximum value of F_r will be when θ is 90 deg. For shorter cutoffs, and for internal-combustion engines where the cylinder pressure drops rapidly during the early portion of the expansion stroke, several values of the piston pressure, corrected for inertia effects, must be used with the corresponding values of θ to determine the maximum load on the rod.

The rod may be assumed to be uniform in cross section without any appreciable error, in which case the resultant inertia force

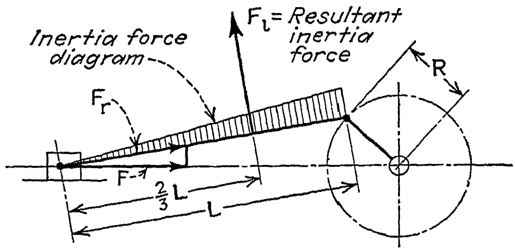


FIG. 322.—Forces acting on the connecting rod.

due to the acceleration of the oscillating rod will act at a distance $2L/3$ from the wrist pin and will have a magnitude

$$F_i = \frac{12Wv^2}{2gR} \sin \theta \tag{427}$$

where W = weight of body of rod, lb.
v = velocity of crankpin, fps.
R = radius of crank, in.

The maximum bending moment caused by this force will be at a distance $0.577L$ from the wrist pin, and its value will be

$$M = 0.774 \frac{Wv^2}{g} \left(\frac{L}{R}\right) \sin \theta, \text{ lb-in} \tag{428}$$

In the case of the steam engine with cutoff at half stroke or later, the stresses due to column action and bending will both be maximum very close to the center of the rod and can be added together directly. With the internal-combustion engine, both stresses must be found at several values of θ and added. The

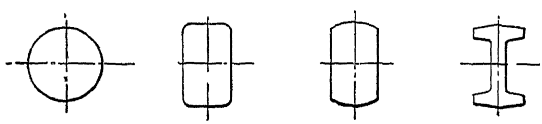


FIG. 323.—Typical connecting-rod sections.

sum of these two stresses must not exceed the permissible stress in tension. The permissible stress should not exceed one-third the yield stress for single-acting engines, or one-sixth the yield stress for double-acting engines, since the stresses are repeated or reversed.

The rod may be round, rectangular, or I sectioned. When the rectangular or I sections are used, the radius of gyration for transverse bending will be less than that in the plane of rotation. However, the pins restrain the bending in this plane, and experience indicates that the column-end constant n may be taken as 2. Using this value and making the column equally strong in both planes, the column equations indicate that the ratio of width to depth of a rectangular section should be approximately 0.7. In high-speed engines where the bending stresses are high, the ratio may be as low as 0.5.

Rods with I sections must be proportional for stiffness to prevent local buckling even when the column and bending equations indicate sufficient strength. A satisfactory section has a width 0.6 times the depth and a flange and web thickness from 0.20 to 0.25 times the depth. Although the rod is assumed to be of uniform section when making the computations indicated, most rods are tapered so that the crankpin end is from 1.10 to 1.15 times the depth computed for the center section.

The stress where the tapered rod joins the wrist-pin rod end should be investigated. The direct stress due to the piston pressure should be added to the bending stress in the same manner as the stresses at the rod center. The bending moment at any section of the rod is

$$M_x = \frac{F_i x}{3L^2}(L^2 - x^2) \tag{429}$$

where M_x = moment at a distance x from the wrist pin, lb-in.

F_i = inertia force from Eq. (427), lb.

All other dimensions should be in inches.

The error introduced by neglecting the weight of the rod ends is very slight, since the heavy rod end is partially balanced about the center of the pin. The side straps of the rod ends are subjected to bending stresses and to repeated tension stresses in double-acting engines. The bending stresses may be determined from the moment M_x, when x is given the proper value. The bending stresses will be most severe at the inner corner or at the section through the wedge-bolt hole. To provide for any inequality of loading and for the localized stresses at the corners and at the bolt hole, each strap should be designed to support two-thirds the total load.

The outer end of solid rods and the cap of marine-type and split-end rods are usually assumed to be simple beams supported at the center. The deflection computed for this beam should not exceed 0.001 in. if the brasses are to be supported rigidly. Cracked brasses may occur when insufficient support is provided. The thickness of the bearing brasses is made approximately 0.25 times the pin diameter.

355. Cranks. The dimensions of the crankpin are determined from the bearing and lubrication requirements discussed in Chap. XIII and then checked for strength and deflection. Crankpins of side-crank engines are cantilever beams on which the load may be assumed to be concentrated at the center of the bearing surface. The maximum bending stress will be at the junction of the pin and the arm. When the pin is pressed or shrunk into the arm, a shoulder is provided next to the arm; and, if the pin diameter is reduced at this point, the smaller diameter is used in the computations. Crankpins of center-crank engines may be treated as simple beams with the load concentrated at the center. In multiple-cylinder engines the torque from each cylinder is transmitted along the shaft and subjects the intervening crankpins to torsional shear. The bending and shear stresses should be combined by the use of Eq. (33). The working stresses used in the design should provide for reversals of bending stress and moderate shock. The deflection of the pins should not exceed 0.001 in.

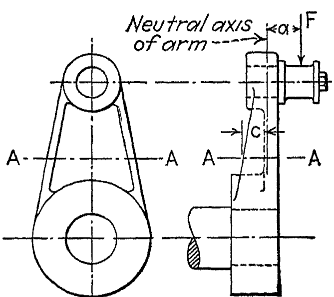


FIG. 324.—Side crank.

The crank arm is subjected to various combinations of loading during each power cycle. When the crank is on the head-end dead-center position, the section AA in Fig. 324 is subjected to a bending moment equal to Fa and to a direct compression by the connecting rod. At the crank-end dead center, the compression may change to a slight tension in single-acting engines. The total stress at either position is

$$s = \frac{F}{A} \pm \frac{Mc}{I} \tag{430}$$

where A = cross-sectional area of arm, sq in.
M = bending moment Fa, lb-in.
c = distance from neutral axis of section to outer fiber of arm, in.
I = moment of inertia of section, $in.^4$

In the crank-end dead-center position, the section of the hub in the plane passing through the shaft center is subjected to direct tension and to the bending moment Fa. When the crank is perpendicular to the connecting rod, the crank is subjected to bending in the plane of rotation and to a torsional moment Fa. The bending and shear stresses caused by these moments should be combined by means of Eq. (34) if the crank is cast iron, and by Eq. (33) if the crank is steel. When the crank is pressed or shrunk on the shaft, the hub acts as a thick cylinder subjected to internal pressure, and the stresses may be determined as outlined in Chap. XX. The outside diameter of the hub is usually made twice the shaft diameter.

356. Flywheels. A flywheel is primarily a rotating energy reservoir. In one group of machines, including punch presses, bulldozers, shears, etc., the flywheel absorbs energy from a power source during the greater portion of the operating cycle so that it may deliver a large amount of energy as useful work in a very short portion of the cycle, with a limited reduction in the speed of the machine. In machines like steam engines, internal-combustion engines, reciprocating pumps, etc., the flywheel is used to smooth out the speed fluctuations caused by the nonuniform flow of power from the pistons during each cycle of operation.

357. Flywheels for Punches and Shears. In order to deliver energy, the rim velocity must decrease, the permissible decrease varying with the class of machine and the time required for a complete operation cycle. Permissible speed reductions usually employed are given in Table 101.

The energy released when the speed drops is

$$\Delta E = \frac{1}{2} M(v_1^2 - v_2^2) = \frac{W}{2g}(v_1^2 - v_2^2) \tag{431}$$

where W = weight of rim, lb.
v_1 = maximum velocity, fps.
v_2 = minimum velocity, fps.

TABLE 101.—COEFFICIENT OF SPEED FLUCTUATION C_s

Driven Machine	Coefficient C_s
Crushers and hammers	0 200
Electric generators	0 003–0 006
Flour-milling machinery	0 020
Machine tools	0 200
Papermaking machinery	0 025
Punches and shears	0 030–0 050
Pumping machinery	0 030–0 050
Transmission by belt	0 030
Transmission by gear train	0 020
Spinning machines	0 030–0 050
Weaving machines	0 025

The velocities should be measured at the radius of gyration of the flywheel, but in general the hub and arms are neglected, since about 95 per cent of the energy is stored in the rim, and the velocities are then measured at the mean radius of the rim.

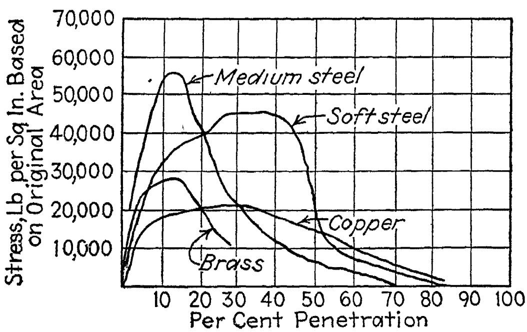


FIG. 325.—Load curves for punch and die with proper clearances.

In Fig. 325, typical curves of the variation of the punching force as a punch or shear passes through the metal are shown. Note that the force increases rapidly to a maximum and then drops off as the punch progresses through the material. The maximum force is used in the design of the pitman rod and other parts of the machine. The work done by the punch is the average force times the thickness of the metal and is measured by the area under the force curve. The metal requiring the maximum force does not necessarily require the most work. The work or energy required to punch the material must be supplied by the flywheel; hence when the required energy is

substituted in Eq. (431) the weight of the required rim can be determined. The capacity of the rim can only be increased by increasing the velocity, the weight, or the diameter; and it is often desirable to mount the flywheel on a high-speed auxiliary shaft in order to keep the diameter and weight within reasonable limits. The velocity at the outer surface of cast-iron flywheels having the arms and hub cast integral with the rim, should not exceed 6,000 fpm.

358. Engine Flywheels. Power delivered by the piston fluctuates over wide ranges during each revolution of the main shaft, whereas the power delivered by the shaft must be nearly uniform in intensity. The flywheel acts as an intrarevolution governor by absorbing the excess energy delivered by the piston during the early portion of the stroke and releasing this energy during the latter part of the stroke when the piston is delivering very little power. In reciprocating compressors, the situation is reversed.

The change in flywheel speed may be designated by a coefficient of fluctuation, thus

$$C_s = \frac{v_1 - v_2}{v_m} \tag{432}$$

The fluctuation of energy in the flywheel is

$$\begin{aligned} \Delta E &= \frac{W}{2g}(v_1^2 - v_2^2) = \frac{W}{2g}(v_1 + v_2)(v_1 - v_2) \\ &= \frac{C_s W v_m^2}{g} \end{aligned} \tag{433}$$

TABLE 102.—COEFFICIENT OF FLUCTUATION OF ENERGY C_e
(For steam engines)

% Cutoff	Single cylinder	Two-cylinder cranks at 90 deg	Three-cylinder cranks at 120 deg
0.10	0 35	0.088	0.040
0.20	0.33	0.082	0.037
0.40	0 31	0 078	0.034
0.60	0 29	0 072	0.032
0.80	0 28	0.070	0.031
1.00	0 27	0.068	0.030

The fluctuation of energy may be found from the indicator card of the engine by plotting the torque delivered to the crankshaft, correcting the piston pressure for the inertia forces. Then ΔE will be represented by the largest area above or below the mean torque line, as illustrated in Fig. 327*a*. For most purposes, a coefficient of fluctuation may be taken from Tables 102 and 103, which have been compiled from the results of various investigators.

TABLE 103.—COEFFICIENT OF FLUCTUATION OF ENERGY C_e
(For internal-combustion engines)

Type of engine	Number of cylinders	Angle between cranks	C_e	
			4-stroke cycle	2-stroke cycle
Single acting	1	360	2 35–2.40	0.95–1.00
twin	2	360	0.92–1.04	0.75–0 85
opposed	2	180	0.92–1.04	0.75–0 85
tandem	2	0	0.92–1.04	0.75–0 85
twin	2	180	1.50–1.60	0.20–0.25
opposed	2	360	1.50–1 60	0.20–0 25
vertical	3	120	0.60–0 75	0.15–0 18
vertical	4	180 and 90	0.15–0.20	0.075–0 10
vertical	6	180 and 60	0.10–0.12	0.016–0.02
Double acting	1	....	1.50–1.60	0.20–0.24
twin or tandem	2	....	0.18–0 20	0 18–0 20
twin-tandem	4	90	0.08–0.09	0.07–0 08

The average energy delivered by the engine shaft per revolution, is

$$E_a = \frac{33{,}000 \times \text{hp}}{N} \tag{434}$$

and the energy fluctuation is

$$\Delta E = C_e \times E_a = C_e \frac{33{,}000 \times \text{hp}}{N} \tag{435}$$

Combining this with Eq. (433), the weight required in the flywheel rim is

$$W = \frac{C_e}{C_s} \times \frac{33{,}000 \times \text{hp} \times g}{Nv_m^2} = 1{,}062{,}000 \frac{C_e \times \text{hp}}{C_s N v_m^2} \tag{436}$$

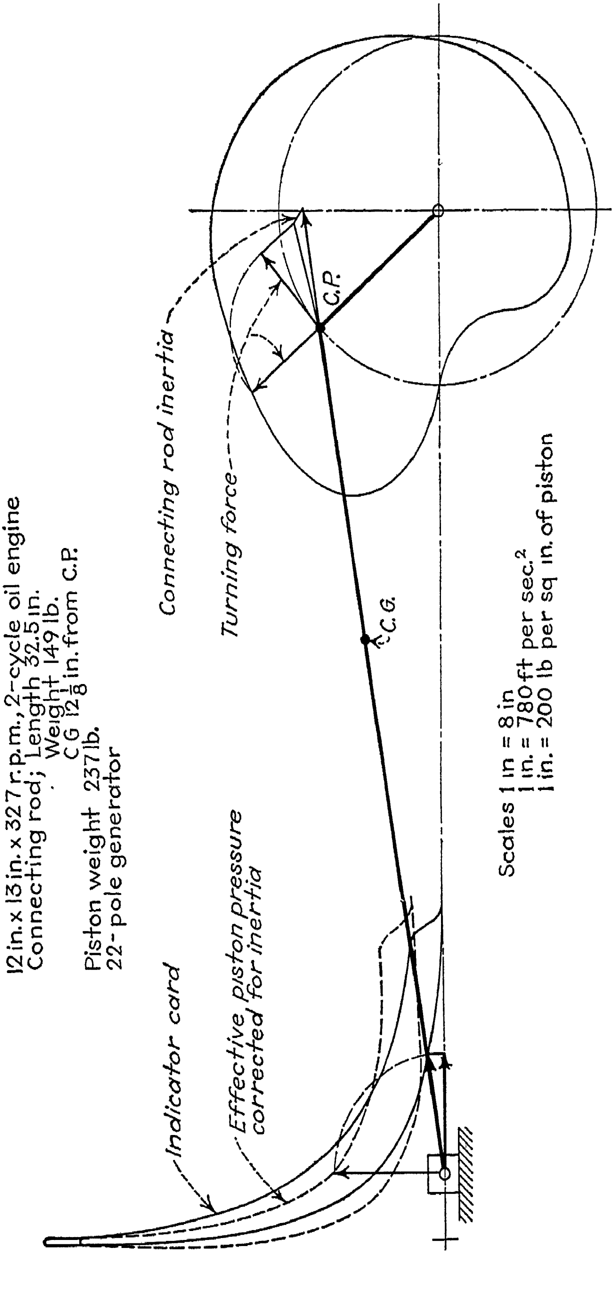


FIG. 326.—Turning-effort diagram, for an internal-combustion engine.

A more exact solution is often required for the flywheels of engines driving electric generators. Since the flywheel is alternately absorbing and delivering energy, the speed varies

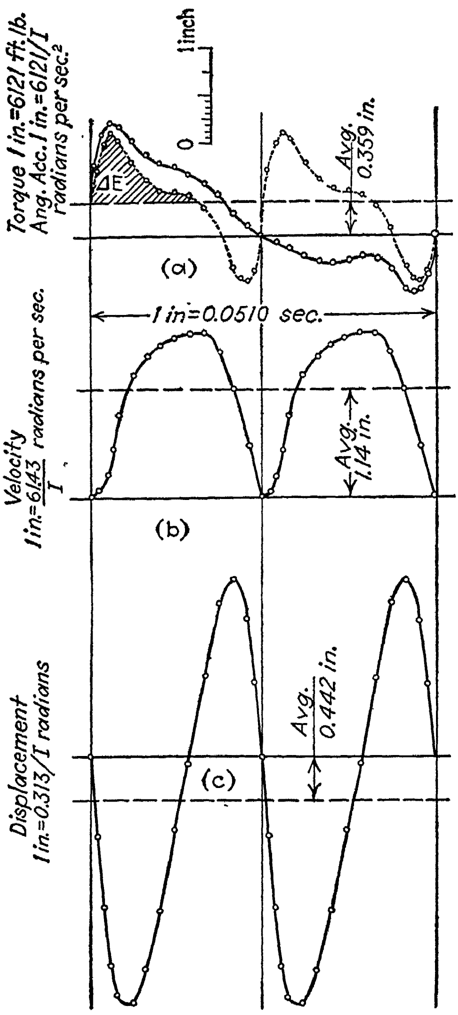


FIG. 327.

during each revolution, displacing the flywheel ahead of and behind its mean position, *i.e.*, the position it would occupy if the angular speed remained uniform. Proper voltage regulation

requires that the maximum angular displacement shall be limited to $2\frac{1}{2}$ or 3 electrical degrees. The problem is one of supplying sufficient flywheel mass to absorb the energy fluctuations with a predetermined angular displacement. The method can best be shown by means of a definite problem.

Example. The indicator card and the corresponding torque diagram from a two-cycle two-cylinder semidiesel oil engine are shown in Fig. 326. The torque diagram is replotted in Fig. 327*a* with the crankpin circle developed into a straight line. It is known that

$$T = I\alpha$$

and

$$\alpha = \frac{T}{I}$$

where T = torque, ft-lb.
I = mass moment of inertia of flywheel, ft-lb sec.^{2}
α = angular acceleration, radians per sec.^{2}

Since I is a constant, the torque diagram also represents the acceleration, the acceleration scale being the torque scale divided by I. It is also known that

$$\omega = \int \frac{d\alpha}{dt}$$

and

$$\theta = \int \frac{d\omega}{dt} = \int\int \frac{d\alpha}{dt} \qquad (437)$$

where ω = angular velocity, radians per sec.
θ = angular displacement, radians.
t = time, sec.

Hence, if the acceleration curve (Fig. 327*a*) is integrated twice, a curve of angular displacement is obtained. The integration is performed graphically by the method explained in Art. 138. The engine in this case is driving a 22-pole generator, which corresponds to 11 complete cycles per revolution. The maximum permissible angular displacement is $2\frac{1}{2}$ electrical degrees, which is $\frac{5}{22}$ mechanical degree, or 0.00396 radian. The maximum displacement, scaled from the mean height line of Fig. 327*c*, is $0.706/I$ radians. Equating the two values of displacement and solving,

$$I = \frac{0.706}{0.00396} = 178.3$$

This is the mass moment of inertia required in the rim of the flywheel. The construction of this engine limits the flywheel diameter to 65 in., and the rim may be square. Hence the mean radius is $(65 - b)/2$ in., and

$$I = \frac{WR_m^2}{g} = \frac{450 \times 2\pi(65 - b)b^2}{2 \times 1{,}728} \times \frac{(65 - b)^2}{144 \times 4g} = 178.3 \qquad (438)$$

from which

$$b = 5 \text{ in.}$$

359. Flywheel Rims. All parts of the rim are subjected to centrifugal force whose magnitude at the mean circumference is

$$F_c = \frac{12wbv_m^2}{gR_m} \quad \text{psi} \qquad (439)$$

where w = weight of material, lb per cu in.
b = rim thickness, in.
v_m = mean velocity, fps.
R_m = mean radius, in.

This force acting on a rotating rim without restraining arms produces a hoop stress or centrifugal tension equal to

$$s_c = \frac{12wv^2}{g} \quad \text{psi} \qquad (440)$$

This stress will be greatest at the outer surface where the velocity is the greatest.

The section of the rim between the arms may be considered to be a fixed-end beam with the centrifugal force acting as a distributing load. The maximum bending stress is at the arms, and the bending stress will be

$$s_b = \frac{Mc}{I} = \frac{12\pi^2 D_m wv_m^2}{gbn^2} \qquad (441)$$

where D_m = mean diameter, in.
n = number of arms.

This would be the stress if the arms were rigid. However, it has been found experimentally that the arms stretch approximately three-fourths the amount required if the rim expanded freely under the action of the centrifugal force. Assuming this amount of stretch, the actual stress in the rim at the junction with the arms will be approximately

$$s = \frac{3}{4} s_c + \frac{1}{4} s_b = \frac{wv_m^2}{g}\left(9 + \frac{3\pi^2 D_m}{bn^2}\right) \qquad (442)$$

Although this equation is often used to compute the stress in flywheel and pulley rims, it should be used with extreme cau-

tion, because the shrinkage stresses at the junction of the rim and arms are very severe if the wheel is not carefully designed and thoroughly annealed. Since the shrinkage stresses cannot be determined, it is best to rely on experience and limit the rim velocity to values that are known to be satisfactory.

Rim velocities should be limited to 6,000 fpm with cast iron when the rim, arms, and hub are cast in one piece. Cast steel may run at 8,000 fpm. In order to eliminate the shrinkage stresses and permit higher rim velocities, the wheel may be cast in sections and bolted together; the rim may be cast one day with a larger number of wrought steel spokes inserted in the mold, and the hub cast onto these the following day. The last type of wheel has been used successfully at velocities of 12,000 fpm. Flywheels having rims and hubs cast independently and bolted together by means of steel disks or webs may be run at velocities of 15,000 fpm.

The arms are designed to support the full torque capacity of the machine (not necessarily that of the shaft, which may be made large to provide stiffness), the method of computation being the same as that used for the arms of gears in Art. 298. The arms are subjected to complete reversals of stress, and the factor of safety should be at least 8. When the flywheel is used on punches and other machines subjected to severe shock, the factor of safety should be as high as 15.

360. Rotating Thin Flat Disks.* The stresses in thin disks of uniform thickness may be found if it is assumed that the stress is uniformly distributed over the thickness of the disk. The general equations for the radial and tangential stresses at any point of radius r are

$$s_r = 0.00000355wN^2\left(R_o^2 + R_i^2 - \frac{R_o^2R_i^2}{r^2} - r^2\right)(3 + m) \qquad (443)$$

and

$$s_t = 0.00000355wN^2\left[(3 + m)\left(R_o^2 + R_i^2 + \frac{R_o^2R_i^2}{r^2}\right) - (1 + 3m)r^2\right] \qquad (444)$$

* For a complete analysis see

TIMOSHENKO and LESSELLS, "Applied Elasticity," Westinghouse Night School Press.

TIMOSHENKO, S., "Theory of Elasticity," McGraw-Hill Book Company, Inc.

where w = weight of material, lb per cu in.
N = angular speed, rpm.
R_o and R_i = outer and inner radii, in.
r = radius at any point under consideration, in.
m = Poisson's ratio (0.3 for steel).

The radial stress is zero at the two plate edges where r is equal to R_o or R_i and is maximum when r is equal to $\sqrt{R_oR_i}$.

The equations indicate that the tangential stress is always greater than the radial stress at the same point in the plate. The tangential stress is a maximum at the inner edge of the plate where its value is

$$s_{t\,\max} = 0.0000071wN^2[(3 + m)R_o^2 + (1 - m)R_i^2] \qquad (445)$$

When the disk is solid, R_i in Eq. (444) becomes zero, and the maximum stress is

$$s_{t\,\max} = 0.00000355wN^2(3 + m)R_o^2 = s_{r\,\max} \qquad (446)$$

The serious effect of a small hole at the center of the disk is shown by making R_i small enough so that the term R_i^2 in Eq. (445) becomes negligible. Then

$$s_{t\,\max} = 0.0000071wN^2(3 + m)R_o^2 \qquad (447)$$

which is just twice the maximum stress in a solid disk.

When the disk is loaded so that there are tension or compression forces distributed around the plate edges, the effect of these must be superimposed on the stresses given by Eqs. (443) and (444). The stresses due to these distributed loads may be found from the thick-cylinder Eqs. (393) and (394) and added to those found for the disk without loads at the edges.

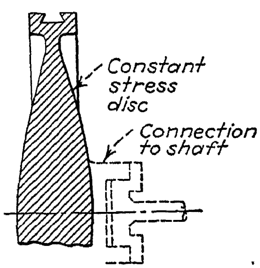


FIG. 328.—Variable-thickness disk for uniform stress distribution.

To permit the use of higher speeds, the disks of many high-speed machines are made with a variable thickness in an attempt to keep the stresses uniform throughout, thus using the material more efficiently. An analysis of this type of disk is beyond the scope of this text, and the reader is referred to discussions* based on the theory of elasticity.

* TIMOSHENKO, S., "Theory of Elasticity," McGraw-Hill Book Company, Inc.

361. Machine Frames. All the moving members of an operating machine must be supported, guided, and held in accurate alignment by means of a frame that is subjected to reactions resisting the forces applied to the moving members as well as the inertia forces. In most machines, the forces acting on the frames, and the stresses induced, are so complicated that no accurate mathematical analysis is possible. In many machines where accurate alignment must be maintained, rigidity is more important than the actual strength. Since it is possible to compute the stresses and deformation only in the very simplest frames, the judgment and experience of the designer must govern the design.

Frames are generally intricate in shape and are therefore cast, cast iron being the most common material, with steel used for the heavy-duty frames. Since the advent of welding, an increasing number of frames are being designed to be built of simple rolled steel shapes assembled by welding.

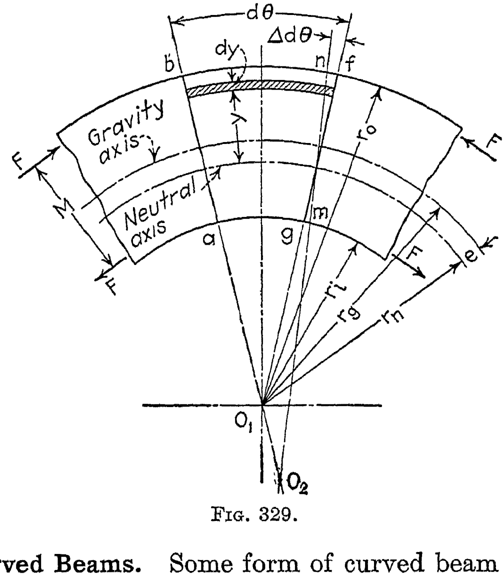
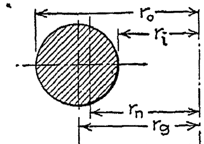
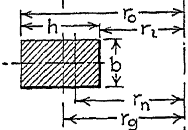
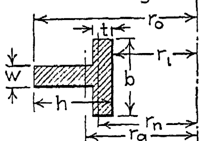
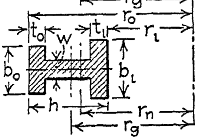
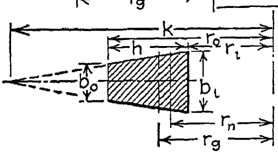


FIG. 329.

362. Curved Beams. Some form of curved beam is found in the frames of machines such as punches, presses, planers, and boring mills, as well as in crane hooks and similar devices. The neutral axis of a straight beam coincides with the gravity axis, and the stress in any fiber is proportional to the distance

TABLE 104.—VALUES OF r_n FOR CURVED BEAMS

Shape of section	Radius of neutral surface $= r_n$
	$r_n = \frac{(\sqrt{r_o} + \sqrt{r_i})^2}{4}$
	$r_n = \frac{h}{\log_e \frac{r_o}{r_i}}$
	$r_n = \frac{(b - w)t + wh}{(b - w) \log_e \frac{r_i + t}{r_i} + w \log_e \frac{r_o}{r_i}}$
	$r_n = \frac{(b_i - w)t_i + (b_o - w)t_o + wh}{b_i \log_e \frac{r_i + t_i}{r_i} + w \log_e \frac{r_o - t_o}{r_i + t_i} + b_o \log_e \frac{r_o}{r_o + t_o}}$
	$r_n = \frac{\left[\frac{r_o + r_i}{2} - r_o - \left(\frac{b_o}{b_i} + 1\right)h\right]h}{h - \left[r_o + \left(\frac{b_o}{b_i} + 1\right)h\right] \log_e \left(\frac{r_o}{r_i}\right)}$

of that fiber from the gravity axis; but if the beam is curved, these statements are not true. Assume (as in the theory of straight beams) that planes normal to the gravity axis before bending will remain planes normal to the gravity axis after bending. In Fig. 329, the lines ab and gf represent these planes when the beam is not stressed. When a bending moment M is applied to the beam the plane gf rotates relative to ab, through the angle $\Delta d\theta$ to the position mn, compressing the fibers on the convex side and elongating those on the concave side.

The shortening of the fiber at a distance y from the neutral axis is $y\Delta d\theta$, and the unit stress in this fiber is

$$s = E\delta = \frac{Ey\,\Delta d\theta}{(r_n + y)d\theta} \tag{448}$$

The total load on a strip of thickness dy and having a cross-sectional area dA, is

$$dF = s\,dA = \frac{Ey\,dA\,\Delta d\theta}{(r_n + y)d\theta} \tag{449}$$

From the conditions of equilibrium, the summation of forces over the entire cross section must be zero, and the summation of the moments of these forces about the neutral axis must equal the applied bending moment. Hence

$$\int dF = \frac{E\,\Delta d\theta}{d\theta}\int \frac{y\,dA}{r_n + y} = 0 \tag{450}$$

from which

$$\int \frac{y\,dA}{r_n + y} = 0 \tag{451}$$

If the shape of the cross section is known, dA can be evaluated in terms of y and dy, and the radius of curvature of the neutral axis determined from this equation.

Taking moments about the neutral axis,

$$\begin{aligned}\int y\,dF &= \frac{E\,\Delta d\theta}{d\theta}\int \frac{y^2\,dA}{r_n + y} = M \\ &= \frac{E\,\Delta d\theta}{d\theta}\int \left(y - \frac{yr_n}{r_n + y}\right) dA \\ &= \frac{E\,\Delta d\theta}{d\theta}\int y\,dA \\ &= \frac{E\,\Delta d\theta}{d\theta}(-Ae) = M \end{aligned} \tag{452}$$

where e = the distance from the gravity axis to the neutral axis.

Combining Eqs. (448) and (452),

$$s = -\frac{M}{Ae}\left(\frac{y}{r_n + y}\right) \tag{453}$$

which is the general equation for the stress at any fiber at a distance y from the neutral axis. At the outer fibers, y is equal to c_o and $-c_i$, respectively, and the maximum bending stresses are

$$s_o = \frac{-Mc_o}{Ae(r_n + c_o)} \tag{454}$$

and

$$s_i = \frac{Mc_i}{Ae(r_n - c_i)} \tag{455}$$

The value of r_n depends upon the shape of the beam and is obtained from Eq. (451). Also e equals $r_g - r_n$, c_o equals $r_o - r_n$, and c_i equals $r_n - r_i$.

For convenience, values of r_n for common beam sections are given in Table 104. Direct solution of the required dimensions of curved beams is very difficult, and the best procedure is to assume the dimensions and then compute the stresses. A few trial solutions will indicate the proper dimensions.

363. Deflection of Curved Beams. When the radius of curvature of the gravity axis is large, the deflection equations for straight beams may be applied without serious error. Space is not available for an extended discussion of the deflection theory, and the reader is referred to texts on the theory of elasticity and to texts on advanced applied mechanics.

PROBLEMS FOR ASSIGNED WORK

Many of the problems in this group require the use of mechanical engineering handbooks and other sources of information with which the young engineer must become familiar. A variety of problems have been assembled to bring out specific points and methods. It is impossible, in the space permitted for this portion of the book, to provide problems covering every phase of design, and problems from other sources may be used to advantage.

Chapters I and II

1. Determine the stress produced by a compressive load of 15,000 lb in a cylinder, whose length is 8 in. and outside and inside diameters are 4 and 3 in., respectively.

2. A bar 18 in. long having a cross section $\frac{3}{4}$ by 2 in. is subjected to a tensile load of 45,000 lb. The modulus of elasticity is 30,000,000 psi. Determine the unit stress in tension and the total elongation of the bar.

3. A piece of cast iron 12 in. long and 3 in. on one side is to sustain a compressive load of 15 tons. The stress is limited to 3,000 psi and the total deformation to 0.020 in. The modulus of elasticity may be taken as 10,000,000 psi. Determine the dimension of the remaining side of this piece.

4. The piston of a steam engine is 18 in. in diameter and its stroke is 26 in. The steam pressure is 250 psi gauge. Determine the required diameter of the piston rod if the permissible stress is 10,000 psi.

5. Steel weighs 480 lb per cu ft and has an ultimate strength of 80,000 psi. What is the maximum length of steel rod that could be hung vertically from its upper end without rupturing?

6. A steel bolt $1\frac{1}{2}$ in. in diameter has a hexagonal head $1\frac{3}{16}$ in. thick and $2\frac{3}{8}$ in. across the flats. The diameter at the root of the threads is 1.234 in. Determine the tensile stress in the body of the bolt and at the root of the threads, the shear stress in the head, and the compression in the head when an axial load of 10,000 lb is applied.

7. A copper tube of $1\frac{1}{2}$ sq in. cross-sectional area is slipped over a bolt of 1 sq in. area. The length of the tube is 20 in., and the pitch of the threads on the bolt is $\frac{1}{8}$ in. The modulus of elasticity of the steel is 30,000,000 psi and of the copper 15,000,000 psi. If the tube is compressed between the head and nut of the bolt, what will be the unit stress produced in both parts by one-fourth turn of the nut?

8. Three beams of equal size and shape are made from cast iron, soft steel, and nickel steel, respectively. Which of these beams will be the stiffest? State your line of reasoning in determining your answer.

9. A 3- by 4-in. beam 6 ft long supports a concentrated load of 2,000 lb at its center.

a. Draw to scale the shear diagram.

b. Draw to scale the bending-moment diagram.

c. Determine the maximum bending stress if the 4-in. side is parallel to the load-application line.

d. Determine the maximum bending stress if the 3-in. side is parallel to the load-application line.

10. A beam is to be rectangular in section with depth twice the width The distance between supports is 6 ft, and there are two loads: 6,000 lb 18 in. from the left support, and 5,000 lb 40 in. from the left support.

a. Draw shear and bending-moment diagrams.

b. Using steel with E equal to 30,000,000 psi and an ultimate strength of 75,000 psi, determine the beam dimensions if the maximum stress is to be 15,000 psi.

c. Using cast iron with E equal to 10,000,000 psi and an ultimate strength of 30,000 psi, determine the beam dimensions if the maximum stress is 5,000 psi.

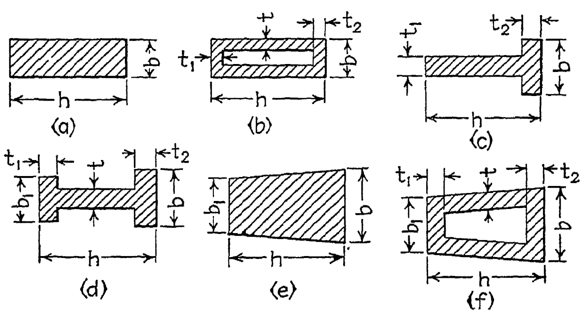


FIG. P-1.

11. A beam of I section 3 in. wide and 6 in. deep with all sections 1 in thick is supported at two points 8 ft apart. There is a concentrated load of 500 lb 6 ft from the left support. A uniformly distributed load of 100 lb per ft is carried on the portion of the beam to the left of the 500-lb load.

a. Draw to scale the shear diagram.

b. Draw to scale the bending-moment diagram.

c. What is the distance from the left support to the section of maximum bending moment?

d. Determine the maximum bending stress.

e. Determine the maximum shear stress.

12. A beam of I section is 2 in. wide and 4 in. deep with all sections $\frac{1}{2}$ in. thick. It is supported at points 5 ft apart and carries a concentrated load of 400 lb at a distance of 2 ft from the left support.

a. Determine the horizontal shear in the vertical section just to the left of the load and at distances of 0, 1, and 2 in. from the neutral axis.

b Determine the horizontal shear in the vertical section just to the right of the left support and at distances 0, 1, and $1\frac{1}{2}$ in. from the neutral axis.

13. A machine-steel shaft 10 ft long is to be held rigidly at one end and is to withstand a torsion moment of 200 lb acting at a distance of 2 ft from the axis of the shaft. The torsion stress is limited to 6,000 psi and the total twist to $1\frac{1}{2}$ deg. The modulus of rigidity may be taken as 12,000,000 psi. Determine the required shaft diameter.

14. A steel shaft $1\frac{1}{2}$ in. in diameter and 2 ft long is held rigidly at one end and has a handwheel 18 in. in diameter keyed to the other end. The modulus of rigidity of steel is 12,000,000 psi.

a. What load applied tangent to the rim of the wheel will produce a torsional shear stress of 10,000 psi.

b. How many degrees will the wheel turn when this load is applied?

c. Will there be any bending stress in this shaft? If so, what will be its magnitude?

15. A hollow bronze tube has an outer diameter of 2 in. and an inner diameter of $1\frac{1}{2}$ in. and is 12 in. long. A crank 15 in. long is keyed to one end, and the other end is held rigidly. The modulus of rigidity is 10,000,000 psi.

a. What force must be applied to the end of the crank to produce a torsional shear of 5,000 psi.

b. What will be the angular movement of the end of the crank when this load is applied?

16. A car weighing 3,600 lb is equipped with a full-floating axle, which prevents bending stress on the axle. The maximum load on a rear wheel is 1,250 lb. The tires are 29 in in diameter and the coefficient of friction between the tires and the road is 0.65. The engine is capable of developing a torque of 250 lb-ft. The total gear reduction is 4.56 in high, 13 2 in low, and 16.7 in reverse. The minimum diameter of the axle is $1\frac{1}{16}$ in.

a. Determine the torsional stress in the axle when the car is running in high, low, and reverse gears if the tires do not slip.

b. Determine the torsional stress in the shaft when the tires slip.

c. Since there are high local stresses at the sharp corners at the bottom of the splines, the stress at the root of the splines should not exceed 50 per cent of the stress in the straight portion of the axle. Determine the minimum root diameter of the splines.

17. If the unit shear stress is limited to 6,000 psi and the angular twist to 0.1 deg, what is the greatest torsional moment that can be transmitted by a $1\frac{1}{2}$-in. square steel bar 2 ft in length?

18. A pair of spur gears having diameters of 36 in. and 12 in., respectively, revolve at 50 and 150 times per minute, respectively, and transmit 20 hp. Find the diameters of the shafts for a maximum torsional stress of 6,000 psi.

19. If a material has a modulus of elasticity in tension equal to 15,000,000 psi and a Poisson's ratio of 0.26, what is its modulus of elasticity (modulus of rigidity) in shear?

20. A carbon-steel specimen 0.505 in. in diameter broke when the applied load was 13,000 lb and passed the yield point when the load was 6,000 lb.

a Determine the yield stress and the ultimate strength.

b. What was the approximate carbon content of this steel, assuming that it had not been heat-treated?

21. A carbon-steel tensile specimen $\frac{7}{8}$ in. in diameter was tested to destruction. The yield point was found at 33,000 lb and the specimen broke at 60,000 lb When the load was 20,000 lb, the elongation was 0.0087 in. in 8 in. When the load was 36,000 lb the elongation was 0.022 in. in 8 in.

a. Determine the yield stress and the ultimate strength.

b. Determine the modulus of elasticity.

c. What was the approximate carbon content of this steel, assuming that the steel had not been heat-treated?

22. A bar of oil-quenched and drawn S.A.E. 1045 steel had a Brinell hardness number of 210

a. What are the probable ultimate strength and yield stress in tension?

b. What is the probable percentage elongation?

c. What are the probable ultimate strength and yield stress in shear?

23. A punch press has a capacity of 100 tons. How many $\frac{3}{4}$-in holes can be punched at one time in $\frac{1}{2}$-in. plates of:

a. S.A.E. 1025 steel, annealed?

b. S.A.E. 2345 steel, annealed?

c. Monel metal?

24. A bar of oil-quenched and drawn S.A.E. 2345 steel has a Brinell hardness number of 300.

a. What are the probable ultimate strength and yield stress in tension?

b. What is the probable percentage elongation?

c. What are the probable ultimate strength and yield stress in shear?

25. A machine member made of S.A.E. 2345 steel is to have an ultimate strength of 180,000 psi and an elongation of 17 per cent in 2 in. Specify the heat treatment that you would give this steel. What would be the probable Brinell hardness?

26. A beam of S.A.E. 1020 steel, 10 ft long, has an I section 4 in. deep and 2 in. wide, with the web and flanges $\frac{1}{4}$ in. thick. This beam is to be replaced by one made of 17SRT aluminum alloy. Assume that in each case the stress is to be one-half the yield stress.

a. Determine the dimensions of the aluminum beam to support the same load.

b. Determine the dimensions if the aluminum beam is to have the same stiffness as the steel beam.

c. Determine the relative weights of these three beams.

Chapter III

27. A bar of steel 2 in. in diameter and 3 ft long is subjected to a tensile load of 40,000 lb. What are the dimensions of the bar after the load is applied?

28. A block of cast iron 2 by 3 by 4 in. in size has a compressive load of 10,000 lb applied to the 3- by 4-in. face and a compressive load of 15,000 lb applied to the 2- by 3-in. face.

a. What is the maximum equivalent stress according to the maximum-strain theory?

b. What will be the maximum equivalent stress if the 10,000-lb load is made tension instead of compression?

29. A block of pure cast copper 1 by 2 by 3 in. in size is subjected to a compressive load of 1,000 lb on the 1- by 3-in. face. The maximum allowable direct tensile stress is 2,000 psi.

a Using the maximum-shear theory, what is the maximum tension load that can be applied to the 1- by 2-in. face?

b. Using the maximum-strain theory, what is the maximum tension load that can be applied to the 1- by 2-in. face?

30. A bar of steel 40 in. long and 2 in. in diameter is subjected to a tensile load of 5 tons. Determine the maximum tension and the maximum shear stresses.

31. A machine member of bronze is subjected to external forces producing a direct shearing stress of 8,000 psi, and a direct tensile stress of 12,000 psi. Determine the maximum principal stress (tension) and the maximum shear stress induced in this machine member.

32. The shaft of a vertical turbine-generator in a hydroelectric plant has an outside diameter of 12 in., and an inside diameter of 10 in. The shaft transmits 5,000 hp at 750 rpm. The total weight of the shaft and turbine runner supported by the shaft is 60 tons. Determine the maximum tensile and shearing stresses in the shaft if the shaft is supported from bearings at the upper end.

33. The connecting rod of an engine is a column with the ends free to turn on the crankpin and the crosshead pin. In the direction of the axis of these pins, however, the column may be considered to approach fixed-end condition, the end factor being 3. Determine the ratio of the depth to the thickness of a rectangular connecting rod.

34. Set up a straight-line equation for the rupturing load on round-end columns of structural steel, assuming the rupturing stress to be equal to the yield stress in compression, 35,000 psi.

35. A column of machine steel has a cross section of $2\frac{1}{2}$ by 6 in. and is loaded through ball joints at both ends. Determine the load that will cause a stress of 15,000 psi, assuming the yield stress to be 33,000 psi and the length to be 42 in.

36. An 8-in. square yellow-pine column is fixed rigidly at both ends. If the yield stress is assumed to be 2,000 psi, what load may be applied if the stress is not to exceed 1,000 psi? Assume the modulus of elasticity to be 1,000,000 psi and the length to be 10 ft.

37. A hollow cast-iron column is 6 in. in diameter outside, and 1 in. thick. It is fixed at one end and loaded through a guided pin joint at the other end. Assume the yield stress to be 8,000 psi and the modulus of elasticity to be 10,500,000 psi. Determine the maximum stress when a load of 10 tons is applied. The length is 15 ft.

38. The piston rod of a steam engine may be considered to be a column with one fixed and one guided end. Determine the size of rod necessary for an 18-in. cylinder using steam at 350 psi pressure. The rod is 40 in. in length and is made of steel having an ultimate strength of 70,000 psi and an elastic limit of 38,000 psi The permissible working stress is 8,000 psi.

39. A circular cast-iron support is to withstand 5,200 lb in compression. The member is 10 ft long, and it must not be shortened more than 0 004 in. in 10 in. and must have a factor of safety of 8. Assume the ultimate strength of the material to be 96,000 psi in compression and 25,000 psi in tension and the modulus of elasticity to be 15,000,000 psi. Determine the diameter of the support if it is hollow with the inside diameter equal to 0.8 of the outside diameter

40. The piston rod of a steam engine is made of nickel steel of 100,000 psi ultimate strength and 70,000 psi elastic limit. This rod is 60 in long and has fixed ends The maximum load carried by the piston rod is 50,000 lb. Assuming a permissible stress of 16,000 psi, determine the rod diameter.

41. The pole supporting several power lines has an outside diameter of 5 in, an inside diameter of $4\frac{1}{2}$ in, and a height of 40 ft. The total load supported by the pole is 1,800 lb. The wind load on the pole is 8 lb per ft length of pole.

a Determine the stress in the pole due to column action.
b. Determine the stress in the pole due to wind.
c What is the maximum stress in the pole?

42. A punch press has a frame similar to Fig P-2 with a cross section at *AA* like Fig. P-1*b*. The dimensions are a equals 4 in., b equals 6 in, and t_1 equals t_2 equals 1 in. Determine the dimension h so that the maximum tensile or compressive stress will not exceed 6,000 psi $F = 10$ tons; $t = 1$ in.

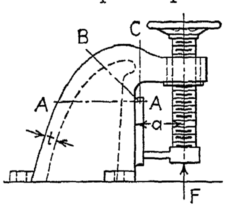


FIG. P-2.

43. A short prism has a rectangular cross section 2 in. by 4 in. A compressive load of 2,400 lb is applied 1 in from the short side and $\frac{1}{2}$ in. from the long side. Determine the stress at each corner.

44. A $\frac{3}{4}$-in. rivet is driven hot, and the average temperature of the metal is 900 F. The length after driving is $1\frac{1}{4}$ in. What will be the tensile stress in the rivet after cooling to 70 F?

45. A rotary kiln made of $\frac{5}{8}$-in. steel plate is 4 ft in diameter and 40 ft long. When operating, the average temperature of the steel is 500 F when the room temperature is 80 F. How much allowance must be made for the expansion and contraction of this kiln?

46. The bus bar in a power-generating station consists of a $\frac{1}{4}$- by 2-in. copper bar rigidly anchored at 3-ft intervals. Power is transmitted over this bus bar at 2,300 volts, and the resistance causes the temperature to rise to 160 F when the room temperature is 80 F. The ultimate strength of the copper is 36,000 psi, and the coefficient of thermal expansion is 0.0000109 per °F. What change in stress is caused by the operation of the plant?

47. A steel bar 1 in. in diameter is inserted inside a copper tube 2 in. in outside diameter and rigidly attached to it. If the completed bar is 12 in. long, what will be the unit stress in each material when the temperature has been raised 200 F?

48. A rod of rolled brass 2 in. in diameter has a soft steel core 1 in. in diameter. The rod is 10 in. long at 70 F and is unstressed.

a. If the ends are held rigidly, what will be the stress in each metal when the temperature is raised 200 F?

b. If the bar acts as a solid rod and the rod is free to elongate, what will be the stresses when the temperature rises 200 F?

49. A 6-in. steam pipe has an outside diameter of 6.625 in., an inside diameter of 6.065 in, and a total length of 200 ft. This pipe was installed when the temperature was 80 F.

a. What will be the pipe length when filled with steam at 300 F if the end is free to move?

b. What will be the stress in the pipe if the ends are rigidly anchored? Neglect column action.

50. A steel pole 6 in. in diameter and 10 ft high supports a transformer weighing 2,000 lb, whose center of gravity is 18 in. from the center of the pole If the pole is made of steel $\frac{3}{8}$ in. in thickness, determine the maximum stress in the pole.

51. A straight tension rod as designed for a certain machine was found to interfere with another member of the machine. Clearance was provided by bending the rod so that the inner edge of the offset part was $\frac{3}{4}$ in. from the center line of the applied loads. The applied tensile load was 6,000 lb, and the rod was made of 2-in. round steel. Determine the stress in the original straight rod and the stresses after offsetting.

52. A C-clamp frame, Fig. P-3, has a rectangular cross section $\frac{3}{8}$ by 2 in. The center line of the screws is 3 in. from the neutral axis of the frame. What load must be applied by the screws to produce a tensile stress of 12,000 psi? What will be the compressive stress at this load?

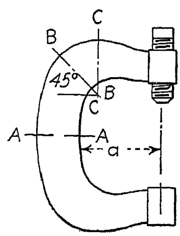


FIG. P-3.

53. A small punch has a capacity sufficient to punch a $\frac{5}{8}$-in. hole in a $\frac{1}{4}$-in. steel plate whose ultimate strength in tension is 90,000 psi. The punch frame is shaped like a C-clamp with a T cross section (Fig. P-1*c*). The dimensions are t_2 equals $1\frac{1}{4}$ in., t_1 equals 1 in., and b equals 6 in The center line of the punch is $6\frac{1}{2}$ in. from the inner surface of the flange. Determine the depth of leg h if the stress is not to exceed 14,000 psi.

54. The frame shown in Fig P-2 is part of a straightening press capable of exerting a pressure of 8 tons. Determine the maximum tension and compression stresses in section *A-A* and *A-B*. The cross sections are similar to Fig. P-1*b* and b equals 10 in., h equals 15 in. for section *A-A* and 12 in for section *A-B*, t equals t_1 equals 1 in., and t_2 equals $1\frac{1}{2}$ in. The distance a is 8 in.

55. The piston of a steam engine is 30 in. in diameter and weighs 2,000 lb. The piston is supported at the center of the piston rod. The piston rod is

6 in. in diameter and is supported at one end by the crosshead and at the other end by the tail-rod slide. The total steam pressure on one side of the piston is 125,000 lb.

a. Considering the piston rod to be a simple beam supported at points 5 ft on each side of the piston, determine the stress due to bending only.

b. Determine the direct stress due to the axial thrust on the piston.

c. Determine the maximum tensile stress due to bending and axial thrust.

56. A machine member made of brass is subjected to a tensile stress of 3,000 psi and to a cross shear of 2,000 psi. Determine the maximum tensile stress and the maximum shear stress.

57. A flange coupling on a line shaft is held together by four $\frac{3}{4}$-in. bolts, arranged on a bolt circle 6 in. in diameter. Each bolt transmits 5,000 lb in shear, and the tensile load on the bolt due to tightening is 12,000 lb. Determine the maximum shear stress and maximum tensile stress?

58. The shaft of a 50-hp 850-rpm direct-current motor is 31 in. from center to center of the bearings and is $3\frac{1}{2}$ in. in diameter at the bearings. If the magnetic pull on the armature is 2,000 lb and is concentrated midway between the centers of the bearings, determine the maximum shear and the maximum tensile stress in the shaft.

59. An aluminum plate $\frac{1}{2}$ by 4 by 6 in. in size has a tensile load of 3,000 lb applied parallel to the 6-in side and a tensile load of 2,000 lb applied parallel to the 4-in. side.

a. What is the maximum normal stress?

b. What is the maximum shear stress?

c. If an additional load of 10,000 lb in compression is applied to the flat faces, what will be the maximum shearing stress?

60. Rubber of 50 durometer hardness is used in a cylinder $2\frac{1}{2}$ in. diameter and $1\frac{1}{2}$ in. long to support a load of 500 lb.

a. What will be the deflection?

b. If 65 durometer rubber is used, what will be the deflection?

61. A rubber slab 3 by 3 in. is $\frac{3}{4}$ in. thick and has a durometer hardness of 55.

a. What will be the deformation under a load of 1,800 lb?

b. What load is required to produce a deformation of $\frac{1}{8}$ in.?

62. A rubber slab 2- by $4\frac{1}{2}$-in. is $\frac{7}{8}$ in. thick and has a durometer hardness of 55.

a. What deformation will be produced by a load of 1,800 lb?

b. What will be the deformation if the temperature is raised to 140 F?

63. Compare the deflection in a 3- by 4-in. rubber slab with that of a $1\frac{1}{2}$- by 8-in. slab when loaded to 600 psi, if both are $\frac{7}{8}$ in. thick and have a 40 durometer hardness.

64. A hollow cylinder of 45 durometer rubber is vulcanized to inner and outer steel sleeves. The outer diameter is 4 in. and the inner diameter 2 in. The length of the bond area is 5 in. on each sleeve. The maximum shear stress in the rubber is to be 30 psi.

a. What axial load can be supported?

b. What will be the axial deflection?

65. Solve Prob. 64 if the bond length on the outer diameter is decreased to 4 in.

66. Two slabs of 55 durometer rubber are each vulcanized to a central steel plate and to an outer steel plate forming a double sandwich. Each slab is 2 in. wide, and 3 in. long and $\frac{3}{4}$ in. thick. What deflection will be produced by a load of 400 lb applied to the center plate parallel to the 3 in. dimension?

67. A 50 durometer rubber cylinder is vulcanized to outer and inner steel sleeves. The inner sleeve is held stationary and a torque is applied to the outer sleeve. The outside and inside diameters of the cylinder are 5 and 3 in., respectively, and the length is 6 in. at the outside and 8 in. at the inside diameters.

a. What torque will produce an angular deformation of 10 deg?

b. If the torque is applied at the end of a single arm of radius 6 in., what will be the radial deflection of the outer cylinder?

Chapter IV

68. The piston of an automobile engine is held to the connecting rod by a wrist pin having an external diameter of $\frac{13}{16}$ in. and an internal diameter of $\frac{7}{16}$ in. The length of the bearing in the rod end is 1 in. and the bearing in each boss of the piston is $\frac{5}{8}$ in. The maximum load transmitted is 3,200 lb.

a. Determine the bearing pressure between the rod end and the wrist pin.

b. Compute the maximum bending stress in the pin, assuming that it is a simple beam with a uniform load.

c. Compute the maximum deflection of the pin.

d. If the pin is made of S.A.E. 3245 steel, what is the apparent factor of safety?

69. A Diesel engine developing 1,850 hp at 105 rpm is used on a ship to drive the propeller. The shaft of the propeller is to be made of $3\frac{1}{2}$ per cent nickel steel, heat-treated, with an ultimate strength in tension of 130,000 psi, and an ultimate in shear about 70 per cent of the ultimate in tension. Angular twist is limited to 1 deg in 20 diameters.

a. Using an apparent factor of safety of 6, find the diameter of solid shaft required.

b. Using the same data, find the diameter of hollow shaft required if the outside diameter is twice the inside diameter.

c. What is the percentage of saving in weight by using the hollow shaft?

70. An automobile has a rear axle $1\frac{1}{8}$ in. in diameter at the smallest section where bending is negligible. The engine develops 75 hp at 3,000 rpm. The gear reduction between the engine and the rear axle is 12 4 in low gear and 4.3 in high gear. The transmission efficiency is 88 per cent.

This axle is made of heat-treated nickel steel (S A.E. 2340) having an ultimate strength of 150,000 psi and a yield stress of 125,000 psi in tension.

a. Determine the working stress when in low gear and in high gear assuming that the engine is developing its full power.

b. What is the apparent factor of safety?

71. A straight link in a machine is subjected to a tension load of 8,000 lb. The cross section is circular.

a. Determine the diameter required if the load is steady and the material is cast iron; if the material is S.A.E 1025 steel, annealed; if the material is Tobin bronze.

b. Determine the diameter required if the load is repeated and completely reversed and the material is cast iron; if the material is S.A.E. 1025 steel, annealed.

72. A stationary machine member made of carbon steel having a yield stress of 36,000 psi is subjected to a tensile stress of 10,000 psi. If the factor of utilization is to be 0.60, what shear stress can be applied?

73. The load on the piston rod of a steam engine varies from zero to a maximum during each stroke. The engine runs at 300 rpm, 8 hr per day, 300 days per yr for 10 yr, is double acting, and has an 8- by 12-in. cylinder. Steam is admitted to the cylinder at 200 psi. The piston rod is to be made of S.A E 1045, annealed

a What apparent factor of safety should be used?

b. What would be the allowable stress used in designing the rod?

74. A 12- by 14-in. by 350-rpm single-acting oil engine has a compression pressure of 325 psi and a maximum explosion pressure of 600 psi. The connecting rod for this engine is to be made of S.A E. 2145 annealed steel. What apparent factor of safety should be used in designing the body of this rod and what should be the design stress?

75. The load on the piston rod of a pump varies from 10,000 lb compressive to 8,000 lb tensile. Determine the proper design stress to be used in this design.

76. A certain material has a yield stress of 40,000 psi in tension, and an endurance limit of 36,000 psi in reversed stress. The applied load varies from 5,000 lb in tension to 30,000 lb in tension. If the utilization factor is to be 0.40, determine the allowable design stress.

77. The steel used in a machine member subjected to impact has an ultimate strength of 110,000 psi, a yield stress of 80,000 psi, and an endurance limit of 60,000 psi in reversed stress. It is estimated that the impact will produce stresses three times as large as the equivalent stress under static loading. Determine the allowable design stress.

78. A machine member made of carbon steel is subjected to a loading that produces a tensile stress varying from 2,000 to 10,000 psi and a shear stress varying from 5,000 to 12,000 psi. The ultimate strength of the material is 80,000 psi, the yield stress 45,000 psi, and the endurance limit in reversed stress 40,000 psi. Determine the design stress and the factor of utilization.

79. A spring subjected to torsional stress is made of hard-drawn brass wire. The ultimate strength in tension is 75,000 psi and the yield stress 26,000 psi. The modulus of elasticity is 14,000,000 psi.

a What is the probable endurance limit in reversed torsion?

b The load on the spring varies rapidly from 75 to 100 lb. What is the permissible design stress if the load is applied with minor shock?

80. A spring subjected to bending stresses is made of hard-drawn brass. The ultimate strength in tension is 95,000 psi and the yield stress is 32,000 psi. The modulus of elasticity is 14,000,000 psi.

a. What is the probable endurance limit in reversed bending?

b. The load carried by this spring varies rapidly from 300 to 700 lb. Determine the permissible design stress, considering the shock to be negligible.

c. Determine the design stress if the load is applied with moderate shock.

81. The cast iron to be used in a certain machine member has an endurance limit of 10,000 psi in reversed stress, and 58,000 psi in compression alone.

a. If this member is subjected to a variable load producing a maximum compressive stress of 12,000 psi, what are the permissible endurance range and the corresponding tensile stress if the utilization factor is to be 0 25?

b. What are the permissible endurance range and the corresponding compressive stress if the maximum load produces a maximum tensile stress of 2,000 psi and the utilization factor is 0 25?

82. The metal on the outside of a hydraulic-pump plunger is subjected to a radial compressive stress of 1,000 psi, a tangential tensile stress of 1,500 psi, and an axial compressive stress of 800 psi. What is the equivalent working stress, and what is the utilization factor if the yield stress is 16,000 psi?

83. A tension member subjected to a load of 30,000 lb has a cross section $\frac{1}{2}$ by 6 in. and is pierced by a central hole $\frac{1}{2}$ in. in diameter.

a. What is the average stress at the section through the hole?

b. What is the probable maximum stress at the hole?

84. A steel shaft subjected to torsion transmits 25 hp at 300 rpm. At one end this shaft is increased in diameter by $\frac{1}{2}$ in. and the two sections are joined by a fillet of $\frac{1}{4}$-in. radius.

a. Determine the shaft diameter if the direct torsion stress is not to exceed 6,000 psi.

b. What will be the probable stress at the fillet?

85. A rectangular beam of cast iron has a cross section $1\frac{1}{4}$ by 3 in. The supports are 15 in. apart, and there is a central load of 1,000 lb. At a distance of 4 in. from one support there is a semicircular groove in the lower surface. The groove radius is $\frac{1}{16}$ in.

a. What is the bending stress at the load?

b. What is the probable stress at the groove?

86. In the photoelastic study of the beam in Fig 37, the fringe constant of the material is 87.7 psi per order of interference. Find the principal stress in the beam at the ninth fringe and compare this value with the theoretical stress if this point is 1.9 in. from the support.

Chapter V

87. Two steel plates $\frac{1}{2}$ by 6 in. are connected by a double-strap triple-riveted butt joint with $\frac{5}{8}$-in. rivets. The inner row of rivets contains 3 rivets with a pitch of 2 in. The second row contains 2 rivets with a pitch of 2 in. The third row contains 1 rivet. The back pitch is $1\frac{1}{2}$ in. and the distance from the outer and inner rivets to the plate edges is 1 in. The cover plates are $\frac{5}{16}$ in. thick and the corners are cut away at 45 deg, starting at the middle or second row of rivets. When the total load transmitted by the plates is 40,000 lb, determine the following unit stresses:

a. Compression in main plate.
b. Compression in cover plates.
c. Tension in main plate at outer row.
d. Tension in main plate at middle row.
e. Tension in main plate at inner row.
f. Tension in cover plates at outer row.
g. Tension in cover plates at middle row.
h. Tension in cover plates at inner row.
i. Shear in rivets.
j. Shear in main plate between inner row of rivets and plate edge.
k. Shear in cover plate between outer rivet and plate edge.

88. A double-riveted lap joint is made of steel having an ultimate strength of 55,000 psi in tension and 95,000 psi in compression. The rivets have a strength of 44,000 psi in shear. The plates are $\frac{3}{4}$ in. thick. Rivets 1 in. in diameter are used in $1\frac{1}{16}$-in. holes spaced $3\frac{1}{4}$ in. on centers. Determine the manner in which this joint will fail and at what load.

89. If a double-strap triple-riveted joint with equal width straps has a higher efficiency than a similar joint with unequal-width straps, why is the latter joint generally used in boiler construction?

90. A triple-riveted butt joint with unequal width straps is made of boiler steel having an ultimate strength of 55,000 psi in tension and 95,000 psi in compression. The rivets have a strength of 44,000 psi in shear. The plates are $\frac{7}{8}$ in. thick and the straps $\frac{5}{8}$ in. thick. Rivets $1\frac{1}{8}$ in. in diameter are used in $1\frac{3}{16}$-in. holes spaced $3\frac{3}{4}$ in. on centers at the inner row. Determine the load at which this joint will fail and the manner of failure.

91. Determine the efficiency of a quadruple-riveted, unequal-strap butt joint for $\frac{7}{8}$-in. boiler plate. The cover straps are $\frac{5}{8}$ in. thick. Rivets $1\frac{1}{4}$ in. in diameter are used in $1\frac{5}{16}$-in. holes, with a 4-in. pitch on the inner rows. Sketch this joint and determine the back pitch between all rows.

92. A pressure vessel subjected to an internal pressure of 200 psi is made of $\frac{7}{8}$-in. boiler steel with triple-riveted, unequal-strap butt joints on the longitudinal seam. The straps are $\frac{5}{8}$ in. thick. The rivets of the two inner rows have 4-in. pitch, and the outer-row rivets have 8-in. pitch. Rivet holes are $1\frac{5}{16}$ in. in diameter.

a. Determine the efficiency of this joint.

b. Determine the largest-diameter vessel that could be used with this joint.

93. Two $\frac{3}{4}$-in. plates are joined by a triple-riveted unequal-strap butt joint. The straps are $\frac{1}{2}$ in. thick, the rivets are $1\frac{1}{4}$ in. in diameter, and the pitch is $8\frac{1}{2}$ in. The plates form a pressure vessel 60 in. in diameter.

a. What is the probable method of failure?

b. Determine the efficiency of the joint.

c. What is the permissible internal pressure?

94. The drum of a water-tube boiler is 1 in. thick and 48 in. in diameter. It has a triple-riveted joint with unequal butt straps, $1\frac{7}{16}$-in. rivet holes, and an outer pitch of $8\frac{1}{2}$ in. The openings for the water tubes are 4 in. in diameter and 6 in. on centers. The section at the tube holes is reinforced with two plates each $\frac{5}{8}$ in. thick. What is the maximum pressure for which this boiler can be used?

95. Design a quadruple-riveted unequal-strap butt joint for the longitudinal seam of a water-tube boiler drum subjected to 500 psi pressure. This drum is to be 48 in. in diameter. Make a dimensioned sketch of this joint.

96. Design the longitudinal joint for a 50-in. steam boiler that is to carry a steam pressure of 750 psi. Sketch this joint giving all dimensions.

97. A 50-in. drum is filled with steam at 450 psi at 800 F. The steel plates have an ultimate strength at room temperature of 65,000 psi in tension and the rivets have a strength of 50,000 psi in shear. Design a triple-riveted joint for this drum. Sketch the joint and give all dimensions.

98. A 10-ft drum is subjected to an internal pressure of 300 psi at 1000 F. The steel plates have an ultimate strength of 70,000 psi in tension at room temperature and the rivets have a strength of 50,000 psi in shear. Design a quadruple-riveted unequal-strap butt joint for this drum. Sketch the joint and give all dimensions.

99. A steel tank 54 in. in diameter is used for storing air at 200 psi pressure. Design a double-riveted butt joint with unequal cover plates for the longitudinal seam and a lap joint for the girth seam. Sketch the intersection of these two joints showing all details and dimensions.

100. The accumulator for a forging press is 48 in. in diameter and contains oil at 1,500 psi at 125 F. Design the longitudinal and girth joints for this accumulator tank. Sketch the intersection of the joints showing all details and dimensions.

101. A gear is built up of a forged-steel rim and hub and a web of steel plate joining them. The web is riveted to the rim by means of 36 rivets arranged in two concentric circles of 16 and 20 in. diameters. All rivets are $\frac{3}{4}$ in. in diameter. The pitch diameter of the gear is 26 in., and the turning force applied at the pitch line is 8,500 lb. Determine the maximum shear stress assuming that all rivets in each row are equally loaded and in single shear.

102. The operating arm of a brake lever is attached to the brake shoe by six rivets arranged in two parallel rows. The distance between the rows is 3 in., and the pitch in each row is $2\frac{1}{2}$ in. A load of 500 lb is applied at a distance of 24 in. from the center of gravity of the rivets. Determine the rivet diameter required if the shear stress is not to exceed 6,000 psi.

103. The bracket in Fig. P-4 is to carry a load F equal to 10,000 lb. If the rivets are $\frac{5}{8}$ in. in diameter, what is the maximum shearing stress developed in the rivets?

104. The load F in Fig. P-4 is 7,500 lb. If the two rivets on the vertical center line are omitted, what size rivets are required if the shear stress is not to exceed 4,000 psi?

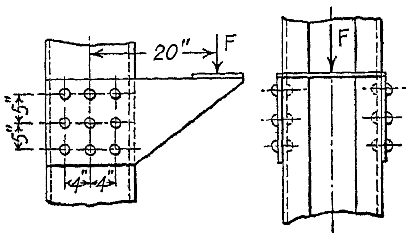


Fig. P-4.

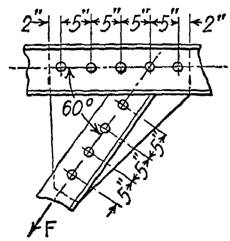


Fig. P-5.

105. In Fig. P-5 the rivets are $\frac{3}{4}$ in in diameter. What load may be applied at F if the permissible shear stress is 10,000 psi?

Chapter VI

106. Two $\frac{1}{2}$-in. plates are placed so they overlap 2 in. and are joined by a single lap weld placed normal to the load. The weld is a $\frac{1}{2}$-in fillet weld 6 in. long. Determine the total static load that this joint will sustain.

107. Two $\frac{5}{8}$-in. plates are placed so they overlap and are welded with two $\frac{5}{8}$-in. fillet welds each 8 in long and placed normal to the load. Determine the total static load that this joint can sustain.

108. What load can be sustained by the joint in Prob. 107 if the welds are placed parallel to the load line?

109. Two $\frac{3}{4}$-in. plates 12 in. wide are joined by a butt weld. Determine the static load that can be sustained if the weld is planed off level with the plates.

110. Determine the load that can be sustained by the joint in Prob. 109 if the load is applied with moderate shock.

111. A steel rod whose ultimate strength is 75,000 psi is to have threaded ends welded on. The load to be carried is 10,000 lb Assume the permissible working stress for the rod to be 15,000 psi.

a. Determine the diameter of rod required if forge welding is used.

b. Determine the diameter of rod required if a metallic-arc butt weld is used.

112. A $\frac{3}{8}$- by 4-in. steel plate is welded to a steel gusset plate by means of parallel fillet welds. Determine the length of welds required if a load of 15,000 lb is to be transmitted.

113. A cast-iron beam with an I section is 8 ft long and 6 in. deep with flanges 4 in. wide. Flanges and web are 1 in. thick.

a. What load placed at the center of this beam will produce a bending stress of 4,000 psi? Assume that E is 15,000,000 psi.

b. Design a welded steel built-up I-beam of the same outside dimension to carry the same load. Assume a permissible stress of 12,000 psi and E equal to 30,000,000 psi. Give size of fillet weld required.

c. Determine the deflection produced in each of the beams.

d. If cast iron costs 6 cts per lb and steel costs 3 cts. per lb, compare the material costs of the two beams.

e. Assuming the cost of the welding to be 20 cts. per ft, compare the cost of the two beams.

114. Two plates $\frac{1}{2}$ in. thick and 6 in. wide are joined by a double lap joint having $\frac{1}{2}$-in. fillet welds. Determine the unit shear and tensile stresses on the faces of the weld and the maximum combined stresses when a static tensile load of 20,000 lb is applied.

115. A 6- by 4- by $\frac{1}{2}$-in. angle is welded to a steel plate by two fillet welds along the edges of the 6-in. leg. The angle is subjected to a tension load of 60,000 lb. Determine the lengths of the welds required if the load is applied with heavy shock.

116. A $3\frac{1}{2}$- by 4- by $\frac{3}{8}$-in. structural-steel angle is used as a tension member to support a moderate shock load of 40,000 lb. The 4-in. leg of this angle is welded to a steel plate by means of two $\frac{3}{8}$-in. parallel welds. Determine the length of each weld.

117. A 6- by 4- by $\frac{1}{2}$-in. angle is welded to its support by two $\frac{1}{2}$-in. fillet welds along the back and edge of the 6-in. leg. A load of 4,500 lb is applied normal to the gravity axis of the angle at a distance of 15 in. from the center of gravity of the welds. Assume each weld to be 3 in. long and determine the maximum shearing stress in the welds.

118. A 4- by 4- by $\frac{3}{8}$-in. angle is used as a strut and is subjected to a load of 40,000 lb. Determine the lengths of welds required along the angle edge and back if the maximum shear stress in the fillet welds is to be 3,000 psi.

119. In Prob. 117 an additional weld is made along the end of the 6-in. leg. Determine the maximum shearing stress in the welds.

120. The bracket shown in Fig. P-4 is welded to the column by $\frac{1}{2}$-in. fillet welds along the top and bottom of the plates. Determine the load that can be applied at F if the maximum shear stress in the welds is to be 4,000 psi.

121. The bracket shown in Fig. P-4 is welded to the column by $\frac{3}{8}$-in. fillet welds along all four edges of each side plate. Determine the load that can be applied at F if the maximum shear stress in the welds is to be 3,000 psi.

122. The riveted joint shown in Fig. P-5 is changed to a welded joint.

a. When $\frac{3}{8}$-in. fillet welds are used along both edges of the channel, what load may be applied at F so that the shear stress in the welds will be 8,000 psi.

b. The tension member is a 5- by 5- by $\frac{3}{8}$-in. angle. What length welds should be used to join the angle and gusset plate under the conditions mentioned?

Chapter VII

123. When a regular hexagonal nut is tightened on its bolt the axial pull produced on the bolt causes a direct tensile stress across the root section of

the bolt and a shear stress across the root of the threads. Using a standard $1\frac{1}{2}$-in finished nut and National Coarse Threads determine if the bolt will rupture by tension before the threads shear off. Assume s_s equals 0.75 s_t.

124. A generator weighing 3,000 lb is furnished with an eyebolt in the housing for lifting purposes. Assume the bolt to be made of S.A.E. 1020 steel with National Coarse Threads.

a. Determine the required bolt diameter.

b. Determine the depth to which the bolt should extend into the cast-iron motor housing.

125. A casting weighing 3 tons is lifted by means of a $1\frac{1}{4}$-in. eyebolt having National Standard Coarse Threads. The bolt extends $1\frac{5}{8}$ in. into the casting.

a. Determine the direct tensile and the direct shear stresses in the threaded portion of the bolt.

b. If the bolt is made of S.A.E. 1025 steel, what is the apparent factor of safety?

126. An electric motor weighs 1,800 lb and is provided with an eyebolt screwed into the cast-iron frame for lifting purposes.

a What size bolt with National Coarse Threads should be used, assuming ordinary bolt steel?

b. How far should the bolt extend into the casting?

127. By experiment it is found that a mechanic in tightening up nuts will put an initial tension on bolts equal to 16,000 lb per in. of diameter.

What will be the tensile stress due to tightening when using $\frac{3}{8}$-, $\frac{1}{2}$-, $\frac{3}{4}$-, 1-, and 2-in. National Coarse Thread bolts. Plot the tightening stress against the bolt diameter.

128. The head of a steam cylinder 24 in. in diameter is subjected to a steam pressure of 200 psi. The head is held in place by 16 National Coarse Thread bolts $1\frac{1}{4}$ in. in diameter. A copper gasket is used to make the joint steam tight. Determine the probable stress in the bolts.

129. A 12-in. steam cylinder has its head bolted on by 12 bolts 1 in. in diameter arranged on a $13\frac{1}{2}$-in. bolt circle. The steam pressure is 125 psi, and the joint is made with a hard gasket and through bolts. Determine the probable unit stress in the bolts. Are these bolts large enough if made of steel having an ultimate strength of 70,000 and a yield stress of 38,000 psi?

130. The cylinder of an airplane engine is $4\frac{3}{4}$ in. in diameter. The maximum gas pressure in the cylinder is 500 psi. Determine the number of $\frac{1}{2}$-in. nickel-steel bolts with National Coarse Threads required to hold this cylinder to the crankcase, assuming the ultimate strength of the nickel steel to be 110,000 psi.

131. The head of a steam cylinder 24 in. in diameter is subjected to a steam pressure of 200 psi. The head is held on by means of 16 National Coarse Thread bolts $1\frac{1}{4}$ in. in diameter arranged on a 27-in. bolt circle. No gasket is used. Determine the probable stress in the bolts.

132. An 18- by 24-in. by 175-rpm steam engine carries a maximum cylinder pressure of 200 psi, gauge. The cylinder head is held in place by 1-in. studs and is fitted with a paper gasket $2\frac{1}{2}$ in. wide. Determine the number and pitch of studs required and the probable stress in the studs.

133. A $12\frac{1}{2}$- by 13-in. by 327-rpm semi-Diesel oil engine has a compression pressure of 325 psi and a maximum explosion pressure of 525 psi. The head is held in place by eight bolts, $1\frac{1}{8}$ in. in diameter, arranged on a $15\frac{1}{2}$-in. bolt circle. A thin asbestos gasket is used.

a. Determine the stress due to the applied load.

b. Determine the stress due to tightening.

c. Compare the bolt size with that determined by Eq. (82).

134. Standard 10-in. pipe flanges for steam pressures up to 125 psi have an outside diameter of 16 in. and are provided with 12 bolts $\frac{7}{8}$ in. in diameter, arranged on a $14\frac{1}{4}$-in bolt circle. A hard asbestos gasket is used.

a. Determine the stress in the bolts due to steam pressure.

b. Determine the stress due to tightening the bolts.

c. What is the factor of safety?

d. What is the factor of safety as a ratio between the steam pressure that would cause failure and the actual steam pressure?

135. A $2\frac{15}{16}$-in. line shaft is supported by a wall bracket so that the shaft center is $7\frac{1}{4}$ in. from the wall. The bracket is supported by three $\frac{3}{4}$-in. National Coarse Thread bolts. Two bolts are $5\frac{1}{2}$ in. below the shaft center, and the third bolt is 17 in. below the shaft center. The lower edge of the bracket is 2 in. below the lower bolt. The shaft puts a vertical load of 4,500 lb. on the bracket.

a. What is the unit shearing stress on the bolts, assuming the shear to come on the body of the bolt?

b. What is the unit tensile stress in each bolt?

c. What is the maximum combined shearing stress on the bolts?

d. What is the maximum combined tensile stress on the bolts?

136. The inside diameter of the stator of a 100-kw 950-rpm motor is 38 in. Starting torque may be assumed to be 200 per cent of the running torque. The maximum belt pull on the motor shaft is 2,000 lb. The shaft center is 24 in. above the base line. There are four foundation bolts spaced 24 in. on centers, axially, and 42 in. on centers normal to the axis. The width of the base is 48 in. Bolt steel has an ultimate strength of 60,000 psi with an elastic limit of 30,000 psi. Determine the load on each bolt and the proper diameter of these bolts.

137. A 2-in. square-thread steel screw is used in a common screw jack. There are $2\frac{1}{4}$ threads per inch. The body of the jack is made of cast iron. The coefficient of friction on the threads is 0.15. The thrust collar at the top has an inside diameter of $1\frac{1}{2}$ in. and an outside diameter of 3 in. The coefficient of friction for the collar is 0.25. The load to be lifted by the jack is 2 tons.

a. Determine the efficiency of the screw alone.

b. Determine the efficiency of the screw and thrust collar.

c. Determine the pull necessary on the end of a 2-ft lever to operate this screw jack.

138. In the machine frame shown in Fig. P-6, the guide post is bolted to the base by means of eight bolts, four on each side.

a. Determine the proper size of bolts using ordinary bolt steel, considering that the load is applied with heavy shock.

b. Determine the proper size of bolts using steel having an ultimate strength in tension of 110,000 psi.

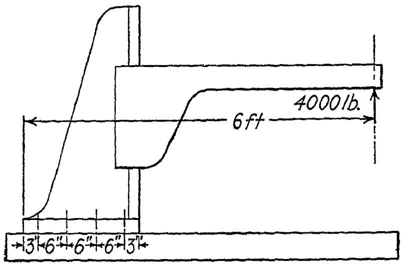


FIG. P-6.

139. The lead screw of a lathe has a 2-in. Acme thread, $2\frac{1}{2}$ threads per in. To drive the tool carriage this screw must exert an axial pressure of 600 lb. The thrust is carried on a collar $4\frac{1}{2}$ in. in outside and $2\frac{1}{4}$ in. in inside diameter.

a. The lead screw revolves 30 rpm. Determine the efficiency of the screw and collar, assuming a coefficient of friction of 0.15 for the threads and 0.10 for the thrust collar.

b. Determine the hp required to drive this screw.

140. In a large gate valve used in a high-pressure water line the gate weighs 1,000 lb, and the friction, due to water pressure, resisting opening is 500 lb. The valve stem is $1\frac{1}{2}$ in. in diameter and fitted with three square threads to the inch. The valve stem is non-rotating and is raised by a rotating wheel with internal threads acting as a rotating nut on the valve stem. This wheel presses against a supporting collar of $1\frac{5}{8}$ in. inside diameter and 3 in. outside diameter. Assume the coefficient of friction for the threads to be 0.15 and for the collar 0.25.

a. Determine the efficiency of the screw and collar.

b. Determine the torque or turning moment that must be applied to the wheel to raise the valve gate.

141. *a.* Determine the force necessary at the end of a 10-in. wrench used on the nut of a $\frac{3}{4}$-in. 10-NC bolt in order to produce a tensile stress of 25,000 psi in the bolt. Assume the coefficients of friction of the threads and the nut to be 0.15.

b. Determine the efficiency of the nut and bolt.

c. Assuming the bolt to be made of S.A.E. 1015 steel, find the allowable stress by Seaton and Routhwaite's equation and explain the difference between this stress and that in part (*a*).

142. A $2\frac{1}{2}$-in. bolt with National Coarse Threads is 18 in. long. The threaded portion is 5 in. long. The maximum stress in the bolt is limited to 10,000 psi. This bolt is subjected to severe shock loads.

a. How much energy, in foot-pounds, can this bolt absorb?

b. What will be the total elongation produced?

c. If the unthreaded portion is turned down to the root diameter of the thread, how much energy can be absorbed?

d. What will be the total elongation produced in part (*c*)?

e. What conclusions would you draw concerning bolts subjected to shock loading?

143. A split nut is held from rotating but is propelled along a 3-in. Acme screw having a single thread of 2 threads per inch. The nut advances against a load of 12,000 lb. A thrust collar of 4 in outside diameter and 3 in. inside diameter is used on the screw. Assume the coefficient of friction in the thread to be 0.12 and on the collar 0.08.

a. Determine the horsepower required to drive the screw.

b. Same as (*a*) using a double-thread screw.

144. The load on a screw jack is exactly supported by friction in the nut when lowering. Prove that the efficiency of this screw when raising the load is 50 per cent.

145. A load of *W* lb falling 0.10 in. creates an impact tensile load on a $1\frac{1}{2}$-in National Coarse Thread bolt. The bolt is 12 in. long between the head and nut, and $1\frac{1}{2}$ in. is threaded. A maximum direct tensile stress of 18,000 psi is permissible.

a. When a full-diameter bolt is used, what is the permissible load *W*?

b. When the bolt shank is reduced to the root diameter of the threads, what is the permissible load *W*?

Chapter VIII

146. A belt pulley is fastened to a $2\frac{15}{16}$-in. shaft, running at 200 rpm, by means of a key $\frac{3}{4}$ in. wide by 5 in. long. The permissible stresses in the key are 8,000 psi in shear and 14,000 psi in compression.

a Determine the horsepower that can be transmitted.

b. What depth of key is required?

147. A 48-in. cast-iron pulley is fastened to a $4\frac{1}{2}$-in. shaft by means of a $1\frac{1}{8}$-in. square key 7 in. long. The key and shaft are S.A.E. 1030 steel, annealed.

a. What force acting at the pulley rim will shear this key?

b. What force acting at the pulley rim will crush the cast-iron keyway if the strength of cast iron is 24,000 psi in tension and 96,000 psi in compression?

c. What should be the maximum force applied at the rim of the pulley if the load is applied with moderate shock?

148. A key $\frac{15}{16}$ in. wide, $\frac{5}{8}$ in. deep and 12 in. long is to be used on a 200-hp, 1160-rpm squirrel-cage induction motor. The shaft diameter is $3\frac{7}{8}$ in. The maximum running torque is 200 per cent of the full-load torque. Determine the maximum shearing and compressive stresses on the key and the maximum direct shearing stress on the shaft, considering the effect of the keyway.

149. A flange coupling for a 4-in. 0.25-C steel shaft has to transmit the full strength of the shaft. Neglecting the weakening effect of the keyway, what will be the dimensions of the key used to connect the shaft and coupling? Use 0.25-C steel for the key.

150. A shaft and key are made of the same material and the key width is one-fourth the shaft diameter.

a. Considering shear only, determine the minimum key length in terms of the shaft diameter.

b. The shearing strength of the key material is 75 per cent of its crushing strength. Determine the thickness of the key to make the key equally strong in shear and crushing

c. How do the dimensions determined disagree with standard practice? Give reasons why standard practice gives better key design.

151. A 4-in. shaft rotating at 100 rpm transmits 300 hp. Power is taken off through a gear whose hub is 8 in. long. The key is made of steel having an ultimate shearing stress of 50,000 psi. Using a factor of safety of 5, determine the width of key desired.

152. A gear with a hub 3 in. long is fastened to a $1\frac{15}{16}$-in. commercial steel shaft by means of a $\frac{1}{2}$-in. square feather key. What force is required to move the gear along the shaft when transmitting the full torque capacity of the shaft? Assume the coefficient of friction to be 0.15.

153. This problem is the same as Prob. 152, except that two keys located at 180 deg from each other are used. The torque transmitted remains the same.

154. A 12-in. gear transmitting 50 hp at 120 rpm is to be fastened to a shaft of S.A.E. 1020 steel with a permanent fit by means of an S.A.E. six-spline fitting. The hub is 1.5 times the shaft diameter. Determine the spline dimensions, using a factor of safety of 5.

155. The transmission gears of an automobile are carried on a $2\frac{1}{4}$-in. S.A.E. 10-spline shaft and slide when under load. The hub length of each gear is $1\frac{5}{8}$ in. Determine the total horsepower that can be transmitted at 3,000 rpm with 800 psi permissible pressure on the splines.

156. An automobile engine has a torque capacity of 200 lb-ft. The clutch is attached to its driven shaft by means of a six-spline S.A.E. fitting that is to slide when not under load. Determine all dimensions of the shaft and spline, assuming S.A.E. 2330 steel of 250 Brinell hardness, moderate shock, and a maximum pressure on the splines of 1,200 psi.

157. Two 6-in. shafts are connected by a flanged coupling. Each coupling hub is fitted with a $1\frac{1}{2}$- by $1\frac{1}{2}$-in. key 6 in. long. The coupling halves are bolted together with six 1-in. bolts arranged on a bolt circle 11 in. in diameter. The shaft, key, and bolt materials have an ultimate strength in tension and compression of 68,000 psi and an ultimate strength in shear of 50,000 psi. The load is applied with shock and an apparent factor of safety of 6 is desired.

a. Determine the probable method of failure.

b. Determine the horsepower that may be safely transmitted at 100 rpm.

158. A 12-in. lever is fixed to a $1\frac{1}{2}$-in. shaft by means of a taper pin passed through its hub perpendicular to the axis, the mean diameter of the pin being $\frac{3}{8}$ in. What pull on the end of this lever will cause a shearing stress on the pin of 9,000 psi, and what torsion stress will this produce in the shaft?

159. A tensile load is transmitted through a $1\frac{3}{4}$-in. rod fitted with a flat cotter key $\frac{7}{16}$ by $1\frac{5}{8}$ in in cross section. The edge of the opening for the cotter is $1\frac{3}{8}$ in. from the rod end. The opposite end of the rod is threaded, and the permissible stress in the threaded portion is 9,000 psi. Determine the weakest part of the joint if all parts are made of S.A.E. 1020 steel.

160. A piston rod of wrought iron is keyed to a cast-iron piston 24 in. in diameter by a cotter of machine steel. Determine the two diameters of the rod, the dimensions of the cotter, and the piston-boss dimensions Steam pressure is 150 psi, gauge.

$s_t = 3{,}500$	$s_c = 15{,}000$	$s_s = 3{,}000$	for cast iron
$s_t = 10{,}000$	$s_c = 12{,}000$	$s_s = 8{,}000$	for wrought iron
$s_t = 12{,}000$	$s_c = 15{,}000$	$s_s = 10{,}000$	for steel.

161. A standard rod-end yoke designed to be welded to a rod $\frac{1}{2}$ in. in diameter is made of 0.25-C steel. The knuckle pin is made of 0.50-C steel $\frac{1}{2}$ in in diameter. The pin bosses are $\frac{15}{16}$ in. in diameter, and each is $\frac{9}{32}$ in. thick. The distance between the inner surface of the bosses is $\frac{9}{16}$ in. Determine all the stresses set up in rod, pin bosses, and pin when a tensile load of 3,000 lb is applied.

Chapter IX

162. A machinery shaft is subject to torsion only. The bearings are 8 ft apart. The shaft transmits 250 hp at 200 rpm Allow a shear stress of 6,000 psi after an allowance for keyways.

a. Determine the shaft diameter for steady loading.

b. Determine the shaft diameter if the load is suddenly applied with minor shocks.

163. The shaft of a 50-hp 850-rpm direct-current motor is 31 in. from center to center of bearings and is $2\frac{1}{2}$ in. in diameter.

a. If the magnetic pull on the armature is 1,500 lb concentrated midway between the bearings, determine the maximum shear and the maximum tensile stress in the shaft.

b. If the shaft extends beyond the bearings and carries a 6-in. gear $7\frac{1}{2}$ in. from the bearing center, determine the maximum shear and the maximum tensile stress in the shaft.

c. What kind of material would you select for this shaft?

164. *a.* What is meant by cold-rolled shafting?

b. What effect has a keyway on the strength of a shaft?

c. What effect has a keyway on the torsional rigidity of a shaft?

d. Is a hollow shaft stronger or weaker than a solid shaft of the same weight?

e. Has a hollow shaft more or less resistance to bending than a solid shaft of the same diameter?

165. A punch press has sufficient capacity to punch six holes 1 in. in diameter in a $\frac{3}{4}$-in. boiler plate. The maximum force exerted acts on the driving crank with a moment arm of $1\frac{1}{4}$ in. Considering torsion only,

determine the required size of shaft, using S.A.E. 2345 steel heat-treated to a hardness of 250 Brinell.

166. A 15-hp 1,725-rpm motor drives a centrifugal pump through a single set of 5:1 reduction gears. Determine the diameters of the shafts on the motor and pump if ordinary steel shafting with standard keys is used

167. This problem is the same as Prob. 166 except that a 15-hp internal-combustion engine replaces the motor.

168. The cross shaft of the braking system of an automobile is shown in Fig. P-7. The brakes are identical Assume that $A = 15$ in., $B = 33$ in., $C = 44$ in., and $d_1 = 1$ in. Determine d_2 so that the forces applied to the brakes will be equal.

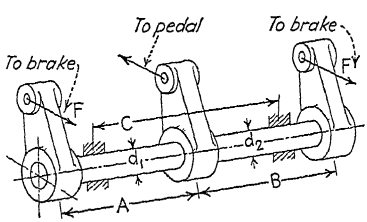


FIG. P-7.

169. A 75-hp motor operating at 90 rpm is to drive a lathe in which the loads may be considered to be suddenly applied and the shocks of a medium intensity. The shaft is fitted with a standard key. Considering torsion only, what shaft diameter is required if the shaft material has a yield stress of 35,000 psi in tension?

170. A machinery shaft supported on bearings 8 ft apart is to transmit 250 hp at 200 rpm while subjected to a bending load of 1,000 lb located at a distance of 2 ft from one bearing. Allow a shearing stress of 6,000 psi and a bending stress of 12,000 psi.

a Determine the shaft diameter for steady loading.

b. Determine the shaft size if the transverse load is steady and the torsional load is suddenly applied

171. A steel shaft 2 in. in diameter is 6 ft between supports. A vertical load of 500 lb is applied 18 in. from one bearing, and a load of 400 lb at 30 deg with the vertical and perpendicular to the shaft is applied at a distance of 30 in. from the same bearing. What is the maximum tensile stress in the shaft?

172. A factory line shaft is 150 ft long and is to transmit 100 hp at 200 rpm. The allowable fiber stress in shear is 7,000 psi, and the maximum allowable twist is 1 deg in a length of 20 diameters. Determine the required shaft diameter.

173. The shaft joining a turbine and generator is 6 in. in diameter. Neglecting the weakening effect of the keyway, the allowable stress may be taken as 6,000 psi in shear. A key is used to hold the coupling flange to the generator shaft. This key is fitted on all four sides, and the allowable stresses may be taken as 6,000 psi in shear and 14,000 psi in bearing. The width of the key is to be $d/4$.

a Determine the horsepower that can be transmitted by the shaft at 1,000 rpm.

b. Determine the required width, depth, and length of the key.

174. A shaft 24 in. between bearings supports a 20-in. pulley 10 in. to the right of the left-hand bearing, and the belt drives a pulley directly below.

Another pulley 15 in. in diameter is located 5 in. to the right of the right-hand bearing, and the belt is driven from a pulley horizontally to the right. The coefficient of friction for the belts is 0.30 and the angle of contact 180 deg. The maximum tension in the belt on the small pulley is 800 lb.

Find the shaft diameter, allowing $s_t = 8{,}000$ psi and $s_s = 6{,}000$ psi.

175. A mild carbon-steel shaft transmitting 15 hp at 210 rpm is supported on two bearings 27 in. apart and has keyed to it two gears. An 18-tooth, $14\frac{1}{2}$-deg involute three-pitch gear is located 5 in. to the right of the right-hand bearing and delivers power to a gear directly below the shaft. An 80-tooth, four-pitch gear is located 6 in. to the right of the left-hand bearing and receives power from a gear directly over it.

Calculate the diameter of the shaft, assuming working stresses of $s_t = 12{,}000$ psi and $s_s = 10{,}000$ psi.

176. A steel shaft is supported by bearings 4 ft apart A gear B, 12 in in diameter, is located 10 in. to the left of the right-hand bearing and is driven by a gear A, directly behind it. A belt pulley C, 24 in in diameter, is located 18 in. to the right of the left-hand bearing and drives a 24-in. pulley directly behind it. The ratio of belt tensions is 2:1, with the tight side on top. The gear teeth are 15-deg involute form.

The stresses in the shaft are not to exceed 16,000 psi in tension and 8,000 psi in shear. The deflection at the gear is not to exceed 0 008 in., and the torsional deformation is not to exceed 1 deg in 20 diameters. The gears transmit 50 hp, and the shaft rotates at 200 rpm Determine the required shaft diameter. Assume the bending load to be gradually applied and the torsion load to be applied with minor shocks.

177. The maximum torque delivered by a certain truck engine is 600 ft-lb. The over-all efficiency of the gearing and drive shafts is 85 per cent. The weight on each rear wheel is 3,000 lb. The total speed-reduction ratio between the engine and rear axle is 12 5:1. The tires are 29 in. in diameter, and the coefficient of friction of rubber on pavement is 0.8.

a Can the tires be made to slip?

b. What diameter of axle must be used if the torsional stress is not to exceed 10,000 psi? The axle is made of nickel steel having an ultimate tensile strength of 115,000 psi.

178. A shaft 48 in. long is supported at the ends by simple bearings. A vertical load of 2,000 lb is applied 10 in. from the left end, a load of 3,000 lb acting down and forward at an angle of 60 deg with the vertical is applied 24 in. from the left end, and a load of 2,500 lb acting down and forward at an angle of 30 deg with the vertical is applied 40 in. from the left end. A torque of 10,000 lb-in. is applied at the first load, a torque of 6,000 lb-in. is taken off at the second load, and a torque of 4,000 lb-in. is taken off at the third load.

a. Determine the maximum bending moments, graphically.

b. Determine the required shaft diameter if the allowable stress is 10,000 psi in tension and 6,000 psi in shear, and the maximum deflection at any point of loading is 0.01 in.

179. During the design of a certain ship it is found that the propeller shaft will be required to transmit 2,000 hp at 110 rpm. The designer has the

choice of 3 steels, A, B, and C, having the properties and prices shown in the table. Prices are per pound after heat-treating and complete machining, *i.e.*, prices per pound of completed shaft.

Steel	Cost per lb, dollars	Ult. strength in tension, psi	Ult. strength in shear, psi
A	0 15	70,000	50,000
B	0 195	90,000	62,500
C	0 24	135,000	95,000

The factor of safety to be used is 12, and the angular twist is to be limited to 1 deg in 20 diameters

a. If weight is the chief consideration what material and what shaft diameter would you use?

b. What diameter shaft would you use if price is the chief consideration?

c. Same as (*a*) using a hollow shaft whose inside diameter is $\frac{3}{4}$ the outside diameter. The hollow shaft costs 2 cts per lb more than the solid shaft.

d. Same as (*b*) using the hollow shaft.

e. Comparing (*b*) and (*d*), what percentage of weight and of cost could be saved by using the hollow shaft?

180. The shaft of a certain engine is 24 in. long between the bearings For 8 in. from the left-hand bearing, the diameter is 2 in. For 6 in from the right-hand bearing, the diameter is $2\frac{1}{4}$ in. The center portion of the shaft is 3 in in diameter. A load of 1,000 lb is concentrated at a point 10 in. from the left-hand bearing, and a load of 2,000 lb at a point 15 in from the left-hand bearing.

a Determine the maximum bending stress in the shaft.

b. By graphical solution, determine the maximum deflection in the shaft.

181. A 2-in. steel shaft 40 in. long is simply supported at the ends It carries a disk *A*, weighing 100 lb, 15 in from the left-hand bearing and a second disk *B*, weighing 75 lb, 25 in from the left-hand bearing.

a. Determine the critical speed of the shaft alone.

b. Determine the critical speed of the shaft and disk *A*.

c. Determine the critical speed of the shaft and disk *B*.

d. Determine the critical speed of the shaft and both disks.

e. What error in critical speed would there be if the weight of the shaft is neglected?

182. Determine the critical speed of a small electric motor designed to run at 1,200 rpm if the shaft is $\frac{3}{4}$ in. in diameter and 18 in. between supports. The rotating element may be considered as being a single disk, its weight of 50 lb being concentrated at the center. Is the speed of this motor satisfactory?

183. It has been found that certain troubles in large machines are due to the fact that a shaft with a keyway has different flexibility in different radial directions. It has been proposed to cure this trouble by machining more than one keyway in the shaft

What is the smallest number of keyways necessary and how should they be distributed around the shaft in order to make its flexibility the same in all directions? Prove your answer.

184. A 3-in. shaft supported on bearings 5 ft apart carries a 3,000-lb disk $1\frac{1}{2}$ ft from the left-hand bearing and a 4,000-lb disk $2\frac{1}{2}$ ft from the left-hand bearing.

a. Determine the critical speed when self-aligning ball bearings are used.

b. Will an operating speed of 1,800 rpm be satisfactory for this shaft?

c. Determine the critical speed if sleeve bearings are used and fitted tight enough so that they may be considered rigid supports.

Chapter X

185. Two $1\frac{7}{16}$-in. shafts are connected by a flange coupling. The flanges are fitted with six bolts of S.A.E. 1020 steel on a 5-in. bolt circle. The shafts run at 350 rpm and transmit a torque of 8,000 lb-in. Assume a factor of safety of 5.

a. What diameter bolts should be used?

b. How thick should the flanges be?

c. Determine the key dimensions.

d. Determine the hub length.

e. What horsepower is transmitted?

186. A plain flange coupling for a 3-in. shaft has the following dimensions: bore, 3 in.; hub diameter, $5\frac{5}{8}$ in.; hub length, $3\frac{3}{4}$ in.; flange diameter, 10 in.; flange thickness, $1\frac{1}{16}$ in.; bolt diameter, $\frac{3}{4}$ in.; bolt-circle diameter, $8\frac{1}{4}$ in.; number of bolts, six; and key, $\frac{3}{4}$ in. square. All parts are made of S.A.E. 1020 steel, annealed. This coupling is rated at 50 hp at 100 rpm.

a. Determine the bearing, shearing, and tensile stresses in all parts of the coupling.

b. What factor of safety does this coupling have?

187. A jaw clutch for a $4\frac{3}{16}$-in. shaft has three jaws with radial faces. The dimensions are: inside diameter of jaws, $4\frac{3}{8}$ in.; outside diameter, $11\frac{1}{2}$ in.; axial height of jaws, 2 in.; and key, 1 by 1 by $6\frac{3}{8}$ in. Assume $\frac{1}{8}$ in. clearance between the jaws and a working stress in the shaft of 6,000 psi.

a. What horsepower can be transmitted at 100 rpm?

b. Determine the shearing and bearing stresses in the key and the bearing stress on the jaw faces.

188. A disk clutch consists of two steel disks in contact with one asbestos-fabric-faced disk having an outside diameter of 10 in. and an inside diameter of 8 in. Determine the horsepower that can be transmitted at 1,000 rpm if the coefficient of friction is 0.35 and the disks are pressed together by an axial force of 2,000 lb.

189. A multiple-disk clutch is to be used on machine tools. There are 8 driven disks having an outside diameter of 3 in. and an inside diameter of $2\frac{1}{4}$ in. The disks are metal and run in an oil spray. The coefficient of friction may be taken as 0.02, and the permissible unit pressure as 100 psi

a. Determine the axial pressure required.

b. Determine the horsepower that can be transmitted at 600 rpm

190. A six-cylinder engine is rated at 60 hp at 2,000 rpm. The maximum torque is developed at 1,200 rpm and 40 hp. The multiple-disk clutch consists of fabric-faced disks of $8\frac{1}{2}$ in. outside diameter and $6\frac{1}{4}$ in inside diameter in contact with five driven disks. The coefficient of friction is 0.20. Determine the axial pressure necessary to engage this clutch.

191. A gasoline-engine-driven tractor is equipped with a multiple-disk clutch with six driven disks faced with asbestos clutch fabric whose coefficient of friction is 0.25. The disks are 8 in. in inside diameter and 12 in. in outside diameter. The construction of the clutch requires five springs and limits the spring diameter to $1\frac{1}{2}$ in., and the length when the clutch is engaged to $2\frac{3}{4}$ in. When the clutch is disengaged the clutch springs are $2\frac{1}{2}$ in. long, and the pressure is 45 per cent higher than when engaged. The maximum stress in the springs is not to exceed 70,000 psi. Find the horsepower that can be transmitted at 1,200 rpm, allowing 10 psi pressure on the disks.

192. A cone clutch has a face angle of 15 deg with a maximum diameter of 24 in. and a face width of 3 in. The coefficient of friction is 0.20 and the permissible pressure on the cone surface 12 psi.

a. What torque may be transmitted?

b. What horsepower may be transmitted at 800 rpm?

c. What axial pressure must be exerted at this power?

193. An engine developing 40 hp at 1,250 rpm is fitted with a cone clutch built into the flywheel. The cone has a face angle of $12\frac{1}{2}$ deg and a maximum diameter of 14 in. The coefficient of friction is 0.20. The normal pressure on the clutch face is not to exceed 12 psi.

a. Determine the face width required.

b. Determine the spring pressure required to engage this clutch.

194. A flat-rim clutch 10 in. in diameter uses four friction blocks, 2 in. wide and 6 in. along the circumference. Coefficient of friction = 0.55. If this clutch rotates at 200 rpm, what pressure is required on each block if 6 hp is transmitted?

195. A clutch uses four friction blocks acting on the outside of a drum 12 in. in diameter. Each block is 2 in. wide and extends 6 in. along the circumference. The coefficient of friction is 0.45. The clutch transmits 6 hp at 150 rpm. What pressure is required on each brake block?

196. A friction clutch consists of two maple blocks acting on the inside of an 18-in. pulley. The diameter of the clutch surface is 16 in. The blocks are 4 in. wide, and each subtends 60 deg of the circumference. The coefficient of friction is 0.55. The clutch is rated at 5 hp at 100 rpm.

a. Determine the total radial force required on each block if the starting torque is twice the running torque.

b. Determine the maximum unit contact pressure.

197. A block clutch has four wooden shoes each contacting with 75 deg of the inside of a 12-in. drum. The coefficient of friction is 0.30 and the maximum contact pressure is not to exceed 35 psi. This clutch is to transmit 15 hp at 250 rpm. Determine the required width of shoes.

198. The drum of a band clutch is 18 in in diameter. The 2-in. steel band is lined with asbestos brake lining having a coefficient of friction of 0.30. When the clutch is engaged the arc of contact is 340 deg and the maximum pull on the end of the band is 150 lb. What horsepower can be transmitted at 250 rpm?

199. A lever-release single-plate clutch is faced with molded asbestos disks having an outside diameter of 10 in. and an inside diameter of 6 in. The coefficient of friction is 0.35 and the permissible unit pressure 30 psi.

a. Determine the total spring pressure required.

b. Determine the torque capacity of the clutch in pound-feet.

c. What horsepower engine, running at 2,400 rpm, would this clutch serve with an overload allowance of 75 per cent for starting?

200. A double-plate lever-release clutch is required for a motor having a maximum torque capacity of 300 lb-ft. The width of the facings is limited to one-fourth their outside diameter, the allowable unit pressure is 35 psi, and the coefficient of friction is 0 35. There are nine pressure springs having an outside diameter of $1\frac{1}{4}$ in. The movement of the pressure plate during release is to be $\frac{3}{32}$ in., and the effective pedal movement, $4\frac{1}{2}$ in. An overload torque capacity of 100 per cent is to be provided for starting and shock loading. Determine the outside and inside facing diameters.

201. A single-plate clutch is to have a maximum capacity of 75 hp at 1,800 rpm. The clutch facing has a coefficient of friction of 0.40 and a permissible pressure of 30 psi. The clutch is engaged through 12 springs of $1\frac{1}{4}$ in. mean diameter. The springs compress $\frac{1}{16}$ in. for disengagement, with an increase in pressure of 10 per cent. Determine the diameters of the clutch facing if the inner diameter is 0 7 of the outer diameter.

202. An automobile engine has a maximum-torque capacity of 285 lb-ft at 3,600 rpm. The clutch used with this engine has molded asbestos facings having a coefficient of friction of 0 35. The unit pressure on the clutch facings is limited to 30 psi. There are to be nine pressure springs with an outside diameter of $1\frac{5}{16}$ in. When the clutch is released, the pressure plate moves $\frac{3}{32}$ in. and the spring pressure increases 15 per cent. Determine the outside and inside diameters of the clutch facings for a single-plate lever-release clutch if the facing width is limited to one-half the mean radius. Allow 75 per cent overload capacity for starting.

Chapter XI

203. Determine the force with which the brake shoe of a Diesel-electric train must be pressed against the wheel to absorb 250,000 ft-lb of energy in 20 sec if the mean velocity of the wheel relative to the brake shoe is 45 fps and the coefficient of friction is 0.2.

204. A simple brake band (Fig. 150) has a 30-in. drum fitted with a steel band $\frac{5}{32}$ in. thick lined with brake lining having a coefficient of friction of

0.25 when not sliding. The arc of contact is 245 deg. This brake drum is attached to a 24-in. hoisting drum that sustains a rope load of 1,800 lb. The operating force has a moment arm of 60 in., and the band is attached 5 in. from the pivot point

a. Find the force required to just support the load.

b. What force will be required if the direction of rotation is reversed?

c. What width of steel band is required if the tensile stress is limited to 7,500 psi?

205. A simple band brake operates on a drum 24 in. in diameter that is running at 200 rpm. The coefficient of friction is 0 25. The brake band has a contact of 270 deg, and one end is fastened to a fixed pin and the other end to the brake arm 5 in. from the fixed pin. The straight brake arm is 30 in. long and is placed perpendicular to the diameter that bisects the angle of contact.

a. What is the minimum pull necessary on the end of the brake arm to stop the wheel if 45 hp is being absorbed? What is the direction of rotation for this minimum pull?

b. What width steel band $\frac{3}{32}$ in. thick is required for this brake if the maximum tensile stress is not to exceed 8,000 psi?

206. The brake shown in Fig. P-8 is fitted with a cast-iron brake shoe. The coefficient of friction is 0.30. The braking torque is to be 3,000 lb-in.

a. Determine the force F required with counterclockwise rotation.

b. Determine the force F required with clockwise rotation.

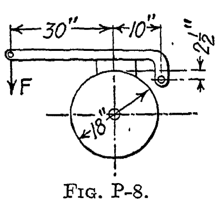


FIG. P-8.

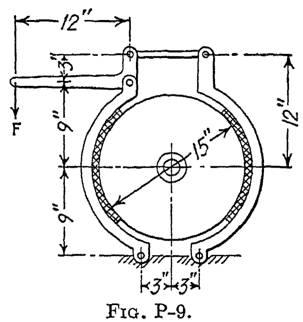


FIG. P-9.

207. In the brake of Prob. 206, where must the pivot point be placed to make the brake self-energizing with counterclockwise rotation?

208. A differential brake band has an operating lever 9 in. long. The ends of the brake band are attached so that their operating arms are $1\frac{1}{2}$ and 6 in. long. The brake-drum diameter is 24 in., the arc of contact 300 deg, the brake band $\frac{1}{8}$ by 4 in., and the coefficient of friction 0.22.

a. Find the least force required at the end of the operating lever to subject this band to a stress of 8,000 psi.

b. What is the torque applied to the brake-drum shaft?

c. Is this brake self-locking? Prove your answer.

209. An elevator brake is constructed as shown in Fig. P-9. Each brake shoe is 5 in. long. The coefficient of friction is 0 30, and the permissible pressure 50 psi average. The operating force F is 100 lb.

a. Determine the braking torque for clockwise rotation.

b. Determine the braking torque for counterclockwise rotation.

c. Determine the required width of the brake shoes.

210. The rope drum of an elevator hoist is 2 ft in diameter, and the speed of the elevator is 600 fpm. This drum is fitted with a brake drum 3 ft in diameter having 4 cast-iron brake shoes each subtending an arc of 45 deg on the brake drum. This elevator weighs 4,000 lb loaded, and the brake is to have sufficient capacity to stop the elevator in 12 ft. The coefficient of friction of cast iron on cast iron may be taken as 0.20.

a. Determine the radial pressure required on each brake shoe.

b. If the allowable pressure on the brake shoe is 50 psi, determine the width of shoes required.

c. How much heat is generated in stopping this elevator?

211. A brake on a 40-hp 3,000-lb automobile consists of a pair of aluminum shoes faced with asbestos fabric and working inside a steel drum 15 in. in diameter. The brake shoes are pinned together at the bottom $6\frac{1}{2}$ in. from the brake center and separated by a cam at the top. Each brake facing extends through 160 deg of the circumference. Assume the coefficient of friction to be 0.35 and the maximum pressure to be 100 psi. The tires are 30 in. in diameter, and the coefficient of friction of rubber on roads is 0.60. Each brake wheel supports 35 per cent of the weight of the car, and the brakes are to be sufficiently strong to slide the tires. Find the width of brake lining and the pressure required at the cam to operate the shoes.

212. The brake shown in Fig. 148 has a 20-in. drum and brake shoes subtending 90 deg of drum. The pressure on each brake shoe is 200 lb, and the coefficient of friction is 0.30. Determine the proper location of the pin supporting the brake shoe.

213. A mine hoist is equipped with a block brake having two 60-deg shoes each 12 in. wide and faced with asbestos blocks. The cast-steel brake drum is 12 ft in diameter by 14 in. wide and is attached to a rope drum 10 ft in diameter. This mine is 1,000 ft deep, and the weight of cage handled by the brake is 20,000 lb. This hoist operates continuously, lowering in $1\frac{1}{4}$ min, raising in $1\frac{1}{2}$ min, and resting while loading and unloading 15 sec per round trip. Assume the radiating surface to be 20 per cent greater than the exposed surface of the brake drum. Assume the coefficient of friction to be 0.3.

a. Determine the probable operating temperature of the brake.

b. Determine the probable variation in temperature during each cycle of operation.

214. A brake similar to Fig. 146 is on a shaft driven at 1,150 rpm by a 100-hp motor. Assume a coefficient of friction of 0.35, angle of contact of each block 90 deg, and a maximum unit pressure of 20 psi.

a. Determine the width of brake shoe and the load on the spring S.

b. Will this brake overheat in continuous service? State your reasoning in full.

215. The brake shown in Fig. P-9 has shoes each subtending 90 deg of drum. The coefficient of friction is 0.30, and the braking force F is 125 lb. Determine the maximum and minimum radial pressures on the brake shoes. Each shoe is 5 in. wide.

Chapter XII

216. A coiled compression spring of oil-tempered steel wire has seven active coils of $\frac{7}{16}$-in wire wound in a coil of $3\frac{1}{4}$ in. outside diameter. The spring is used to produce axial pressure on a clutch. The free length is $7\frac{1}{2}$ in. With the clutch engaged, the length is $5\frac{5}{8}$ in. Determine the stress in the wire and the pressure exerted against the clutch plate.

217. A coiled spring having $8\frac{1}{2}$ active coils of $\frac{3}{8}$-in. steel wire has an outside diameter of $3\frac{1}{4}$ in, a free length of 8 in. and an operating length of 5 in. Determine the stress in the spring and the pressure exerted.

218. The plunger of an oil pump is held against the operating cam by a spring made of No. 10 B.W.G steel wire coiled with an outside diameter of $\frac{7}{8}$ in. There are five active coils. The open length is $1\frac{3}{16}$ in. What are the length and stress when a load of 30 lb is applied?

219. The valve springs of an automobile engine are made of No 8 W M G. wire with an outside diameter of $1\frac{1}{8}$ in. The spring pressure is 43 lb at a spring length of $2\frac{1}{4}$ in. (valve closed) and 96 lb at a length of $1\frac{29}{32}$ in. (valve open). There are seven effective coils.

a. Determine the maximum stress in the wire.

b. Determine the modulus of rigidity of the wire material.

c. Determine the open length of the spring.

220. A rod of 0.12 in diameter is coiled into a tension spring of 20 effective turns with a mean radius of 1 in The modulus of rigidity is 12,000,000 psi.

a. Determine the stress due to a load of 3 lb.

b. Determine the corresponding spring elongation.

c. If the initial compression in the coils was 1 lb, what would the stress and elongation be when the 3-lb load is applied?

221. The valve spring of a gasoline engine is $1\frac{19}{32}$ in. long when the valve is open, and $1\frac{29}{32}$ in. when the valve is closed. The spring loads are 50 lb with the valve closed and 80 lb with the valve open. The inside diameter of this spring cannot be less than 1 in.

a. Determine the required wire size for a maximum operating stress of 60,000 psi.

b. Determine the number of active coils required.

c. Determine the spring length when completely closed, assuming squared and ground ends.

d. Determine the pitch to which this spring should be wound.

222. A gas-engine valve spring is loaded to 75 lb when the valve is closed and 115 lb when the valve is open. The valve lift is $\frac{5}{16}$ in. The outside diameter is to be from $1\frac{1}{2}$ to $1\frac{3}{4}$ in., and the permissible stress is 60,000 psi.

a. Determine the wire diameter, outside diameter, and number of effective coils.

b. Determine the open or free length of the spring, assuming the ends to be squared and ground.

223. An engine valve spring exerts a pressure of 65 lb when the valve is open and 40 lb when the valve is closed. The spring has an outside diameter of $1\frac{1}{2}$ in. The valve lift is $\frac{3}{8}$ in. The permissible stress is 65,000 psi. Determine the wire size and the number of effective coils required.

224. A $\frac{1}{2}$- by 1-in. rectangular wire forms a spring of 6 effective coils with a 3 in. outside diameter. A 3,000-lb load is supported.

a. Determine the maximum stress if the long side of the wire is parallel to the spring axis.

b. Determine the deflection at this load.

c. What load will produce the same stress if the wire is wound with the short side parallel to the spring axis?

225. A coil spring is made of 8 effective coils of $\frac{1}{4}$- by $\frac{1}{2}$-in. wire and has an outside diameter of $2\frac{1}{2}$ in. The permissible stress is 50,000 psi.

a. Determine the load that can be supported if the wire is coiled with the long side parallel to the axis.

b. Determine the load if the wire is coiled with the short side parallel to the axis

c. Determine the deflection in each case.

226. Determine the wire diameter, the number of effective coils, and the open length of the springs to be used in the clutch of Prob. 191, allowing $1\frac{1}{2}$ noneffective coils. Select Washburn and Moen or Roebling gauge wire.

227. A Diesel engine weighing 160,000 lb is mounted on 12 springs in order to protect the building from vibration. The springs have $2\frac{1}{2}$ effective coils and are made of S.A.E. 3245 steel, oil-quenched and drawn to 1000 F.

a. What size of square steel and what outside diameter would you use for these springs?

b. What will the spring deflection be when the engine is not running?

228. A $\frac{1}{4}$-in. round wire is coiled to form a conical spring with 10 effective coils and an outside diameter of 3 in. at one end and 2 in. at the other.

a. Determine the load that will produce a stress of 50,000 psi.

b. Determine the deflection at this load.

229. A clutch spring is made of $\frac{3}{16}$- by $\frac{5}{16}$-in. rectangular wire wound to form a conical spring with outside diameters of $5\frac{3}{4}$ and $3\frac{5}{8}$ in. at the ends. There are $3\frac{1}{2}$ effective coils. The long side of the wire is parallel to the axis.

a. What clutch pressure is required to produce a spring stress of 50,000 psi?

b. Determine the free length of the spring if the operating length is $1\frac{1}{8}$ in.

c. Determine the spring stress if the clutch plate moves $\frac{1}{4}$ in. in disengaging.

230. The spring pressure in the clutch of Prob. 200 is not to increase more than 15 per cent during release. Oil-tempered steel spring wire and an allowable stress equal to 80 per cent of the endurance limit in shear is used.

a. Determine the spring-wire diameter and the number of effective coils.

b. Determine the clutch-pedal pressure required.

231. Determine the spring-wire size and the number of effective coils for the clutch in Prob. 201, allowing a stress of 60,000 psi.

232. The maximum unit stress in the spring wire of the clutch in Prob. 202 is to be approximately 60 per cent of the yield stress in shear.

a. Determine the diameter of oil-tempered steel wire required.

b. Determine the number of effective coils required, the total number of coils, and the open length of the spring if squared and ground ends are used.

c. If the clutch pedal moves 4 in. during the release movement, what pedal pressure is required?

233. Determine the natural vibrating frequency of the spring in

a. Prob. 216.

b. Prob. 217.

c. Prob 218.

d. Prob. 220.

234. A laminated spring is made of six graduated and two full-length leaves, each 0.134 in. thick and $\frac{3}{4}$ in. wide. The effective length of the spring is 30 in. The permissible stress is 40,000 psi.

a. Determine the central load that may be carried, assuming no initial stress in the spring.

b. Determine the deflection.

235. A semielliptic laminated spring is made of No. 10 B.W.G. steel 2 in. wide. The length between supports is $26\frac{1}{2}$ in. and the band is $2\frac{1}{2}$ in. wide. The spring has two full-length and five graduated leaves. A central load of 350 lb is carried.

a. Determine the maximum stress in each set of leaves for an initial condition of no stress in the leaves.

b. Determine the maximum stress if initial stress is provided to cause equal stresses when loaded.

c. Determine the deflection in part (*a*).

d. Determine the deflection in part (*b*).

236. A cantilever spring has an effective length of 21 in. and has two full-length and eight graduated leaves, each $2\frac{1}{4}$ in. wide. The spring is to sustain a load of 900 lb with a stress of 50,000 psi in all leaves.

a. Determine the thickness of the spring leaves and give the B.W G. number.

b. Determine the deflection at full load.

237. A truck spring has 10 leaves of graduated length. The spring supports are $42\frac{1}{2}$ in. apart and the central band is $3\frac{1}{2}$ in. wide. The central load is to be 1,200 lb with a permissible stress of 40,000 psi. Determine the width and thickness of the steel spring material and the deflection when loaded. The spring should have a ratio of total depth to width of about $2\frac{1}{2}$.

238. A locomotive spring has an over-all length of 44 in. and sustains a load of 16,000 lb at its center. The spring has 3 full-length leaves and 15 graduated leaves with a central band 4 in. wide. All leaves are to be stressed to 60,000 psi when fully loaded. The ratio of total spring depth to width is to be approximately 2.

a. Determine the width and thickness of the leaves.

b. Determine the initial space that must be provided between the full-length and graduated leaves before the band is applied.

c. What load is exerted on the band after the spring is assembled?

239. A disk spring is made of $\frac{1}{8}$-in. sheet steel with an outside diameter of 5 in. and an inside diameter of 2 in. The spring is dished $\frac{3}{16}$ in. The maximum stress is to be 80,000 psi.

a. Determine the load that may be safely carried.

b. Determine the deflection at this load.

Chapter XIII

240. A 3-in. bearing using oil with an absolute viscosity of 70 centipoises running at 300 rpm gives satisfactory operation with a bearing pressure of 200 psi. The bearing clearance is 0.005 in. Determine the unit pressure at which this bearing should operate if the speed is changed to 400 rpm.

241. The bearing in Prob. 240 is given a total clearance of 0.003 in. What change should be made in the oil?

242. The specific gravity of a lubricating oil was found to be 0.875 at 100 F. What is the probable specific gravity at 160 F?

243. The bearing described in Prob. 240 is kept flooded with oil. Determine the probable coefficient of friction and the horsepower loss in the bearing.

244. A bearing $2\frac{1}{2}$ in. in diameter and $3\frac{1}{2}$ in. long runs at 450 rpm. The total clearance is 0.004 in. Oil of 60-centipoise viscosity at the operating temperature is used. Determine the maximum safe working load on this bearing.

245. A bearing $1\frac{15}{16}$ in. in diameter and $2\frac{3}{4}$ in. long running at 500 rpm operates satisfactorily with a total clearance of 0.003 in. The total load is 600 lb, and at the operating temperature of 150 F the value of ZN/p was found to be 30. Select an oil suitable for this bearing.

246. Design a bearing and journal to support a load of 1,200 lb at 800 rpm using a hardened-steel journal and a bronze-backed babbitt bearing. An abundance of oil, having a viscosity of 250 sec Saybolt at 100 F and a specific gravity of 0.90 at 60 F, is provided. The bearing is relieved for 20 deg from the normal to the load line. Assume an oil temperature of 185 F and a clearance of 0.0015 in. per in. of diameter.

247. Use the same conditions as Prob. 246, but assume a clearance of 0.00075 in. per in. of diameter.

248. Use the same conditions as Prob. 246, but assume a clearance of 0.0005 in. per in. of diameter.

249. Use the same conditions as Prob. 246, except that an oil groove is located at 45 deg from the load line.

250. Use the same conditions as Prob. 249, except that the clearance is 0.00075 in. per in. of diameter.

251. The main bearing of a stationary steam engine is $6\frac{1}{2}$ in. in diameter by 9 in. long and sustains a load of 15,000 lb. Perfect lubrication cannot be maintained. Determine the probable horsepower loss in this bearing when running at 200 rpm.

252. A line shaft running at 250 rpm is supported on six $2\frac{7}{16}$- by 7-in. ring-oiled bearings. Determine the probable horsepower loss in this shaft if the average pressure on the bearings is 75 psi.

253. The wrist pin of a gas engine is to sustain a load of 3,500 lb. Design a pin for this engine and check for strength and deflection.

254. Determine the probable coefficient of friction and the horsepower loss in a film-lubricated bearing 4 in. in diameter and 8 in. long with a total clearance of 0.005 in when S.A E. 40 oil is used and the operating oil-temperature is 160 F. The total load on the bearing is 4000 lb at 400 rpm.

255. A steam engine has an overhanging crank 12 in. long. The maximum load on the crank is 20,000 lb, and the average load is 14,000 lb The permissible stresses are $s_t = 8{,}000$ psi and $s_s = 6{,}000$ psi. The engine runs at 150 rpm.

a Determine proper dimensions for the crankpin.

b. Determine the horsepower loss in the bearing.

c. How much heat must be dissipated by this bearing?

256. A steam turbine running at 1,800 rpm has a bearing 4 in. in diameter and 7 in long supporting a load of 8,000 lb. Assuming film lubrication, what will the probable operating temperature be in a room where the temperature is 80 F?

If the bearing is artificially cooled to maintain an oil temperature of 160 F, how many gallons of water must be circulated per minute allowing 15 F rise in water temperature?

257. Determine the dimensions of the crankpin of a 12- by 18-in by 300-rpm center-crank type steam engine operating on steam at 200 psi. Neglect the angularity of the connecting rod and assume permissible stresses of $s_t = 6{,}000$ psi and $s_s = 4{,}500$ psi.

258. Derive an expression for the horsepower loss in a thrust collar, in terms of the unit pressure, the coefficient of friction, the revolutions per minute, the inside diameter, and the outside diameter. Assume the wear perpendicular to the bearing surface to be uniform over the entire surface.

259. A bearing cap is to withstand a total load of 2,000 lb The journal is 4 in. in diameter by 10 in. long. There are four holding bolts, two on each side of the shaft, placed on 8-in. centers. Assume a factor of safety of 8 and that the cap is made of alloy cast iron with an ultimate strength of 42,000 psi in tension. A $\frac{3}{8}$-in. oil hole is provided Determine the thickness of the bearing cap

260. The main bearing of an engine-generator set is 8 in. in diameter by 12 in. long and supports a load of 12,000 lb. The bearing is ring-oiled, and the coefficient of friction may be taken as 0.0125. The engine rotates at 85 rpm in a room where the temperature is 80 F.

a. Determine the probable operating oil temperature.

b. Determine the horsepower loss in the bearing.

261. A ship traveling 18 mph requires 2,500 hp at the propeller, which turns at 80 rpm. The thrust collars on the propeller shaft are 18 in. in outside diameter and 12 in. in inside diameter. Assume the coefficient of friction to be 0.08 and the permissible pressure on the bearing surfaces to be 50 psi.

a. Determine the number of thrust collars required.

b. Determine the power lost in the bearing.

262. The stator of a large synchronous motor is cylindrical on the outside and rests on two bearings as shown in Fig. P-10. The bearings are segments of cylindrical surfaces. The stator weight is W lb. It is desired to rotate the stator through a small angle by means of a force F. Derive an equation for the force F and the reactions at A and B. Discuss the effect of varying the angle α.

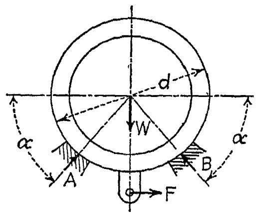


FIG P-10.

263. Same as Prob. 262 except that the stator is subjected to a clockwise torque T.

264. The seat of a thrust bearing is a portion of a spherical surface whose radius equals the diameter of the bearing. Derive an equation for the frictional moment in terms of the total thrust force F, the coefficient of friction f, and the bearing diameter d. Assume the pressure to be uniformly distributed over the projected area of the bearing.

265. Same as Prob. 264 except that the wear is assumed to be uniform in the direction parallel to the bearing axis.

Chapter XIV

266. Determine the force necessary to move a weight of 5,000 lb supported on four steel wheels of 2 ft diameter, assuming the coefficient of rolling friction to be 0 008

267. The dome of an observatory, weighing 8,000 lb, rests on 12 cast-iron balls of 3 75 in. diameter. The balls run in cast-iron upper and lower races of 15 ft pitch diameter. The coefficient of rolling friction may be taken as 0 01. The dome is rotated at $\frac{1}{4}$ rpm by means of a gear of 6 in. pitch diameter keyed to a vertical shaft at the axis of the dome. Determine the horsepower of the motor required.

268. Same as Prob 267 except that the 6-in. gear drives a 200-in. gear built integral with the ball race

269. The weight on each wheel of a motor-driven traveling crane is 6,000 lb. The wheels are 2 ft in diameter and are carried on journals 4 in. in diameter and 8 in. long. The coefficient of rolling friction is 0.02 and of sliding friction 0.05. Determine the horsepower of the motor required to drive a four-wheel crane traveling 500 fpm.

270. A locomotive, weighing 60 tons, has four pairs of driving wheels. The coefficient of friction between the wheels and rails is 0.20. The drivers are 48 in. in diameter with journals 6 in. in diameter by 9 in long. The coefficient of rolling friction is 0.01. The coefficient of journal friction is 0 03.

a. Determine the maximum pull that the locomotive can exert.

b. What force is required to move the locomotive forward, neglecting wind resistance?

c. What is the resistance per ton of weight?

d. Assuming the same resistance ratio for the train, what total load can the locomotive pull on level tracks?

e. What total train weight could the locomotive pull on a 10 per cent grade?

271. A rotating tube mill in a cement mill consists of a horizontal cylinder 7 ft 6 in. in diameter. This mill weighs 50 tons when loaded and is supported on 16-in. rollers, two at each end, carried on 5-in. journals. The rollers are located 30 deg on either side of the vertical center line. The coefficient of rolling friction is 0.0125 and the coefficient of journal friction is 0.06. Determine the horsepower friction loss when the mill revolves at 20 rpm.

272. A steel roller, 6 in. in diameter by 8 in long, rests on a flat steel surface. A load of 20,000 lb is applied to the roller.

a. Determine the maximum pressure between the roller and plate in pounds per square inch.

b. Determine the maximum shear stress in the roller.

273. Same as Prob. 272 except that a 6-in. sphere is substituted for the roller.

274. Two steel rollers 6 in. and 30 in. in diameter and 10 in. long are pressed together with a force of 30,000 lb.

a. Determine the maximum unit pressure of contact.

b. Determine the maximum shear stress produced.

275. Same as Prob. 274 except that spheres are substituted for the rollers.

276. A roller bearing consists of nine $\frac{15}{16}$-in. diameter by 3-in. carbon steel rollers running on a race $2\frac{1}{2}$ in. in diameter. What load can this bearing support at 450 rpm?

277. A radial ball bearing consists of 18 hardened alloy-steel balls $\frac{7}{16}$ in. in diameter arranged on a pitch circle 70 mm in diameter.

a. Using a factor of safety of 10, what is the safe load on a single ball?

b. What is the safe load on the complete bearing using curved races with two-point contact on the balls?

c. What will be the probable friction loss in foot-pounds per minute if the outer race revolves at 300 rpm?

278. A No. 314 deep groove type ball bearing is to operate at 650 rpm with the inner race revolving. The load is steady.

a. What is the bearing capacity from the tables?

b. How many hours of operation should be expected at this capacity?

c. What would be the capacity if the bearing is to operate 10 hr per working day for 5 years?

279. Same as Prob. 278 but using a No. 210 ball bearing.

280. Same as Prob. 278 but using a No. 212 roller bearing and an estimated life of 18 hr per day for 6 months.

281. Select a ball bearing to support a radial load of 2,000 lb for 10 hr per day for 3 yr. The supported shaft is $2\frac{3}{8}$ in. in diameter and rotates at 400 rpm.

282. Same as Prob. 281 but with the addition of a 1,500-lb thrust load.

283. Select a ball bearing to support a radial load of 2,400 lb on a $1\frac{5}{8}$-in. shaft revolving at 600 rpm, 12 hr per day for 6 years.

284. A needle bearing consists of steel rollers $\frac{5}{32}$ in in diameter and $1\frac{1}{4}$ in. long, running on a shaft $1\frac{3}{4}$ in. in diameter. The shaft rotates at 200 rpm.

a. Determine the capacity of this bearing when subjected to a steady radial load.

b. Determine the capacity when the load is applied with heavy shock

285. Determine the number, diameter, and length of the rollers for a needle bearing to support a radial load of 12,000 lb on a journal whose diameter is $1\frac{1}{2}$ in. The shaft rotates at 350 rpm.

286. A shaft is mounted on two bearings 15 in. apart and carries at its middle a gear of 8 in. pitch diameter. The gear causes a 2,000-lb radial load and a 500-lb thrust load on the shaft when rotating at 500 rpm. Allowable stresses in the shaft are 8,000 psi in tension and 6,000 psi in shear.

a. Determine the shaft diameter.

b. Select a ball bearing for each end of the shaft.

Chapter XV

287. A pulley 72 in. in diameter running at 240 rpm drives an 18-in. pulley by means of a two-ply leather belt. The shafts are 18 ft apart and 100 hp is transmitted. The belt is 0.35 in. thick.

a. Determine the width of belt required when cemented joints are used.

b. Determine the belt length.

288. A 36-in. driving pulley and a 48-in. driven pulley are arranged on 10-ft centers. The output of the driven shaft is 150 hp. Assume a belt speed of 4,200 fpm, a coefficient of friction of 0 30, a slip of 1.5 per cent at each pulley, and 5 per cent friction loss at each shaft.

a. Determine the revolutions per minute of each shaft.

b. Determine the difference in belt tensions.

c. Determine the size of machine-wire-laced leather belt required.

d. Determine the required shaft sizes, assuming pure torsion and an allowable stress of 8,000 psi.

e. Determine the over-all efficiency of this transmission.

289. A 25-hp by 870-rpm motor with a 12-in. pulley drives a centrifugal pump at 290 rpm, by means of a medium double leather belt laced with wire by hand. Determine the width and length of the belt required, allowing for 20 per cent overload and 4-ft center distance.

290. A 50-hp 1,200 rpm high-torque squirrel-cage motor is used to drive a punch press. The motor pulley is cast iron 14 in. in diameter. The driven pulley is cast iron 42 in. in diameter. The center distance is 8 ft and is inclined at 55 deg with the horizontal. Select a suitable leather belt.

291. Select a leather belt to drive from a 3-hp 900-rpm alternating-current normal-torque motor to a centrifugal blower operating at 3,300 rpm. The load on the blower is fairly uniform. The cast-iron pulley on the blower is 4 in. in diameter, and the center distance is 72 in.

292. The drive in Prob. 291 could be improved by the use of an 1,800-rpm motor. Select a leather belt for this motor using the other data as given.

293. A 5-hp 900-rpm high-torque squirrel-cage motor drives a plate shear by means of a light double-ply leather belt. A 9-in. motor pulley is used and a 6-in. driven pulley, both of cast iron. The drive is horizontal with a 36-in. center distance.

a. Determine the effective tension required for starting.

b. Determine the probable unit stress in the belt.

294. A ventilating fan having an 18-in. cast-iron pulley is driven from a 25-hp normal-torque motor placed directly below it. The rawhide pulley on the motor is 10 in. in diameter, and the center distance is 5 ft. Select a leather belt for this drive.

295. Using the data in Prob. 294, select a suitable rubber belt.

296. When preparations were being made to install a leather belt to drive a pulp beater at 20 rpm, no information as to the power requirements was available. When a rope was wrapped around the 12-in. cast-iron pulley of the beater, it was found that a pull of 55 lb was necessary to start the machine, and that the pull required to keep the beater running when loaded varied from 90 to 120 lb.

a What effective tension is required to start this machine?

b What average torque is required to keep the machine running?

c What torque is required to start the machine?

d. What size of induction motor running at 720 rpm should be used for this installation?

e What is the effective belt tension required?

f Select a suitable leather belt for this drive if the normal speed of the machine pulley is 960 rpm.

297. Two shafts, 6 ft apart, are connected by an open belt $\frac{1}{4}$ in. thick and 4 in. wide. The driving shaft rotates at 1,750 rpm and carries an 8-in. pulley. The driven shaft rotates at 438 rpm.

a. Determine the horsepower that may be transmitted under steady-load conditions.

b. Determine the additional horsepower that may be transmitted when a 6-in idler is placed 9 in. from the center of the driver on a line parallel to the original belt.

298. A mine ventilating fan running at 120 rpm is driven from a 12-in. pulley on a 50-hp 720-rpm motor. The center distance is 18 ft. Determine the width of six-ply rubber belt required.

299. The main drive in a tile plant is from a 250-hp 550-rpm motor. A 30-in pulley is used on the motor and a 130-in. pulley on the driven shaft. An endless belt with 25 per cent overload capacity is to be used. The center distance is 20 ft.

a. What width of 36-oz. duck eight-ply rubber belt is required?

b. If the belt is $\frac{13}{16}$ in. thick and the coefficient of friction is 0.3, determine the probable working stress.

300. A lathe spindle is driven from a countershaft located 6 ft above and 18 in. behind it and running at 250 rpm. A 12-in. pulley is used on the countershaft and a 6-in. pulley on the lathe. A 3-in. medium two-ply leather belt with machine–wire-laced joints is used. Determine the horsepower that may be transmitted.

301. A leather belt with cemented joints is to transmit 50 hp from a 48-in. pulley to a 36-in. pulley. The center distance is 10 ft, and the driving pulley runs at 250 rpm.

a. Determine the size of belt required if the drive is horizontal.

b. Determine the size of belt required if the center line makes an angle of 60 deg with the horizontal.

302. A ventilating fan fitted with a 16-in. cast-iron pulley is driven from a 10-in. pulley on a 25-hp 1,200-rpm motor placed 5 ft directly below it. Select a rubber belt for this drive.

303. A 10-hp 1,200-rpm motor is arranged for a Rockwood drive as shown in Fig. 225. The pulley diameter is 8 in., and the motor weight is 350 lb. The distance b is 16 in., and the distance c is 18 in. The starting torque on the motor is 250 per cent of rating.

a. Determine the effective tension required for starting the drive and the values of F_1 and F_2.

b. Determine the distance a from the hinge point to the motor center line.

c. Determine the tensions F_1 and F_2 in the belt at rated load.

d. Determine the tensions at zero load.

e. Compare the results of (*a*), (*c*), and (*d*) with the tensions required with a simple open belt drive.

304. This problem is the same as Prob. 303, except that the motor revolves counterclockwise.

305. This problem is the same as Prob. 303, except that the Rockwood drive is replaced by a gravity idler so located that the arc of contact is 220 deg, and the tension on the slack side is 60 lb.

306. The drive from a motor to a centrifugal pump consists of three size B V-belts. The motor pulley has a 4.3 in. pitch diameter, and the pump pulley has a pitch diameter of 15.4 in. The motor runs at 1,200 rpm. What power can be transmitted if the center distance is 3 ft?

307. An oil-field pumping jack is fitted with a gear-reduction unit delivering 260,000 lb-in. torque at 25 rpm. The total reduction in the gears is 40:1. The gear unit is fitted with a 13 in. pitch diameter V-belt pulley and is driven by a 1,200-rpm motor. The center distance is 4 ft. Determine the number of size C belts required.

308. Determine all dimensions and make a dimensioned sketch of a 36-in. cast-iron pulley to be used with a single-ply 8-in. leather belt. The pulley is to rotate at 400 rpm. The permissible stress in the pulley shaft is 6,000 psi in shear.

309. Determine all dimensions and make a dimensioned sketch of a 24-in. cast-iron pulley for a heavy 2-ply leather belt of 6 in. width. The pulley arms are to be elliptical in cross section with the major axis twice the minor axis. The permissible stress is 2,000 psi in the arms and 6,000 psi in the shaft. Maximum rotating speed is 350 rpm.

Chapter XVI

310. A 12-ft sheave running at 100 rpm transmits power to a 6-ft sheave by means of 18 Manila ropes. The sheaves have 45-deg grooves for $1\frac{5}{8}$-in. ropes. The center distance is 40 ft. Determine the power that can be transmitted.

311. Same as Prob. 310 but using cotton ropes.

312. A multiple-rope drive transmits 250 hp from a 12-ft sheave running at 150 rpm to an 8-ft sheave. Both sheaves have 60-deg grooves, and the center distance is 30 ft.

a. Determine the number of $1\frac{1}{2}$-in. Manila ropes required.

b. Determine the amount of sag in both the tight and slack strands.

313. Two shafts located 20 ft apart rotate at 350 rpm. Ten Manila ropes running on 60-in sheaves with 60-deg grooves are used. Determine the size of rope required to transmit 200 hp.

314. Same as Prob. 313 but using cotton rope.

315. A continuous rope drive is used in a flour mill. Each sheave is 6 ft in diameter and has three 60-deg grooves for $1\frac{1}{4}$-in. Manila rope. The rope speed is 4,000 fpm. Determine the horsepower transmitted.

316. The main drive from the engine of a textile mill consists of a continuous rope drive from a 12-ft sheave to an 8-ft sheave running at 160 rpm. Each sheave has six 60 deg grooves. The maximum load on any section of rope is 300 lb. Assume the transmission efficiency of each guide pulley on the tension carriage to be 96 per cent. Call the load on the rope leading from the tension carriage to the first groove of the driven pulley F_1, the load on the first rope leading to the first groove of the driving pulley F_2, etc.

a. Determine the total tension in each strand of rope.

b. Determine the horsepower transmitted.

c. Determine the proper Manila rope size.

317. A $\frac{5}{8}$-in. Manila hoisting rope is used in a block and tackle having one block with two 4-in. sheaves and one single sheave block. The rope passes from the single sheave through the two-sheave block, through the single-sheave block, back to and through the two-sheave block to the point of application of the operating force. Assume an efficiency of 90 per cent for each sheave.

a. Determine the maximum safe load that can be handled.

b. Determine the operating force required to move this load.

c. Determine the over-all efficiency of this block and tackle.

318. A block and tackle consists of one three-sheave block and one four-sheave block. Assume the frictional loss at each sheave to be 3 per cent of the rope tension on the right side of the sheave.

a. Determine the load that can be lifted when the applied rope pull is 150 lb.

b. Determine the required size of Manila rope.

319. Determine the probable bending stress and equivalent bending load in a $1\frac{1}{2}$-in. 6 by 19 steel rope made from a 0.095-in. wire, when it is used on a 92-in. sheave.

320. A $\frac{3}{4}$-in. 6 by 37 steel rope is made of 0 034-in. wire

a. Determine the bending stress when used with a 24-in. sheave.

b. Determine the equivalent bending load.

321. A $1\frac{1}{2}$-in 6 by 19 plow-steel rope is used to lift the cage of a mine hoist. The mine is 600 ft deep, the rope sheave is 12 ft in diameter, and the cage acceleration is 5 ft per sec^2.

a. Determine the equivalent bending load.

b. Determine the load due to the rope.

c. Determine the acceleration load.

d. Determine the useful load with a factor of safety of 5.

322. In an office building the elevator rises 1,200 ft with an operating speed of 1,000 ft per min and reaches full speed in 35 ft Assume a drive similar to Fig. 239, 6 by 19 cast-steel ropes, and 36-in. sheaves. The loaded elevator weighs 2 tons. Determine the number of $\frac{5}{8}$-in. ropes required if the factor of safety is 6.

323. An oil well using $4\frac{1}{2}$-in drill pipe (same dimensions as 4-in. std pipe) is drilled to a depth of 4,000 ft. Assume 50 lb every 40 ft for pipe joints. The rope sheaves are 30 in. in diameter, and the acceleration is 10 ft per sec^2. Determine the number of strands of 1-in. 6 by 37 plow-steel rope required for lifting the string of pipes using a factor of safety of $2\frac{1}{2}$.

324. Given the following information on a traction type elevator:

Lift height—600 ft
Maximum speed—600 fpm
Distance between stops—15 ft
Maximum acceleration—2 ft per sec^2
Weight of car—2,000 lb
Rated capacity—3,000 lb
Counterweight—3,900 lb
Number of ropes—4
Size of ropes (6 by 37)—$\frac{7}{16}$ in. in diameter
Ultimate strength—5.5 tons
Weight per ft—0.3 lb
Diameter of sheaves—24 in.
Coefficient of friction—0 135

a. Determine the actual load on each rope and the factor of safety

b. If the angle of contact is 180 deg, will the friction between the rope and the grooves be sufficient to prevent slipping under the worst conditions?

325. The 70-story tower building of the Rockefeller Center in New York is provided with 75 elevators, 24 of which operate at 1,200 fpm. This is the highest passenger elevator speed used anywhere up to 1933.

The highest rise cars have a total travel of 779 ft and carry a rated load of 3,500 lb, the elevator weighing approximately 3,000 lb. It requires 120 ft to accelerate to full speed.

The ropes used on these elevators are special $\frac{11}{16}$-in. 8 by 19 ropes of 32,400-lb rated strength.

a. Assume the head sheave to be 30 in. in diameter and determine the number of ropes required per elevator with a factor of safety of 8, including the bending stress.

b. If the empty elevator rests with its platform level with the lowest floor, how much will it drop when fully loaded?

326. Select a wire rope for a vertical mine hoist to lift 1,500 tons of ore in each 8-hr shift from a depth of 3,000 ft. Assume a two-compartment shaft with the hoisting skips in balance. Use a maximum velocity of 2,500 ft per min with acceleration and deceleration periods of 15 sec each and a rest period of 10 sec for discharging and loading the skips. A hoisting

skip weighs approximately 0.6 of its load capacity. The factor of safety should be approximately 5.

NOTE: Since there are several unknown quantities, such as pulley diameters, bending loads, etc., you should assume a trial diameter of rope, then select a drum diameter. From the results obtained modify the rope selected until a satisfactory one is found

Chapter XVII

327. A $\frac{7}{16}$-in coil chain is to be used on a crane hoisting drum. The drum is fitted with pockets cast to receive the chain.

a. What load can be raised with a factor of safety of 3?

b. What is the pitch diameter of the smallest drum that should be used?

328. A crane having a capacity of 20 tons uses a coil chain for hoisting. What size chain would you select, using a factor of safety of 3?

329. A "caterpillar"-type tractor uses a chain-type track with 8-in. pitch. The driving sprocket has 18 teeth. The pitch clearance is to be 0.10 in.

a. Determine the pitch diameter of the sprocket.

b. Determine the sprocket speed in revolutions per minute when the tractor travels 2 mph.

330. The head shaft of a bucket-and-chain elevator is 42 in. between bearings and carries a 42-in. sprocket at its center. Closed-end pintle chain is used. The chain pull is 3,000 lb.

a. What size chain is required with a factor of safety of 6? Give the chain type and catalog number.

b. Determine the head-shaft diameter, if made of material having an ultimate strength of 62,500 psi in tension. Assume an apparent factor of safety of $5\frac{1}{2}$.

331. A roller chain operating under steady-load conditions, transmits 5 hp from a shaft rotating at 600 rpm to one operating at 750 rpm.

a. Determine the chain required using at least 15 teeth in the sprockets.

b. Determine the sprocket pitch diameters.

c. Determine the shortest advisable center distance.

d. Determine the number of links of chain required.

332. A 10-hp 1,200-rpm motor drives a line shaft at 250 rpm. The shaft center distance is to be approximately 2 ft. The motor shaft has a diameter of $1\frac{1}{4}$ in. The starting torque of the motor is from 1.75 to 2.00 times the running torque. The load is applied with moderate shock.

a. Select a roller chain for this drive.

b. Determine the sprocket pitch diameters.

c. How many chain links are required, and what is the exact center distance?

333. A roller chain is used to drive the camshaft of an internal-combustion engine. Both shafts rotate at 325 rpm and the center distance is approximately 21 in. The crankshaft is 5 in. in diameter, and the root spaces of the sprockets must clear the shaft by at least $\frac{5}{8}$ in. Three horsepower is

required to drive the shaft. Determine all necessary dimensions for the chain and sprockets and show by calculations that it is safe in every way.

334. A truck equipped with a 50-hp engine uses a roller chain as the final drive to the rear axle. The driving sprocket runs at 225 rpm and the driven sprocket at 100 rpm with a center distance of approximately 3 ft. The chain speed is to be approximately 500 fpm. The transmission efficiency between the engine and the driving sprocket is 85 per cent.

a. Determine the pitch and type of chain to be used.

b. Determine the number of teeth in each sprocket and the pitch diameters.

335. A rotary engine driving a wire-line reel on an oil-well rig develops 250 hp at 1,000 rpm. The reel runs at 50 rpm maximum. The engine may be slowed down to 200 rpm, the torque remaining constant. This is severe service for the chain drive between the engine and reel.

a. Select a suitable roller chain.

b. Determine the number of teeth and the pitch diameter of each sprocket.

336. A silent chain operating under good service conditions transmits 100 hp from a 600-rpm motor to a shaft running at 167 rpm.

a. Determine a suitable number of teeth for each sprocket.

b. Determine the pitch and width chain required for this drive.

337. Same as Prob. 331 but using silent chain.

338. Same as Prob. 332 but using silent chain.

339. Same as Prob. 333 but using silent chain.

340. Select a silent chain and sprockets for the drive from a 60-hp 690-rpm motor to an air compressor running at 135 rpm. The center distance is to be as short as possible, and the driven sprocket is not to exceed 36 in. in outside diameter.

Chapter XVIII

341. Two shafts, connected by full-height $14\frac{1}{2}$ deg true-involute-tooth gears, are to have a 5:1 velocity ratio. Determine the minimum number of teeth required on each gear.

342. A $22\frac{1}{2}$-deg involute rack is to drive a pinion. If both have standard addendums and true-involute-tooth curves, what will be the least number of teeth that can be used on the pinion?

343. A 64-tooth full-depth 20-deg involute gear rotates at 100 rpm and drives a second gear.

a. What is the highest speed at which the second gear can run?

b. How many teeth are on the second gear?

c. If 20-deg stub teeth (A.G.M.A.) are used, at what speed can the second gear run, and how many teeth will it have?

344. A standard $14\frac{1}{2}$-deg involute gear has 60 teeth.

a. If this is the larger gear of a pair, what is the largest speed reduction possible?

b. If this is the smaller gear of a pair, what is the largest speed reduction possible?

345. An iron gear with cast teeth has a pitch diameter of 14 in., 21 teeth, and a 4-in. face width. The permissible working stress is 5,000 psi.

a. What is the diametral pitch?
b. What is the circular pitch?
c. What is the tooth thickness?
d. What is the outside diameter?
e. What is the root diameter?
f. What is the clearance?
g. What is the backlash?
h. What is the permissible pitch-line load?

346. A gear with cast teeth of 1-in pitch and $2\frac{1}{2}$-in. face width safely transmits a pitch-line load of 140 lb when subjected to moderate shock. Determine the pitch of a similar gear to transmit 30 hp under similar conditions and a pitch-line velocity of 1,800 fpm.

347. A one-pitch $14\frac{1}{2}$-deg involute gear has 14 teeth Draw one tooth three times the actual size, and determine graphically the value of the Lewis factor Use radial flanks below the base line and a root fillet equal to $1\frac{1}{3}$ times the clearance.

348. A cast-iron gear of 30 ft $6\frac{3}{8}$ in. pitch diameter and 30-in. face has 192 machine-cut cycloidal teeth. Determine the circular and diametral pitches and the horsepower that can be transmitted at 12 rpm under steady-load conditions.

349. A 24-tooth $14\frac{1}{2}$-deg involute gear of 8 in. diameter and 3-in. face width has teeth cut with milling cutters. The gear transmits 16 hp at 175 rpm.

a. Determine the apparent working stress in the teeth.
b. Determine the equivalent static stress.
c. Select a material suitable for this gear.

350. A pair of carefully cut gears with 20-deg stub teeth is to transmit 30 hp at 300 rpm of the gear with a speed reduction of 10:1. The 3-in. pinion is made of S.A.E. 1035 steel with a hardness of 250 Brinell and drives a cast-iron gear. Determine the diametral pitch and face width required if the load is steady. Use the Lewis equations.

351. Same as Prob. 350 but assuming moderate shock loading.

352. Same as Prob. 350 except that the dynamic load equations are to be used

353. Design a pair of gears with A.G.M.A. stub teeth to transmit 75 hp from a 7-in. pinion running at 2,500 rpm to a gear running at 1,500 rpm. Both gears are to be made of S.A.E. 2345 steel with a hardness of 260 Brinell. Approximate the pitch by means of the Lewis equations. Then adjust the dimensions to keep within the limits set by the dynamic load and wear equations.

354. An 18-tooth Bakelite pinion running at 1,750 rpm drives a 145-tooth cast-iron gear. Teeth of 3-pitch A.G.M.A. stub form are used, and the face width is 6 in.

a. Determine the horsepower that can be transmitted with moderate shock.

b. Give all important machining dimensions of both gears.

355. A 16-tooth rawhide pinion transmits $7\frac{1}{2}$ hp to a 35-tooth cast-iron gear running at 600 rpm. The standard composite tooth form is used.

a. Determine the required pitch, diameter, and face width considering strength only.

b. Will the wear on these gears be satisfactory for continuous operation? Give reasons for your answer.

356. A 20-tooth helical gear has a pitch diameter of 10 in. The helix angle is 23 deg and the pressure angle measured in a plane perpendicular to the axis of rotation is 20 deg. The addendum is 0.8 divided by the diametral pitch.

a. Find the diametral pitch.

b. Find the circular pitch in a plane normal to the teeth.

c. Find the pressure angle in a plane normal to the teeth.

357. A 75-hp motor, running at 450 rpm, is geared to a pump by means of double-helical gearing. The S.A.E. 1035 forged-steel pinion on the motor shaft is 8 in in diameter and drives the good grade cast-iron gear on the pump shaft at 120 rpm. Determine the diametral pitch and the face width.

358. A single-stage turbine running at 30,000 rpm is used to drive a reduction gear, that delivers 3 hp at 3,000 rpm. The gears are 20-deg involute herringbone gears of 28 pitch and $2\frac{1}{8}$-in. effective width The pinion has 20 teeth with a helix angle of 23 deg.

a. Determine the tangential pressure between the gear teeth at the pitch line.

b. Determine the load normal to the tooth surface.

c. Determine the apparent stress in the teeth, by the Lewis equations.

d. Determine the pitch-line velocity, in feet per minute.

e. Determine the equivalent static stress in the teeth.

f. What material would you use for each of these gears?

359. Design the teeth for two herringbone gears for a single-reduction speed reducer to have a velocity ratio of 3.80. The speed reducer is to transmit 36 hp, and the pinion is to have a speed of 3,000 rpm. The helix angle should be 30 deg and the teeth are to be 20-deg stub teeth in the plane of rotation. The length of the face of the pinion should not exceed twice the pitch diameter. The material of the gears is a high-carbon steel, heat-treated to have a yield point of approximately 60,000 psi and a Brinell hardness of 450.

360. A speed reducer for oil-field use is to have herringbone gears and a total reduction of 40:1. No gear is to have less than 18 teeth and all gears are to have the same pitch. The low-speed shaft is to run at 25 rpm and is to deliver 240,000 lb-in. torque. Determine the pitch and face width of all gears when A.G.M.A. stub-tooth herringbone gears are used. Use the Schmitter gear equations.

361. In a standard design, two shafts are connected by gears having 30 and 133 teeth. The gears have 20-deg full-height teeth of 16 pitch. An order for a number of these machines can be obtained if the gear ratio can be changed to 4:1. Examination of the gear housing for clearances indi-

cates that the smaller gear cannot have an outside diameter greater than $2\frac{1}{8}$ in. The shaft center distance cannot be changed. Is it possible to replace the regular drive by two gears cut with standard gear cutters? If it is possible, make a dimensioned sketch of the drive and prove that it will operate satisfactorily.

Chapter XIX

362. A pair of 4-pitch $14\frac{1}{2}$-deg involute machine-cut bevel gears of S.A.E. 3245 steel have a 2:1 reduction. The pitch diameter of the driver is 10 in., and the width of face 2 in. Determine:

a. The pitch angle of the pinion.
b. The pitch angle of the gear.
c. The face angle of the pinion.
d. The face angle of the gear.
e. The cutting angle of the pinion.
f. The cutting angle of the gear.
g. The maximum diameter of the pinion.
h. The virtual number of teeth on the pinion.
i. The equivalent tangential pressure at the large end of the teeth that may be transmitted at 200 rpm of the driver.
j. The pitch diameter at the center of pressure on the pinion.
k. The resultant tooth pressure at the center of pressure.
l The horsepower transmitted.

363. A pair of machine-cut bevel gears made of S.A.E. 1035 steel have 20 and 30 teeth. The $14\frac{1}{2}$-deg full-height tooth form is used. The smaller gear has a 5-in. pitch diameter, a $1\frac{1}{4}$-in face width, and runs at 600 rpm. Using the Lewis equation, determine the horsepower that can be transmitted under steady load and continuous service.

364. A pair of bevel gears have their shafts at right angles. The larger gear has 50 teeth of 4–5 pitch and is made of S.A.E. 1045 steel. The pinion has 20 teeth and is made of alloy steel hardened to 300 Brinell. The pinion runs at 850 rpm and transmits 30 hp. Determine the required face width.

365. The ring gear of a truck differential has 56 teeth of 4-pitch and is made of S.A.E. 2345 steel hardened to 240 Brinell. The pinion has 13 teeth with $1\frac{5}{8}$-in. face width and is made of S.A.E. 3245 steel hardened to 300 Brinell. The teeth are of the Gleason straight-tooth form. Determine the following:

a. The horsepower that may be transmitted at 1,000 rpm of the pinion.
b. The resultant tooth pressure and the radius at which this pressure acts on both gears.
c. The magnitude of the thrusts along the shafts.

366. A pair of bevel gears have a 1:1 velocity ratio, a pitch diameter of 8 in., and a face of $1\frac{1}{2}$ in. and rotate at 250 rpm The teeth are 5-pitch $14\frac{1}{2}$-deg involute and accurately cut. These gears transmit 8 hp. Determine:

a. Outside diameter of gears.
b Equivalent static stress in the teeth.
c. Resultant tooth pressure, tangent to the pitch cone.

d. Resultant radial pressure on the bearings.

e Resultant thrust pressure on the gear shafts.

367. A centrifugal pump submerged in a well is driven at 1,250 rpm by a 25-hp 900-rpm motor through a pair of bevel gears. The gear on the motor shaft is cast steel and has a pitch diameter of 8 in. with a face width of $1\frac{1}{2}$ in. The pinion is made of S.A E. 1040 steel. The teeth are of the $14\frac{1}{2}$-deg full-height form.

a Determine the diametral pitch.

b. Determine the radial pressure on the pinion.

c. Determine the end thrust on the pinion.

d. Determine the end thrust on the gear.

368. The straight-tooth bevel pinion driving the differential of an automobile has 15 teeth and a $1\frac{1}{4}$-in. face width. Gleason-type teeth of 5-pitch are used. The pinion transmits 40 hp at 2,500 rpm The pinion is supported on two bearings placed $1\frac{1}{2}$ and $5\frac{3}{4}$ in. behind the large pitch circle.

a. Determine the probable stress in the teeth.

b. Determine the resultant tooth pressure and the radius at which this pressure acts.

c. Determine the thrust along the axis.

d. Determine the radial pressure on each bearing.

369. This problem is the same as Prob. 459 except that spiral bevel teeth of $12\frac{1}{2}$-deg right-hand spiral (on the gear) are used The pinion rotates counterclockwise.

370. A three-thread worm, rotating at 1,000 rpm, drives a 31-tooth worm gear and transmits 15 hp. The worm has $14\frac{1}{2}$-deg teeth with $\frac{3}{4}$-in. pitch, 2 in. pitch diameter, and an included face angle of 60 deg. The coefficient of friction is 0.05.

a. Determine the helix angle of the worm.

b. Determine the speed ratio.

c. Determine the center distance.

d. Determine the apparent stress in the worm-wheel teeth.

e. Determine the probable efficiency of this worm-gear set.

371. A triple-thread cast-iron worm running at 225 rpm receives $7\frac{1}{2}$ hp through its shaft. The total efficiency of the worm and its bearings is 92 per cent. The velocity ratio is to be 10:1, and the distance between shafts is to be 8 in. The worm is to have a $14\frac{1}{2}$-deg pressure angle and a lead angle greater than 15 deg. Supporting bearings of both worm and worm gear are placed on 6-in. centers.

a. Determine the worm-shaft diameter with a permissible stress of 7,500 psi in torsion.

b. Using a standard circumferential pitch, determine the number of teeth and pitch diameters of the worm and worm gear.

c. Determine the radial, thrust, and tangential pressures on both the worm and worm gear.

d. Determine all bearing pressures.

e. Determine the heat that must be dissipated per minute.

372. A traction-type elevator is driven by a 50-hp 1,200 rpm motor through a worm drive. The worm has four threads of $14\frac{1}{2}$-deg pressure

angle and a pitch diameter of $4\frac{1}{4}$ in The worm gear has 52 teeth of $1\frac{3}{4}$-in. pitch and 4-in. face. The worm bearings are 18 in. center to center, and the worm-gear bearings are 9 in. center to center. Assume the coefficient of friction to be 0 03.

a. Determine center distance of the shafts.

b. Determine the efficiency of the drive.

c. Determine the loads on each bearing.

d. Determine the heat that must be dissipated, in Btu per minute.

Chapter XX

373. *a*. What is the tangential stress in a steel cylinder of 6 in. inside diameter and of $7\frac{1}{2}$ in. outside diameter with an internal pressure of 750 psi?

b. What is the longitudinal stress?

c. What is the maximum shear stress?

d. Assuming the maximum-stress theory to be applicable, determine the equivalent stress.

374. Determine the thickness of a fusion-welded steel drum that is to contain steam at 600 psi pressure and 750 F. The inside diameter of the drum is 54 in.

375. A cast-iron pipe is to deliver water at the rate of 31,000 gpm and a flow rate of $1\frac{1}{2}$ ft per sec. The maximum pressure in the pipe is 125 psi. The permissible stress in the cast iron is 3,000 psi. Determine the pipe diameter and the wall thickness.

376. A cast-iron pipe is 10 in. in inside diameter and the metal is $\frac{3}{8}$ in. thick. The pipe contains water under a head of 250 ft.

a. What is the apparent factor of safety considering water pressure only?

b. Determine the stress caused by bending if the pipe is full of water, horizontal, 24 ft long, and simply supported at the ends.

c. What is the maximum combined tensile stress?

377. A four-inch steel water pipe is subjected to an internal pressure of 175 psi. The permissible stress is 10,000 psi.

a. Determine the required wall thickness.

b. Compare this thickness with that of a standard-weight steel pipe.

c. Can you give any reasons for differences between the thickness determined in (*a*) and that of a standard pipe?

378. Determine the wall thickness required for a 5-in. pipe made of S.A.E. 1020 steel to transmit steam at 500 psi and 800 F.

379. Determine the wall thickness required for a brass condenser tube of 1 in. inside diameter. The vacuum in the condenser is to be 26 in. of Hg and the water pressure is to be 35 psi on the inside of the tube. Allow $\frac{1}{16}$ in. on the wall thickness for corrosion.

380. A steel lap-welded tube 10 ft long with an outside diameter of $6\frac{5}{8}$ in. is subjected to an external pressure of 150 psi. Determine the required wall thickness.

381. A brass condenser tube 1 in. in diameter and 4 ft long has a wall thickness equivalent to No. 16 B.W.G.

a. Determine the collapsing pressure.

b. Determine the permissible working pressure with an apparent factor of safety of 10.

382. A seamless cold-drawn steel tube is 6 in. in outside diameter and 12 ft long. The tube is to be subjected to an external pressure of 110 psi with an apparent factor of safety of 8. Determine the required tube thickness.

383. A Scotch marine boiler operating at 125 psi has plain furnace flues 3 ft in diameter and 7 ft 6 in. long, and fire tubes 4 in. in diameter and 7 ft 6 in. long.

a. Determine the required thickness of the flues.

b. Determine the required thickness of the tubes.

384. A water-tube boiler operating at 225 psi steam pressure has 4-in. steel tubes of No. 9 B.W.G. wall thickness and 18 ft long. Determine the apparent factor of safety.

385. The furnace of an internally fired boiler is 36 in. in diameter and 7 ft long and is subjected to an external pressure of 100 psi.

a. Determine the wall thickness if a plain cylindrical steel flue is used.

b. Determine the wall thickness if a Morrison corrugated furnace flue is used.

386. The tubes of a return-tubular boiler are 4 in. in outside diameter and 10 ft long. The boiler pressure is to be 125 psi. Determine the required wall thickness of seamless steel tubes with an apparent factor of safety of 10. Give your result to the nearest B.W.G.

387. Determine the wall thickness required in Prob. 385 if the furnace flues are of the Fox type and of the Purves type.

388. A cylinder 10 in. in diameter is to be filled with fluid at a pressure of 3,000 psi. The cylinder is made of steel with an ultimate strength of 60,000 psi. Determine the outside diameter of the cylinder using an apparent factor of safety of 5.

389. A closed-end cast-iron cylinder of 8 in. inside diameter is to carry an internal pressure of 2,000 psi with a permissible stress of 3,000 psi.

a. Determine the wall thickness by means of Lamé's, Clavarino's, and the maximum-shear equations.

b. Which result would you use? Give reasons for your conclusion.

390. A steel tank for shipping gas is to have an inside diameter of 8 in. and a length of 40 in. The gas pressure is 1,500 psi. The permissible stress is to be 8,000 psi.

a. Determine the required wall thickness, using the thin-cylinder equations.

b. Determine the thickness, using Clavarino's equations.

391. Same as Prob. 390 except that the pressure is to be 5,000 psi.

392. A single-acting triplex pump has a bore of $2\frac{1}{8}$ in. and a stroke of 8 in. When running at 50 strokes per minute it delivers 18 gal of oil per minute at 1,800 psi.

a. Determine the thickness of the cylinder walls using good cast iron with an apparent factor of safety of 6.

b. Same as (*a*) using cast bronze.

393. A single-acting triplex pump delivers 120 gpm at 275 psi when operating at 29 rpm. The cylinders have a bore of $5\frac{1}{2}$ in. and a stroke of 8 in.

a. Allowing $\frac{1}{8}$ in. for reboring and variation in casting thickness, determine the outside diameter of the cylinder when made of cast iron having an ultimate strength of 30,000 psi. Use an apparent factor of safety of 8.

b. Same as (*a*) using cast iron with an ultimate strength of 18,000 psi.

c. Same as (*a*) using cast bronze.

d. If the metals used in (*a*), (*b*), and (*c*) cost 7, 4, and 20 cts. per lb, respectively, compare the relative costs of the three cylinders

394. A cylinder of 8-in. bore has an outside diameter of 14 in. Using Lamé's equations, determine the maximum tangential and radial stresses under the following conditions.

a. Internal pressure of 2,000 psi.

b. External pressure of 2,000 psi.

c. Internal pressure of 2,000 psi, and an external pressure of 1,000 psi.

395. The cylinder of a hydraulic press has a bore of 18 in., and the working pressure is $2\frac{1}{2}$ tons per sq in. The working stress is limited to 4 tons per sq in. Assume Lamé's equations to apply.

a. Determine the required wall thickness.

b. Plot curves showing the variation of the radial and tangential stresses throughout the wall thickness.

396. Same as Prob. 395, assuming the maximum-shear theory to apply.

397. Same as Prob. 395, assuming Clavarino's equations to apply.

398. A steel ram of a hydraulic press has a 6 in. inside diameter and a 12 in. outside diameter and is subjected to an external pressure of 6,000 psi. Plot curves showing the variation of the stress throughout the wall thickness, computing points for each $\frac{1}{2}$ in. change in radius. How does the maximum stress compare with that obtained by the thin-cylinder formulas?

399. A steel shaft having a diameter of 3.500 in. has a steel ring having a bore of 3.495 in. and an outside diameter of $5\frac{3}{4}$ in. shrunk onto it. Determine the maximum stresses set up in the shaft and in the ring.

400. An aluminum-alloy ring is shrunk onto a steel shaft having a diameter of 2.500 in. The ring has a bore of 2.497 in. and an outside diameter of 3.000 in. Assume the modulus of elasticity of aluminum alloy to be 10,000,000 psi. Determine the probable tangential stress in the ring.

401. Same as Prob. 400 except that the shaft is bored out to $1\frac{1}{2}$ in.

Chapter XXI

402. A flat circular plate of 12 in. diameter is supported around the edge and is subjected to a uniform pressure of 150 psi. The stress is to be limited to 10,000 psi.

a. Determine the thickness of the steel plate required.

b. Determine the maximum deflection and state where it occurs.

403. Same as Prob. 402 except that the load is distributed over a concentric circular area of 6 in. diameter.

404. Same as Prob. 402 except that the load is concentrated on a small central area. The total load is 5,000 lb.

405. A flat circular plate of 10 in. diameter is rigidly supported around the edge and supports a load of 3,000 lb uniformly distributed around the circumference of a circle of 4 in. diameter.

a. Determine the thickness of the steel plate required if the stress is not to exceed 12,000 psi.

b. Determine the maximum deflection in the plate.

406. A cast-iron cylinder is to have a flat head cast integral with the cylinder walls. The internal working pressure is 2,000 psi, and the permissible working stress 3,500 psi. Determine the wall thickness if the inside diameter is 6 in.

407. A tank for holding hydrogen gas under a pressure of 1,000 psi is 10 in. in diameter. One end is closed by a dished head with the inner side convex. The radius of curvature of the head is $8\frac{1}{2}$ in. What should the head thickness be?

408. A 54-in. boiler drum has dished heads of 48-in. radius of curvature. The boiler pressure is 275 psi.

a. Determine the head thickness.

b. If the heads are reversed, what thickness is required?

c. If the radius of curvature is made 36 in., what thickness is required?

409. Design the joint between the head and the boiler shell of Prob. 408 if riveted construction is used.

410. Same as Prob. 409, using welded construction.

411. Determine the wall thickness and the head thickness required for an 18-in. fusion-welded steel drum that is to contain ammonia at 1,000 psi pressure. The radius of curvature of the head is to be 15 in.

412. A water-tube boiler has a drum 42 in. in diameter and a working pressure of 300 psi. The head of the drum is concave inside and has a radius of curvature of 40 in.

a. Determine the required head thickness.

b. If the radius of curvature were 30 in., what head thickness would be required?

413. The head of a steam cylinder 24 in. in diameter is subjected to a steam pressure of 200 psi. The head is held in place by 16 N.C. bolts $1\frac{1}{4}$ in. in diameter. A copper gasket is used to make the joint steam tight. Determine the head thickness and the probable stress in the bolts.

414. In a timber-treating plant, the creosoting cylinder is 9 ft in diameter and 150 ft long. Oil at 200 F and 200 psi pressure are used in this cylinder.

a. Determine the thickness of steel required in the cylinder plates and design the longitudinal joint.

b. Determine the thickness of the flat heads used to close the ends of the cylinder. These heads are held on by bolts arranged on a 10-ft bolt circle.

415. A drum designed to contain CO_2 at 1,000 psi has a 12 in. inside diameter with flat heads flanged and riveted to the shell. Determine

the required head thickness and the number of rivets required. Assume the inside corner radius of the head flange to be 2 in.

416. The flat heads of a cast-iron cylinder are cast integral with the cylinder walls. The inside diameter is 6 in., the working pressure 2,000 psi, and the permissible stress 3,500 psi. Determine the head and wall thickness.

417. A cylinder of 30 in. inside diameter is to contain liquid at a pressure of 400 psi. The cylinder head is made of cast iron and is attached by bolts arranged on a 34-in. circle.

a. Determine the required head thickness.

b. Determine the required number and size of bolts if made of material having an ultimate strength of 70,000 psi.

418. A Scotch marine boiler of 12 ft diameter and 15 ft length is built for a pressure of 100 psi. The head is $\frac{5}{8}$ in. thick and is stayed with through stays screwed in and riveted over. Determine the maximum permissible pitch and the diameter of the stays.

419. The flat head of a Scotch marine type of boiler is $\frac{1}{2}$ in. thick and is designed for 150 psi steam pressure. The head is supported by 12-ft through stays passed through the head and fitted with a nut and washer on each side. Determine the required pitch and diameter of the stays.

Chapter XXII

420. The journal of a generator is $1\frac{7}{8}$ in. in diameter. The bearing is to have a medium fit. Determine the allowance, the tolerance on the shaft, and the tolerance on the bearing bore.

421. The shaft of an 800-rpm turbine is $4\frac{1}{2}$ in. in diameter at the bearing. Give the machining limits for the shaft and the bearing bore to provide a free fit.

422. The outer race of a No. 312 ball bearing is to be given a snug fit, and the inner race is to be given a tight fit. Determine the machining dimensions for each of the following: bearing bore, shaft diameter, bearing outside diameter, and housing bore.

423. A No. 220 ball bearing is to be used with the inner race stationary. Determine the machining dimensions of the bearing bore, the shaft, the bearing outside diameter, and the housing bore.

424. The crank disk on a gear speed-reducing unit is to be given a medium press fit. Determine the machining dimensions for the $5\frac{7}{8}$-in. shaft and for the crank-disk bore.

425. What class of fit would you select for a ring-oiled bearing on a $2\frac{1}{2}$-in. shaft of an electric generator. The shaft rotates at 1,125 rpm, and the unit bearing pressure is 150 psi. Give the machining dimensions of the shaft and the bearing bore.

426. A cast-iron coupling is to be pressed onto a $3\frac{15}{16}$-in. steel shaft. The coupling hub has a length of 6 in. and an outside diameter of $7\frac{1}{2}$ in. The coupling is bored 0.003 in. smaller than the shaft.

a. Determine the pressure between the shaft and coupling.

b. Determine the maximum tensile stress in the coupling hub.

c. Determine the force required to force the coupling on the shaft.

427. A cast-steel crank is to be shrunk onto a 10-in. steel shaft. The outside diameter of the crank hub is $17\frac{1}{2}$ in. The maximum tangential stress in the hub is to be 20,000 psi. The coefficient of friction between the hub and shaft is assumed to be 0.15.

a. Determine the required bore of the crank.

b. Determine the probable value of the normal pressure between the shaft and hub.

c. Determine the torque that may be transmitted without using a key.

d. Determine the stress in the shaft due to torsion.

428. The cast-iron crank of a gas engine is shrunk on its shaft. The steel shaft is $4\frac{1}{2}$ in. in diameter, and the crank hub is $8\frac{1}{2}$ in. in diameter. The maximum torque to be transmitted is 100,000 lb-in. Assume the coefficient of friction to be 0.10.

a. Determine the shrinkage allowance to give a tangential stress of 7,500 psi in the hub.

b. Determine the normal pressure between the hub and shaft after assembly.

c. Determine the required hub length if no key is used.

d. To what temperature must the hub be raised if 0.003 in. clearance is required for assembly.

429. An aluminum-alloy ring is shrunk onto a steel shaft. The shaft diameter is 2.500 in. The ring has an inside diameter of 2.497 in. and an outside diameter of 3.000 in. Determine the probable tangential stress in the ring, assuming the modulus of elasticity of the aluminum alloy to be 10,000,000 psi.

430. The cylinder of a high-pressure pump is made in the form of a compound cylinder with outer part shrunk on. Both parts are of steel. The bore of the pump cylinder is 5 in., the inner cylinder has an outside diameter of $9\frac{1}{2}$ in., and the outer cylinder has an outside diameter of 13 in. Determine all machining dimensions so that the tangential stress at the cylinder bore will vary between the limits of 5,000 and 7,500 psi in compression, after assembly.

a. What will be the maximum tensile stress in the outer cylinder?

b. What will be the maximum tangential stress in the inner and outer cylinders when the pump pressure is 5,000 psi?

Chapter XXIII

431. A flywheel running at 300 rpm is to absorb 50,000 ft-lb of energy. The maximum velocity at the mean radius is to be 5,000 fpm and is to vary not more than 5 per cent. Determine the weight, width, and depth of the rim.

432. The speed of a flywheel changes from 300 rpm to 290 rpm in 0.05 sec. The rim weighs 2,500 lb and has a mean diameter of 60 in.

a. Determine the energy given up during the change in speed.

b. Determine the average torque required to cause this change in speed.

433. A cast-iron flywheel rotates at 180 rpm. The rim weighs 5,000 lb and has a 10 by 10 in. section.

a. Determine the outside and inside diameters.

b. Determine the radius of gyration.

c. Determine the peripheral speed in feet per minute.

d. Determine the unit stress due to centrifugal force.

e. Determine the kinetic energy in the rim.

f. How much energy is released when the speed changes to 170 rpm?

g. If the speed change occurs in one-fourth revolution, what is the angular deceleration?

h. What torque must be applied to the shaft to produce this deceleration?

i. If the shearing stress is limited to 6,000 psi, determine the shaft diameter.

434. The speed of a flywheel changes from 360 to 340 rpm in 0.06 sec, releasing 60,000 ft-lb of energy. The maximum velocity at the mean radius is 6,000 fpm. Determine the weight, width, and depth of the rim.

435. A single-cylinder double-acting steam engine delivers 250 hp at 100 rpm. The maximum fluctuation of energy per revolution is 15 per cent. The speed variation is limited to 1 per cent either way from the mean. The mean diameter of the rim is 8 ft. Determine the required weight of the rim and its cross-sectional dimensions, neglecting the effect of the arms.

436. A cast-iron flywheel running at 40 rpm is to furnish 75,000 ft-lb of energy to a punch during 0.15 revolution with a 15 per cent change in speed. The maximum velocity at the mean radius is to be 1,000 rpm

a. Determine the dimensions of the rim.

b. Determine the horsepower of the motor required to drive this punch, if the over-all efficiency of the machine is 80 per cent.

437. The diameter of a cast-iron flywheel is 20 ft Determine the speed at which this wheel may be operated if the stress in the rim, neglecting any effect of the arms, is not to exceed 1,000 psi. What would be the probable stress if the effect of the arms is considered?

438. Determine the dimensions of a box-type piston for a 24- by 36-in. by 125-rpm double-acting steam engine using steam at 200 psi.

439. Design a cast-iron piston for a single-acting internal-combustion engine with a $5\frac{1}{2}$-in cylinder bore, $7\frac{1}{2}$-in stroke, and 15-in. connecting rod. The maximum gas pressure is 500 psi, the engine runs at 600 rpm, develops a brake mep of 100 psi, and uses 12,000 Btu per hp-hr. Make a dimensioned scale drawing of this piston and check for heat flow.

440. This problem is the same as Prob. 439 except that the piston is to be of aluminum alloy.

441. The piston rod of the engine in Problem 440 is $3\frac{15}{16}$ in. in diameter and 44 in. long.

a. Determine the maximum stress considering the rod to be in direct compression.

b. Determine the maximum stress considering the rod to be a column with both ends fixed.

c. Do you consider these stresses to be reasonable, the rod being made of annealed S.A.E. 1035 steel? State your reasoning in arriving at your answer.

442. Determine the dimensions of the body of a 60-in. rectangular-section connecting rod for a 20- by 30-in. by 150-rpm double-acting steam engine. Use S.A.E. 2140 forged steel. The steam pressure is 300 psi.

443. A single side crank is used with the engine described in Prob. 439. Design a cast-iron crank for this engine with a maximum tensile stress of 3,000 psi and a torsion stress of 2,500 psi.

444. Determine the direct tension and the bending stress in a chain (see Fig. P-11) made from $\frac{1}{2}$-in. rod when the load F is 10,000 lb.

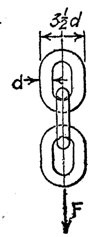


FIG. P-11.

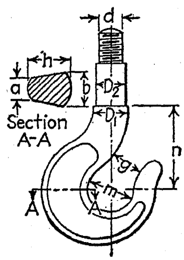


FIG. P-12.

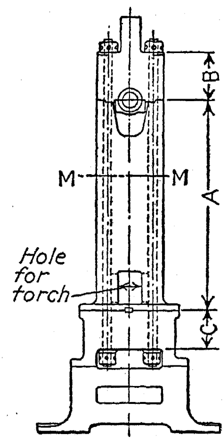


FIG. P-13.

445. If one side of a $\frac{3}{4}$-in. chain link similar to Fig. P-11 breaks, what will be the maximum stress when F is 20,000 lb?

446. A chain hook similar to Fig. P-12 is used on a 10-ton crane. The main dimensions are $a = 1\frac{3}{8}$ in., $b = 2$ in., $d = 1\frac{3}{4}$ in., $g = 3$ in., $h = 2\frac{3}{4}$ in., $m = 3\frac{1}{4}$ in., $n = 6$ in., $D_1 = 2\frac{1}{4}$ in., and $D_2 = 2$ in. Determine the maximum tension or bending stress in this hook.

447. Design a crane hook similar to Fig. P-12 for a 5-ton crane using a permissible stress in tension or bending equal to 8,000 psi.

448. The frame of a 30-ton punch press is of the C type with an inside radius of 10 in. The distance from the center line of the punch to the inside of the curved part of the C frame is 24 in. The frame is to be cast with a cross section similar to Fig. P-1*f* with the cast walls all 2 in. thick. Make $h = 2\frac{1}{2}b = 4b_1$. Determine the dimensions of the section so that the stresses will not exceed 3,000 psi in tension and 10,000 in compression.

449. The bolts in the press frame shown in Fig. P-13 are 3 in. in diameter, have National Coarse Threads, and are made of S.A.E. 1020 steel, annealed. The cross-sectional area of each cast-iron side column is 30 sq in., and $A = 48$ in., $B = 9$ in., and $C = 5$ in. The initial stress in the body of the bolts is 9,000 psi.

a. Determine the stress in the cast-iron column.

b. The maximum load on the press head is 50 tons. What will be the stress in the bolts when the press is operating?

c. What will be the stress in the columns when the press is operating?

d. What will be the elongation of the columns caused by the press load?

e. What would be the elongation of the columns if the bolts had been omitted and the frame made in one piece with the cross-sectional area increased by the area of the bolts?

f. What would be the stress in the columns for the conditions of part (*e*)?

g. Which method of construction is the better?

450. This problem is the same as Prob. 449 except that the frame is made of cast steel.

451. The bolts of the press frame, shown in Fig. P-13, are $2\frac{1}{2}$ in. in diameter and made of S.A.E. 1045 steel, annealed. The area of the cross section of the cast-iron frame is 24 in. and $A = 36$ in., $B = 8$ in., and $C = 4$ in. The bolts are to have National Coarse Threads extending $1\frac{1}{2}$ in. below the bearing surface of the nut. These bolts are to have an initial stress of 10,000 psi in the shank. When assembled the nuts are screwed by hand to contact the casting; the bolts are then heated until the nuts rise a predetermined amount; then the nuts are again screwed down by hand and the bolts allowed to cool.

a. How much should the bolts be elongated by heating?

b. What will be the stress in the threads?

c. What will be the stress in the frame?

d. How much error will there be in the desired bolt stress if the shortening of the frame is neglected when computing the bolt stretch?

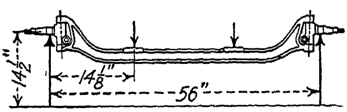


FIG. P-14.

452. The following data refer to a five-passenger automobile whose front axle is shown in Fig. P-14:

Weight of car	3,000 lb
Weight on front wheels	1,350 lb
Average weight of passengers	150 lb
Passenger load on front wheels	20 %
Capacity of front springs per in. of deflection	475 lb
Normal clearance between the spring clips and the frame	2 75 in.
Weight of front axle and wheels	170 lb
Section modulus of axle about the horizontal axis	0.90 in.^{3}

a. Determine the load on each front wheel and on each spring pad under static conditions of loading and a full passenger load.

b. Determine these loads when road conditions cause the spring clips to just touch the frame.

c. Determine the bending stress in the central portion of the axle under the static and under the dynamic conditions of parts *a* and *b*.

453. When turning a corner, the inertia of the car tends to move it in the original direction of motion; and since the wheels are turned at an angle there is a side thrust on the wheels and axle. This side thrust is at a maximum when the momentum of the car is greater than the frictional resistance of the tires, and the car skids. The center of gravity of the load carried on the axle of the car in Prob. 452 is 28 in. above the ground. The coefficient of friction of rubber on a brick pavement is 0.65. The tires are 29 in. in diameter.

a. Determine the total friction force between the tire and the road under the conditions just described. The car is carrying the full passenger load.

b. Determine the vertical reactions R_1 and R_2 at the wheels

c. Determine the friction force between each tire and the road.

d. Consider the vertical loads and the side thrust due to skidding and determine the maximum bending stress in the axle.

454. When a certain passenger car is fully loaded, the weight on each front wheel is 700 lb and the weight on each spring pad is 500 lb. When the car makes a turn and skids, the wheel load is 1,200 lb and the side thrust between the tire and road is 800 lb. The center of the wheel spindle is 15 in. above the road, and the wheel reaction passes $\frac{3}{16}$ in. from the inner end of the wheel spindle. If the diameter of the spindle is $\frac{15}{16}$ in., determine the bending stress in the spindle when driving straight ahead and when skidding at a turn.

455. The wheel spindle of the axle shown in Fig. P-14 has a diameter of $1\frac{15}{16}$ in. The center line of the wheel load is $\frac{3}{16}$ in. from the shoulder of the spindle. Determine the bending stress and the shear stress at the shoulder when the car is traveling straight forward, as in Prob. 452, and when turning a corner as in Prob. 453.

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